Proceedings of the 2000 Winter Simulation Conference
J. A. Joines, R. R. Barton, K. Kang, and P. A. Fishwick, eds.

A COMPARISON OF FIVE STEADY-STATE TRUNCATION HEURISTICS FOR SIMULATION

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ABSTRACT

We compare the performance of five well-known truncation heuristics for mitigating the effects of initialization bias in the output analysis of steady-state simulations. Two of these rules are variants of the MSER heuristic studied by White (1997); the remaining rules are adaptations of bias-detection tests based on the seminal work of Schruben (1982). Each heuristic was tested in each of a 168 different experiments. Each experiment comprised multiple tests on different realizations of the sample path of a second-order autoregressive process with known (deterministic) bias function. Different experiments employed alternative process parameters, generating a range of damped and underdamped stochastic responses. These were combined with alternative damped, underdamped, and mean shift bias functions. The performance of each rule was evaluated based on the ability of the rule to remove bias from the mean estimator for the steady-state process. Results confirmed that four of the five rules were effective and reliable, consistently yielding truncated sequences with reduced bias. In general, the MSER heuristics outperformed the three rules based on bias detection, with Spratt's (1998) MSER-5 the most effective and robust choice for a general-purpose method.

1 BACKGROUND

Much has been written about the *start-up* or *warm-up problem* in steady-state, discrete-event simulation. The problem arises because a steady-state operating regime offers no natural boundary conditions for starting or stopping simulation runs. Instead, initial conditions typically are chosen to convenience the analyst and terminating conditions are sought which provide satisfactory interval estimates. It is well known that arbitrary initialization introduces bias in estimators for output statistics such as the steady-state mean. Uncontrolled, the start-up problem yields seemingly precise, but none-the-less inaccurate results.

Researchers and practitioners alike have employed many different techniques to solve this problem. Space does not permit a discussion of the full compliment of these techniques and the interested reader is referred to Wilson and Pritsker (1978), Cash, *et al.* (1992), Goldsman, *et al.* (1994), White (1997) and such excellent standard works as Law and Kelton (2000). Instead, we focus solely on truncation procedures, which seek to eliminate initialization bias in the estimated steady-state mean by discarding an initial subsequence of the sample path judged to be unrepresentative of normal steady-state behavior. Truncation is the technique most commonly used in practice and many simulation languages include run control features to facilitate its application.

In Section 2 we review the five truncation rules selected for testing. Section 3 describes the experimental methodology and Section 4 summarizes the test results. Conclusions are presented in the final section.

2 TRUNCATION RULES

Truncation rules are among the techniques first given serious attention in the literature and remain a standard for reducing initialization bias in practice. Given the sample path $\{Y_i : i = 1,2,...,n\}$, truncation removes the first $d<<n$ observations from the series and computes the truncated mean as

$$\overline{Y}_{n,d} = \frac{1}{n-d} \sum_{i=d+1}^{n} Y_i \qquad (1)$$

The challenge is to determine the observation d^* at which to truncate, which preserves as much of the original sequence as is consistent with the objective of minimizing

bias. The implied assumptions are that (i) the sample output reaches a unique stochastic steady state regardless of the selection of initial condition, Y_1, and (ii) the truncated run length, *n-d*,* is adequate to yield desired estimation precision using some standard analysis procedure (such as batch means).

2.1 Marginal Standard Error Rules (MSER and MSER-5)

The MSER (White 1997) and MSER-5 (Spratt 1998) rules determine the truncation point as the value of d that best balances the tradeoff between improved accuracy (elimination of bias) and decreased precision (reduction in the sample size) for the reserved series $\{Y_i : i = d^*+1, d^*+2,...,n\}$. These methods select a truncation point that minimizes the width of the *marginal* confidence interval about the truncated sample mean. (Note that, because the series of reserved observations is sequentially correlated, the marginal confidence interval is *not* a valid estimator of the truncated mean. As used here, the marginal confidence interval is a measure of the homogeneity of the truncated series reserved for analysis.)

Stated formally, given a finite output series $\{Y_i : i = 1,2,...,n\}$, the optimal truncation point for the sequence is

$$d^* = \arg\min_{n>>d\geq 0}\left[\frac{1}{(n-d)^2}\sum_{i=d+1}^{n}\left(Y_i-\bar{Y}_{n,d}\right)^2\right] \qquad (2)$$

While the MSER heuristic applies equation (2) to the raw output series $\{Y_i\}$, MSER-*m* instead uses the series of $b=\lfloor n/m \rfloor$ batch averages $\{Z_j\}$, where $\lfloor\cdot\rfloor$ is the maximum integer function, and

$$Z_j = (1/m)\sum_{p=1}^{m} Y_{m(j-1)+p} \qquad (3)$$

2.2 Bias Detection Tests

Cash, *et al.* (1992), Nelson (1992), and Goldsman, *et al.* (1994), propose a family of related tests for detecting the presence of bias in an output series. These tests generalize and extend the earlier work Schruben (1982) and Schruben, *et al.* (1983). All of these tests follow the following general procedure.

The output series $\{Y_i : i = 1,2,...,n\}$ is divided into $b=\lfloor n/m \rfloor$ equal batches, each batch comprising a non-overlapping subseries of m successive observations $\{\{Y_{m(j-1)+p} : p = 1,2,...,m\} : j=1,2,\ldots,b\}$. These b batches are then grouped into two (not necessarily equal) sets of b' and $b-b'$ successive batches, respectively. Estimates of the mean and variance of the mean for each subset are computed and a test statistic is formed from the variance estimators. The test statistic is then compared to the critical value of an appropriate F distribution and the null hypothesis of no bias is either accepted or rejected on this basis.

Different versions of this test differ in the form of the variance estimator and test statistic employed. The following three versions where chosen for this study:

2.2.1 The Batch Means Test (BM)

This is the test described in Goldsman, Schruben, and Swain (1994) as earlier presented in Cash, *et al.* (1992). The mean Z_j of each batch is estimated from equation (3) and the variance of the batch means is estimated from

$$V^s_{BM} = \frac{m}{c-a-1}\sum_{j=a+1}^{c}\left[Z_j - \frac{1}{c-a}\sum_{i=a+1}^{c} Z_i\right]^2 \qquad (4)$$

where s=1, a=0, and $c=b'$ for the first set of batches and s=2, $a=b'$, and $c=b$ for the second set of batches. The test statistic is computed as the ratio V^1_{BM}/V^2_{BM} and the critical value of the test is $F_{1-\alpha,b'-1,b-b'-1}$. Following the recommendation of Cash, *et al.* (1992), the batching parameters were set at b=16 and b'=8.

2.2.2 The Maximum Test (MAX)

This is a second test reported in Goldsman, Schruben, and Swain (1994) and earlier presented in Cash, *et al.* (1992). The cumulative batch means for each observation p=1,2,…,m within each batch j=1,2,..,b are computed

$$Z_{j,p} = (1/m)\sum_{t=1}^{p} Y_{m(j-1)+t} \qquad (5)$$

and standardized as

$$S_{j,p} = Z_j - Z_{j,p} \qquad (6)$$

Within each batch, the index K_j of the maximum cumulative sum $S_j = K_j S_{j,p}$ is determined

$$K_j = \arg\max_{1\leq p\leq m}\left\{jS_{j,p}\right\} \qquad (7)$$

and the variance of the cumulative sums is estimated from

$$V^s_{MAX} = \frac{1}{3(c-a)}\sum_{j=a+1}^{c}\frac{mS_j^2}{K_j(m-K_j)} \qquad (8)$$

where s=1, a=0, and $c=b'$ for the first set of batches and s=2, $a=b'$, and $c=b$ for the second set of batches. The test

statistic is computed as the ratio V^1_{MAX}/V^2_{MAX} and the critical value of the test is $F_{1-\alpha,3b',3b-3b'}$. Following the recommendation of Cash, *et al*. (1992), the batching parameters were set at b=8 and b'=6.

Note that the MAX test as stated assumes that the initialization bias is negative. If the bias is positive, then the minimum cumulative sum is required instead of the maximum. This is achieved replacing argmax with argmin in equation (7).

2.2.3 Schruben's Alternative Method (IE)

This is the version of the alternative test proposed Schruben (1982) that appears in the *Handbook of Industrial Engineering* (Nelson, 1992). This procedure is the same as MAX, with the restriction that $b'=\lfloor b/2 \rfloor$. Following Schruben's recommendation, the batching parameters where set at $b=\lfloor n/5 \rfloor$.

3 METHODOLOGY

3.1 Implementing the Heuristics

For the MSER and MSER-5 rules, equation (2) is evaluated over the initial half of the test sequences for candidate truncation points $d=1,\ldots,\lfloor n/2 \rfloor$. If the argument is nonincreasing, resulting in $d^*=\lfloor n/2 \rfloor$, then it is concluded that the sample size n is inadequate. Otherwise, any ties are resolved by selecting the earliest truncation point.

Unlike the MSER rules, as the detection tests do not eliminate bias directly, but simply indicate if bias is present in the current sequence of data. To determine the optimal truncation point d^*, the tests are applied sequentially for candidate truncation points $d=1,\ldots,\lfloor n/2 \rfloor$ until the first instance in which the null hypothesis of no bias is not rejected. If all of the candidate truncation points are rejected, then it is concluded that the sample size n is inadequate.

The MAX and IE rules require additional care in implementation. Note that if the batch mean Z_j is greater than all of the cumulative means $Z_{j,p}$ in batch j, then the index $K_j=m$ in equation (7) for that batch. The difference $m-K_j=0$ for the corresponding term in the sum in equation. (8) and therefore V^s_{VM} is undefined. Nelson (1992) suggests that if V^1_{VM} is undefined (based on data from the first set of batches), then a test for the opposite (positive) bias needs to be applied; if V^2_{VM} is undefined (based on data from the second set of batches), then the sample size n is inadequate. Extensive testing was conducted as a part of this study and Nelson's suggestions appear to be sound.

However, *testing also revealed that a significant opposite bias can exist when V^1_{VM} is defined*. The implication is that one cannot rely on a one-sided test to indicate whether a test for the opposite bias is warranted. Therefore, truncation heuristics based on tests that use the maximum variance estimator (MAX and IE) must test for both positive and negative bias at each iteration. The iterative hypothesis testing ceases when one of the following conditions is met: (i) no significant positive bias and no significant negative bias are detected, (ii) no significant positive (negative) bias is detected and V^s_{VM} is undefined in the test for opposite bias, or (iii) V^s_{VM} is undefined for both tests. *If the rule terminates on the condition (iii), then the rule is inconclusive and fails to recommend a truncation point.*

3.2 Data Sets

The zero-mean, second-order autoregressive process

$$X_i = \Phi_1 X_{i-1} + \Phi_2 X_{i-2} + a_i \tag{9}$$

with initial conditions $Y_1=Y_2=0$, was used to generate a time series representing sample path of a steady-state distribution. The coefficients and corresponding characteristic roots of the difference equation (9) used in the study are given in Table 1. For all of the models considered, the impulse function a_i was standard normal noise.

Table 1: Model Parameters and Characteristic Roots

Model Number	Φ_1	Φ_2	Characteristic Roots
1	0.9	0.0	(0, 0.9)
2	-0.9	0.0	(0, -0.9)
3	0.25	0.5	(-0.59307, 0.84307)
4	-0.25	0.5	(-0.84307, 0.59307)
5	0.75	-0.5	0.375 ± 0.59948 ***i***
6	-0.75	-0.5	-0.375 ± 0.59948 ***i***

Three bias functions were selected to exhibit desired behaviors: exponential (equation 10), mean shift (equation 11), and underdamped oscillations (equation 12).

$$B_i = \begin{cases} Ce^{-0.005(i-1)} & \text{for } i = 1,\ldots,1000 \\ 0 & \text{for } i > 1000 \end{cases} \tag{10}$$

$$B_i = \begin{cases} C/5 & \text{for } i = 1,\ldots,1000 \\ 0 & \text{for } i > 1000 \end{cases} \tag{11}$$

$$B_i = \begin{cases} Ce^{-0.005(i-1)} \sin\left(\dfrac{\pi i}{200} + \dfrac{\pi}{2}\right) & \text{for } i = 1,\ldots,1000 \\ 0 & \text{for } i > 1000 \end{cases} \tag{12}$$

These functions are based on the bias functions used by Cash, *et al* (1992). Three values for the bias coefficient, C=5, 10, and 15, were employed for each of the three functional forms, for a total of nine different instances of the bias function. Bias terms were incorporated into the

sample paths both by superposition (adding the bias function to the autonomous process output)

$$Y_i = X_i + B_i \tag{13}$$

and injection (adding the bias function as a forcing term in the equation of state)

$$Y_i = \Phi_1 Y_{i-1} + \Phi_2 Y_{i-2} + a_i + B_i \tag{14}$$

3.3 Experiments

Each individual experiment generated performance measures for the truncated series determined by each of the five of the truncation heuristics—MSER, MSER5, BM, MAX, and IE—when applied to each of 35 independent time series derived from the simulation of a given combination of model and biasing condition. Each sample path comprised 10,000 serial observations. The length of the sample path was selected so that approximately 90% of the observations would be unbiased; the number of replications was selected to insure the desired statistical precision of the performance measures across replicationa.

Each experiment also generated performance measures for three controls applied to each of the 35 replications. The NULL control is simply the raw time series {Yi: i = 1,2,...,n} from either equation (13) or (14), without truncation. The SS control is simply the raw time series {Xi: i = 1,2,...,n} from equation (9), with no bias introduced. The FIXED control is the series {Yi : i = d+1,...,n} truncated at the observation d at which 99% of the effects of the bias function are no longer present in the observations. (For experiments with bias, the fixed d was typically on the range of 1000 to 1050 observations, as determined from the known zeroing of the bias function for i>1000, together with the known autocorrelation of the process.)

3.4 Experiment Sets

Three sets of experiments were used to evaluate the rules. Experiment Set 1 comprised six individual experiments (one for each model) in which the bias function was set to zero for all observations. The purpose of the unbiased data set is to evaluate whether or not a rule would truncate when no bias was present. Although there may be some minimal truncation due to the stochastic nature of the data, the mean distribution of the heuristics should not differ significantly from the distribution of the unbiased sample mean.

Experiment Set 2 tested all of the heuristics against all combinations of time series and biasing conditions. This experiment set consisted of 108 experiments all (6 models together with each of the 18 biasing conditions).

Experiment Set 3 tested each of the rules on variety of capacitated data sets with nonnegative output values. This set was considered because it is representative of data typical in simulation output, including wait in queue and number in system. The time series were generated in the same manner as the previous experiments, except that any negative value for Y_i was immediately set equal to zero before generating the next observation. The experiment set considered all of the models, bias functions, and magnitudes of bias with the injection biasing method only. This experiment set consisted of 54 experiments.

3.5 Performance Measures

Summary output from a representative experiment is shown in Table 2. For each heuristic and each control, the output includes statistics for the sample means, as well as an estimate of the absolute bias. The bias estimate is calculated as the absolute difference between the grand mean of the sequence reserved by the corresponding method (over all replications) and the grand mean of the unbiased data (SS).

In addition, the *p*-value from a two-sample *t*-test is given. The null hypothesis is that the mean estimated for the corresponding heuristic or control is not significantly different from the mean estimated by the SS control. The test statistic is the probability *p* of a Type I error (the probability of rejecting a true null hypothesis). Typically, if $p<0.05$ then the null hypothesis is rejected at the 95% level, indicating there is a significant difference between the means of the two distributions.

Table 2: Example Summary Table

		SAMPLE MEAN				| BIAS |	t-test	AVG	TRUNCATION POINT			
METHOD	**Inc.**	**low 95%**	**mean**	**up 95%**	**Stdev**	**est.**	**p**	**CT**	**min**	**mean**	**max**	**stdev**
NULL	-	0.9439	0.9791	1.0143	0.1063	0.9957	< 0.0001	-	0	0	0	0
BM	-	0.0534	0.0954	0.1374	0.1268	0.1121	0.0002	133	295	455	655	87
MAX	11	0.1038	0.1836	0.2635	0.1995	0.2003	< 0.0001	273	0	425	1075	208
IE	0	0.0388	0.0882	0.1376	0.1492	0.1049	0.0012	177	265	513	2155	304
MSER	0	-0.0174	0.0225	0.0624	0.1204	0.0392	0.1536	309	470	609	945	95
MSER-5	0	-0.0227	0.0165	0.0557	0.1184	0.0332	0.2215	59	470	634	945	105
FIXED	-	-0.0631	-0.0242	0.0147	0.1175	0.0075	0.7790	-	1045	1045	1045	0
SS	-	-0.0519	-0.0167	0.0185	0.1063	-	-	-	-	-	-	-

The next column reports the computation times (in hundredths of a second) for each of the truncation rules. Average computation times are not given for the NULL, FIXED, and SS controls since these times are negligible. The rules were coded in FORTRAN and run on an IBM RS/6000 with multiple users. The average computation times provide a crude relative comparison of the speed of the algorithms on the specific experimental data for which they are given. No general statements are made about the computational effort. The speed of the IE, MAX, and BM rules are dependent on the character of data to which they are applied and will take significantly longer to execute if there is a large amount of sufficiently biased data. In contrast, the speed of the MSER and MSER-5 depend solely on the number of observations in the data set under consideration.

The final columns in the table report the mean, minimum, maximum, and standard deviation of the truncation points for each method. No truncation point summaries are given for the unbiased (SS) method since its estimates are based on unbiased sample paths. These statistics provide a measure of the efficiency of the corresponding rule. Clearly, one prefers a heuristic that reserves as much unbiased data as possible to preserve confidence in the estimate.

The second column in Table 2 (labeled "Inc.") records the number of inconclusive runs for the corresponding rule or control. As described in Section 3.1, any of the tests is reported inconclusive on a given sample path if it concludes that the run length n is inadequate. In addition, an inconclusive run for the MAX and IE rules results if variance estimator is undefined in both the positive and negative tests for initialization bias. Given the design of this study, inadequate runs lengths were never reported and only the MAX rule yielded inconclusive results.

4 RESULTS

Spratt (1998) provides detailed summaries for all 168 experiments. In this section we summarize these results. All of the rules exhibited satisfactory performance on the unbiased series in Experiment Set 1, although the IE rule tended to overestimate the optimal truncation point and discard more unbiased data.

Experiment Set 2 assessed the robustness and characterized the performance of the rules. The breadth of experiments demonstrated that, overall, the MSER-5 was the most effective rule in mitigating bias. The BM rule was the least effective. The performance of the other rules generally fell somewhere between that of BM and MSER-5.

None of the rules successfully truncated all of the exponential bias function injected into data models 2, 5,and 6 (those with oscillating autocorrelation functions). In these cases the bias disappeared into the process noise and was undetectable, although still present. Although the t-test indicated dissimilarity with the unbiased sample means, enough of the biased data was discarded to reduce the estimated bias by up to 97%--a desirable improvement.

Although the BM rule consistently outperformed the NULL control, it was consistently outperformed by the other heuristics. The BM rules appeared to have (relatively) low sensitivity in detecting bias and rarely truncated enough of the biased series. It is noteworthy, however, that the BM rule also never truncated beyond d^* and its computation time was generally faster than that of the other rules, except the MSER-5.

The principle shortcoming of the MAX rule is that it was often inconclusive, particularly on models where $\Phi_1>0$. In those experiments in which the MAX rule is inconclusive on one or more replications, the standard deviation of the sample means is larger, because of the reduced sample size, resulting in a wider confidence interval and a deceptively large t-test probability. The MAX rule did perform well on data sets containing damped oscillating bias, however, and was generally efficient in its selection of truncation points.

The IE rule was effective in removing damped oscillating bias and mean shift bias, but was generally unsuccessful in dealing with exponential bias, except in the most extreme cases. A major concern with respect to the IE rule is that it occasionally removed an excessive amount of unbiased data relative to the other heuristics.

The MSER consistently outperformed the BM, MAX, and IE rules on models that contained exponential and mean shift bias, but was not as effective in dealing with damped oscillating bias. The rule was highly accurate in locating the optimal truncation point in the former instances. However, MSER occasionally was inconsistent in detecting damped oscillating bias. The performance of the MSER rule decreased as the average bias increased (as indicated by the estimated bias of the NULL method), contrary to what one would expect and desire. This is likely an artifact of the sensitivity of the MSER to individual observations, since this behavior was not observed when the data was batched (MSER-5).

The MSER-5 rule was the most robust rule in removing bias on all models. It was particularly effective in mitigating exponential and mean shift bias. There was not an experiment where another rule truncated effectively and the MSER-5 did not perform either better, or almost as well. Furthermore, MSER-5 did not suffer the inconsistency of the exhibited by MSER: if the bias increased, then the effectiveness of MSER-5 in reducing or removing the bias also increased. In addition to being the most effective rule in the experiment set, the computation time of the MSER-5 was significantly less than that of the other rules, particularly on data sets with significant bias.

In Experiment Set 3, the relative effectiveness of the rules on the capacitated data sets was similar to that

reported for Experiment Set 2, with the following exceptions. The MAX and IE rules were not effective in detecting the capacitated damped oscillating bias. The MSER was more effective in detecting mean shift bias.

5 CONCLUSIONS

The purpose of this research was to evaluate the performance of five well-known truncation heuristics on data generated from systems with a variety of behaviors symptomatic of the startup problem. The rules were automated and applied to data in 168 different experiments, with each consisting of 35 replications of a 10,000-observation sample path. The results of each experiment were analyzed and the performance of each rule was evaluated on the basis of the ability of the rule to remove bias from the mean estimator. Based on this research, the following conclusions are offered.

- The MAX rule is unreliable because the test statistic on which it is based is frequently undefined, yielding an inconclusive result.
- The BM, IE, MSER, and MSER-5 heuristics consistently outperformed the NULL control and, in general, were effective, reliable, and conclusive in mitigating the effects of the start-up problem.
- In general, the truncation rules based on the test for initialization bias were not as effective as the rules based on the MSER.
- The performance of the MSER can be greatly enhanced by batching the observations prior to application, as is evidenced by the MSER-5 heuristic.
- The MSER-5 is the most attractive general-purpose heuristic for mitigating the effects of the startup problem evaluated in this research. It is the most sensitive rule in detecting bias and the most consistent rule in mitigating its effects.

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