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A SIMULATION MODEL TO VALIDATE AND EVALUATE THE ADEQUACY OF AN ANALYTICAL EXPRESSION FOR PROPER SAFETY STOCK SIZING

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ABSTRACT

The purpose of this paper is to validate and test the adequacy of an analytical expression to calculate proper safety stock levels using simulation techniques. The model refers to a periodic review system and a lot-4-lot replenishment policy, with randomness in forecast errors and in order fulfillment. The simulation model is formulated in a spreadsheet environment using MS Excel® and @Risk®. The percentage of periods without stockout is computed and compared to the theoretical value expected by the assumptions inherent to the analytical expression.

1 INTRODUCTION

Formulating an effective inventory planning policy to ensure product availability at the lowest possible cost is not an easy task. The uncertainties inherent to the logistical process, i.e., inaccurate demand forecasting, replenishment lead time and amounts received short of amounts ordered require the building up of safety stocks.

While overstocking involves extra inventory holding costs, the lack of safety stocks may cause sales losses and a higher rate of order filling postponements than desirable, resulting in the deterioration of customer service standards.

The problem of sizing the proper safety stock has been tackled by a number of authors, like Hadley and Within (1963) and Brown (1967), particularly for the classical policy {r,Q}, i.e., economic lot size and reorder point, where Q is the quantity ordered when the on-hand inventory hits r units. In this case, the safety stock is determined for a given level of customer service according to demand fluctuations along the replenishment time, where lead times and demands are stochastic.

The problem in general is not simple to solve analytically, due to the need to determine the percentiles of the demand during the lead time. This lead time demand follows a compound probability distribution, which is the convolution of a random number of periods, each having its own random demand. A common approach for this problem is to assume normality in the distribution of the lead time demand, as shown in Keaton (1995). Thus, it is only needed to estimate the mean and the standard deviation of the compound distribution to evaluate analytically the safety stock.

Nevertheless, according to Tyworth (1992), even if the lead time and the demand are normally distributed, the compound distribution may not be normal. This fact can lead to considerable errors and deviations in determining the necessary safety stock to achieve a certain level of customer service.

Simulation models are usually more precise in determining safety stocks. However, such models can demand a lot of computational effort and time, especially when it is needed to evaluate the safety stock for many products and the parameters of the variables involved change from time to time. For example, the distribution of the lead time can change when the supplier or the transportation mode is changed. The parameters of the demand for a product is another factor that changes through time, being dependent on which stage the product is in its life cycle, as seen in Slack, Chambers, Harland, Harrison and Johnston (1995).

Thus, analytical models are more flexible and can be very practical in industrial problems, automating the evaluation of safety stocks. Simulation models can be used to test and validate the characteristics assumed for the compound distribution present in the problem. Such simulations can indicate that an analytical expression is valid or, if the expected level of customer service is not achieved, that the analytical model should be reformulated.

In this manner, the purpose of this paper is to present simulation models to validate an analytical expression for determining the safety stock in a periodic review planning environment, with lot-4-lot replenishment policy, i.e., orders equal net requirements. Uncertainties are present in demand forecast errors and in the consistency between quantity received and order placed. The latter uncertainty occurs by virtue of quality control failure, supplier fault or production yields different from the expected rates, as shown by Gullu, Onol and Erkip (1999).

2 DESCRIPTION OF THE ANALYTICAL MODEL

Garcia and Machado (2001) formulated an analytical expression to calculate the proper safety stock levels according to the desired customer service rate, in a periodic review system with stochastic forecast error and order fulfillment.

In this model, it is assumed that the quantity ordered at the planning time is received, completed or not, before the demand occurs. Figure 1 illustrates one possible situation.

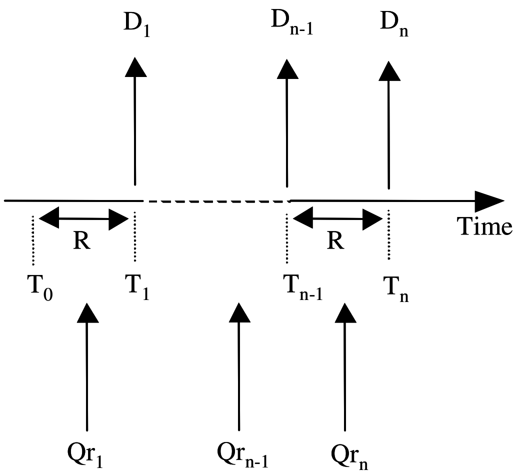

Figure 1: An order is Received between T_{i-1} and T_i to Fulfill Demand at Time T_i

where:

- T_i is the time in which an order is placed to fulfill the demand at time T_{i+n} ;
- R is the period between two planning times, i.e., between two inventory level reviews;
- n is the number of inventory level reviews between the planning period and the occurrence of the demand that was planned to be fulfilled;
- D_i is the demand at time T_i ;
- Qr_i is the quantity received within a period R between T_{i-1} and T_i , which was ordered at time T_{i-n} to fulfill the demand at time T_i .

At this situation, an order is placed at time T_0 to meet the demand at time T_n. At T_0, the forecasted demand is known for each period from T_1 to T_n and the orders placed before. The real demands from T_1 to T_n and the quantities received within that time are unknown at T_0 and should be modeled as random variables.

Considering the simplest case, in which n is equal to 1, such situation can represent the formation of inventory levels in some manufacturing environments, similarly to the approach presented by Hung and Chang (1999). An order is placed at the beginning of a period, the inventory levels rise due to production and at a pre-defined time occurs the demand. The abrupt fall of the inventory level can be caused by consolidation in shipments to other links in the supply chain, such as dealers and wholesalers. Likewise, the demand for raw materials in certain production environments can be concentrated at specific and preset periods according to production scheduling characteristics. The behavior of inventories described is illustrated in Figure 2, in which continuous replenishment along time and replenishment at a single time are shown respectively.

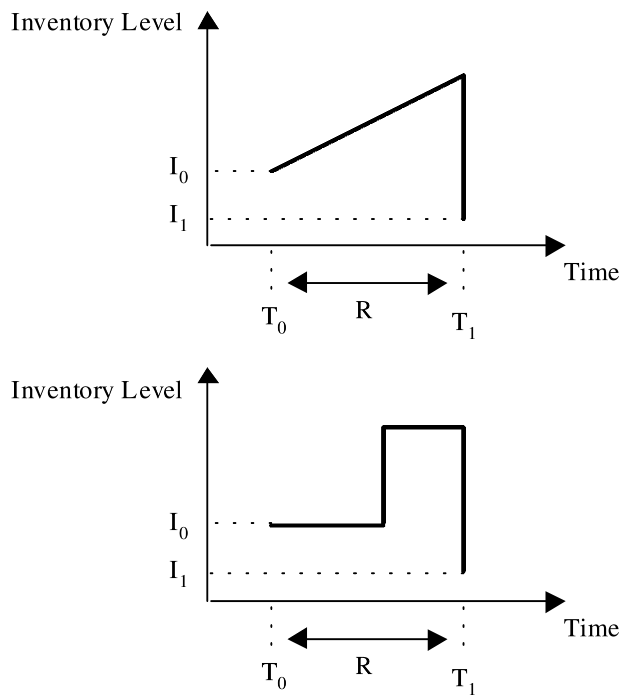

Figure 2: Continuous and Instantaneous Replenishment

where I_0 and I_1 are on-hand inventories at periods T_0 and T_1.

Another situation, analogous to the one presented in Figure 1, is when the quantities ordered at time T_i are received at time T_{i+n} to meet the demand that occurs between T_{i+n} and T_{i+n+1} . Figure 3 illustrates this case.

This case is similar to the periodic review system {R,S} presented in Silver and Peterson (1979). Such situation is more common in managing inventories in dealers and wholesalers, where an order is received at a single time and the demand occurs continuously through time. A sawtooth graphic, as shown in Figure 4, can represent it well. The periodic review is efficient in these cases as it can save costs of processing orders and obtain other scale economies, since all the orders are placed at the same time.

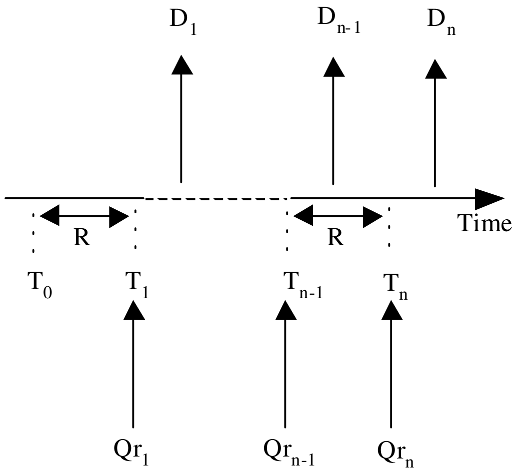

Figure 3: An order is Received at Time T_i to Fulfill the Demand that Occurs between T_i and T_{i+1}

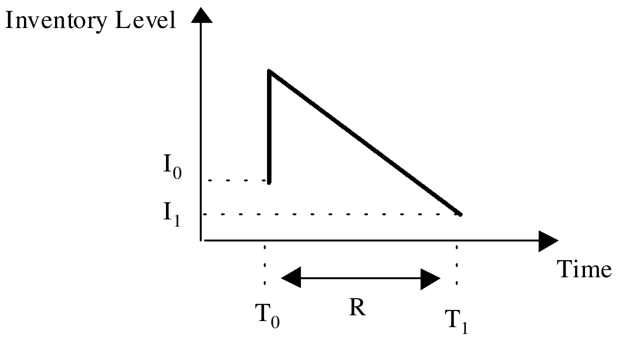

Figure 4: Saw-tooth Representing the Inventory Level Though Time

For the given descriptions of inventory levels behavior, considering the uncertainties in forecasted demand and in order fulfillment, two variables should be defined:

$$r_i = \frac{D_i}{F_i} \tag{1}$$

$$a_i = \frac{Qr_i}{Qo_i} \tag{2}$$

where r_i is the quantifier of the deviation between the demand D_i and its forecast F_i, and a_i is the quantifier of the deviation between the quantity ordered Qo_i and the quantity effectively received Qr_i. All the parameters refer to a certain period T_i.

If we are at time T_0 planning for time T_n , the order placed to fulfill the demand at T_n is:

$$Qo_n = F_n - I_{n-1} + SS \tag{3}$$

where:

- Qo_n is the quantity ordered at T_0 to meet the demand at T_n;
- F_n is the forecasted demand for T_n;
- I_{n-1} is the expected on-hand inventory just before the order Qo_n is received or starts to be received;
- SS is the safety stock required to achieve a certain level of product availability or customer service.

Finally, knowing the statistical parameters of all the r_i and a_i at time T_0, we have the expression for the safety stock SS (Garcia and Machado 2001):

$$SS = \left(NR - F_n + I_{n-1}\right) + k * S_{NR} \tag{4}$$

$$NR = \frac{F_n * \mu_{r_n} - I_{n-1}}{\mu_{a_n}} \tag{5}$$

$$I_{n-1} = I_0 + \sum_{i=1}^{n-1}\left(\mu_{a_i} * Qo_i - \mu_{r_i} * F_i\right) \tag{6}$$

$$S_{NR} = \sqrt{\left(\frac{F_n}{\mu_{a_n}} * \sigma_{r_n}\right)^2 + \left(\frac{NR}{\mu_{a_n}} * \sigma_{a_n}\right)^2 + \left(\frac{\sigma_{I_{n-1}}}{\mu_{a_n}}\right)^2} \tag{7}$$

$$\sigma_{I_{n-1}} = \sqrt{\sum_{i=1}^{n-1}\left[\left(\sigma_{a_i} * Qo_i\right)^2 + \left(\sigma_{r_i} * F_i\right)^2\right]} \tag{8}$$

where:

- NR is the estimated mean of net requirements at time T_n;
- μ_{ri} is the mean of each variable r_i;
- μ_{ai} is the mean of each variable a_i;
- I_0 is the on-hand inventory at time T_0;
- F_i is the forecasted demand for time T_i;
- Qo_i is the order placed to fulfill demand at time T_i;
- S_{NR} is the estimated standard deviation of net requirements at time T_n;
- σ_{ri} is the standard deviation of each variable r_i;
- σ_{ai} is the standard deviation of each variable a_i;
- σ_{In-1} is the standard deviation of I_{n-1};
- k is the constant to determine the customer service, according to the normal distribution.

As seen, this model assumes normality in the net requirements, the compound distribution, what can be a non-reasonable approach. Following, simulation models will be formulated to test the analytical expression and its assumptions, determining in which situations it is valid.

3 A FIRST SIMULATION MODEL (n=1)

First, a simulation model will be presented for the simplest case, with n equals 1. In this manner, the expected on-hand inventory I_{n-1} becomes I_0, a deterministic parameter at T_0, i.e., there is no uncertainty presented in the initial on-hand inventory.

A spreadsheet model is built in MS Excel® and @Risk® to run the simulations. An example of the model is shown in Figure 5.

Period	1	2	3	4	5
Forecasted Demand (F)	100	130	110	85	90
Initial Inventory (I)	20	24	48	20	33
Net Requirements (NR)	89	118	69	72	63
Safety Stock (SS)	44	57	44	37	37
Quantity Ordered (Qo)	124	163	106	102	94
Quantity Received (Qr)	114	160	103	91	88
Actual Demand (D)	110	136	131	78	115
Final Inventory	24	48	20	33	6

Figure 5: Example of the Spreadsheet Model Formulated

Here we consider that the order placed at the beginning of period 1 (T_0) is received before the demand in that period occurs. At the example in Figure 5 the calculations were done considering 1 for the mean of all r_i and 0.15 for its standard deviation, and for a_i a mean of 0.9 and a standard deviation of 0.05. The constant k equals 2 in this example. The net requirements are calculated as in equation 5, the safety stock as in equation 4 and the quantity ordered as in equation 3.

Both r_i and a_i are modeled by normal distributions using @Risk® built in function. The quantity received is equal to the quantity ordered times the random variable a_i, and the demand is equal to its forecast times the random variable r_i. In this manner, in Figure 5 the quantity received (Qr) and the actual demand (D) are the stochastic variables, being the values presented in the figure randomly generated. The final inventory of a period is the quantity received plus the initial inventory minus the demand. The initial inventory of the next period is the final inventory of the previous period. Backorders are accepted in this model, i.e., a negative final inventory should be fulfilled in the next period.

The simulation consisted of one run of 5000 iterations for each scenario, using Latin Hipercube sampling. The differences between the scenarios are the values of the constant k and the parameters of the random variables (means and standard deviations). For each scenario it was computed the percentage of periods without stockouts, a measure of customer service. The results are shown in Table 1.

The values obtained by simulation should be compared to the expected values according to the normal distribution. In the unit normal table, k equals 1 corresponds to 84.13% cumulative probability, k equals 2 to 97.72%

Table 1: Simulation Results – Percentage of Periods without Stockout

	% of Periods without Stockout obtained by the Simulation		
Parameters	**k=1**	**k=2**	**k=3**
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.9\\ \sigma_{a_i}=0.05\end{Bmatrix}$	83.67%	97.35%	99.81%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.8\\ \sigma_{a_i}=0.10\end{Bmatrix}$	82.47%	96.06%	99.34%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.7\\ \sigma_{a_i}=0.15\end{Bmatrix}$	80.52%	93.74%	98.21%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.9\\ \sigma_{a_i}=0.05\end{Bmatrix}$	83.99%	97.60%	99.83%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.8\\ \sigma_{a_i}=0.10\end{Bmatrix}$	83.08%	96.88%	99.74%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.7\\ \sigma_{a_i}=0.15\end{Bmatrix}$	81.71%	95.43%	99.06%

and k equals 3 to 99.87%. Table 2 shows the absolute deviation between the percentage of periods without stockouts obtained by the simulation and the expected values assuming normality in the compound distribution.

Table 2: Comparison of Simulation Results and the Expected Values Assuming Normal Distribution

	Absolute deviation between the simulation results and the expected values assuming normality		
Parameters	**k=1**	**k=2**	**k=3**
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.9\\ \sigma_{a_i}=0.05\end{Bmatrix}$	0.47%	0.37%	0.05%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.8\\ \sigma_{a_i}=0.10\end{Bmatrix}$	1.67%	1.66%	0.53%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.7\\ \sigma_{a_i}=0.15\end{Bmatrix}$	3.61%	3.98%	1.65%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.9\\ \sigma_{a_i}=0.05\end{Bmatrix}$	0.15%	0.13%	0.03%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.8\\ \sigma_{a_i}=0.10\end{Bmatrix}$	1.05%	0.84%	0.12%
$\begin{Bmatrix}\mu_{r_i}=1\\ \sigma_{r_i}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_i}=0.7\\ \sigma_{a_i}=0.15\end{Bmatrix}$	2.42%	2.29%	0.81%

As seen in Table 2, a very good agreement between theoretical and simulated result was achieved. Furthermore, the analytical model tested has smaller deviations from the normal assumption when k assumes higher values, and, as the ratio between the standard deviations of r_i and a_i increases, the deviations decrease, i.e., the analytical model converges to normality.

4 A SIMULATION MODEL FOR n=3

Now a more complex model will be formulated, when n equals 3. In this case, at T_0 an order is placed to fulfill the demand at T_3. The information known at T_0 are the orders in process, which were placed before, the initial on-hand inventory I_0, the forecasted demand for T_1, T_2 and T_3, and the statistics for each r_i and a_i. All other data should be calculated based on this information.

Figure 6 shows an example of the known data and the calculations done to decide how much to order at the beginning of period 1 (T_0) to guarantee, with a certain confidence, the fulfillment of the demand at the end of period 3 (T_3).

Known Data and Analytical Calculations			
Period	1	2	3
Forecasted Demand (F)	90	115	120
Expected Initial Inventory (I)	30	57	41
Orders in Process	130	110	-
Net Requirements (NR)	-	-	88
Safety Stock (SS)	-	-	62
Quantity Ordered (Qo)	-	-	141
Expected Final Inventory	57	41	48

Figure 6: Example of the Known Data and the Necessary Calculations to Decide How Much to Order at T_0

At this example, k equals 2, the mean for all a_i is 0.9 and the standard deviation for all a_i is 0.05. For r_i, three variables were considered: r_1, quantifier of the relation between the demand at T_1 and its forecast at T_0; r_2, quantifier of the relation between the demand at T_2 and its forecast at T_0; and r_3, quantifier of the relation between the demand at T_3 and its forecast at T_0. Since all the forecasts are done at T_0, it is assumed that the more the forecast is far from T_0, the more the uncertainty in r_i increases. Thus, at the example in Figure 6, r_1 standard deviation is 0.05, r_2 standard deviation is 0.10 and r_3 standard deviation is 0.15. The mean of all r_1 was assumed to be 1.

Having done all the necessary calculations, the model consists in simulating the demands, quantities received and final inventories at T_1, T_2 and T_3, besides the initial inventories at T_2 and T_3. Figure 7 shows an example of the variables simulated, according to the data presented in Figure 6.

As in the first simulation model, this simulation consisted of one run of 5000 iterations using Latin Hipercube sampling in @Risk® for each scenario. The differences between the scenarios are the values of k and the parameters of r_i and a_i, which are modeled as normal distributions. The percentages of periods without stockout obtained by the simulation are presented in Table 3.

Simulation			
Period	1	2	3
Real Initial Inventory	30	55	46
Quantity Received (Qr)	114	104	121
Actual Demand (D)	89	113	127
Final Inventory	55	46	39

Figure 7: Example for the Simulated Variables Based on the Calculations Presented in Figure 6

As before, the values obtained by the simulation are compared to the expected values according to the normal distribution. Table 4 shows the deviations between the percentage of periods without stockout obtained in the simulation and the theoretical values according to the normal distribution.

Table 3: Simulation Results Obtained for n=3

	% of Periods without Stockout obtained by the Simulation		
Parameters	**k=1**	**k=2**	**k=3**
$\begin{Bmatrix}\mu_{r_1}=1\\ \sigma_{r_1}=0.05\end{Bmatrix}\begin{Bmatrix}\mu_{r_2}=1\\ \sigma_{r_2}=0.10\end{Bmatrix}\begin{Bmatrix}\mu_{r_3}=1\\ \sigma_{r_3}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_1}=0.9\\ \sigma_{a_i}=0.05\end{Bmatrix}$	83.72%	97.26%	99.82%
$\begin{Bmatrix}\mu_{r_1}=1\\ \sigma_{r_1}=0.05\end{Bmatrix}\begin{Bmatrix}\mu_{r_2}=1\\ \sigma_{r_2}=0.10\end{Bmatrix}\begin{Bmatrix}\mu_{r_3}=1\\ \sigma_{r_3}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_1}=0.8\\ \sigma_{a_i}=0.10\end{Bmatrix}$	83.12%	96.02%	99.44%
$\begin{Bmatrix}\mu_{r_1}=1\\ \sigma_{r_1}=0.05\end{Bmatrix}\begin{Bmatrix}\mu_{r_2}=1\\ \sigma_{r_2}=0.10\end{Bmatrix}\begin{Bmatrix}\mu_{r_3}=1\\ \sigma_{r_3}=0.15\end{Bmatrix}\begin{Bmatrix}\mu_{a_1}=0.7\\ \sigma_{a_i}=0.15\end{Bmatrix}$	81.12%	94.06%	97.96%
$\begin{Bmatrix}\mu_{r_1}=1\\ \sigma_{r_1}=0.10\end{Bmatrix}\begin{Bmatrix}\mu_{r_2}=1\\ \sigma_{r_2}=0.20\end{Bmatrix}\begin{Bmatrix}\mu_{r_3}=1\\ \sigma_{r_3}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_1}=0.9\\ \sigma_{a_i}=0.05\end{Bmatrix}$	84.08%	97.38%	99.84%
$\begin{Bmatrix}\mu_{r_1}=1\\ \sigma_{r_1}=0.10\end{Bmatrix}\begin{Bmatrix}\mu_{r_2}=1\\ \sigma_{r_2}=0.20\end{Bmatrix}\begin{Bmatrix}\mu_{r_3}=1\\ \sigma_{r_3}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_1}=0.8\\ \sigma_{a_i}=0.10\end{Bmatrix}$	83.26%	96.48%	99.54%
$\begin{Bmatrix}\mu_{r_1}=1\\ \sigma_{r_1}=0.10\end{Bmatrix}\begin{Bmatrix}\mu_{r_2}=1\\ \sigma_{r_2}=0.20\end{Bmatrix}\begin{Bmatrix}\mu_{r_3}=1\\ \sigma_{r_3}=0.30\end{Bmatrix}\begin{Bmatrix}\mu_{a_1}=0.7\\ \sigma_{a_i}=0.15\end{Bmatrix}$	81.38%	94.72%	98.34%

Table 4: Comparison of the Simulation Results and the Expected Values Assuming Normal Distribution for n=3

	Absolute deviation between the simulation results and the expected values assuming normality		
Parameters	**k=1**	**k=2**	**k=3**
$\begin{Bmatrix} \mu_{r_1} = 1 \\ \sigma_{r_1} = 0.05 \end{Bmatrix} \begin{Bmatrix} \mu_{r_2} = 1 \\ \sigma_{r_2} = 0.10 \end{Bmatrix} \begin{Bmatrix} \mu_{r_3} = 1 \\ \sigma_{r_3} = 0.15 \end{Bmatrix} \begin{Bmatrix} \mu_{a_1} = 0.9 \\ \sigma_{a_i} = 0.05 \end{Bmatrix}$	0.41%	0.46%	0.05%
$\begin{Bmatrix} \mu_{r_1} = 1 \\ \sigma_{r_1} = 0.05 \end{Bmatrix} \begin{Bmatrix} \mu_{r_2} = 1 \\ \sigma_{r_2} = 0.10 \end{Bmatrix} \begin{Bmatrix} \mu_{r_3} = 1 \\ \sigma_{r_3} = 0.15 \end{Bmatrix} \begin{Bmatrix} \mu_{a_1} = 0.8 \\ \sigma_{a_i} = 0.10 \end{Bmatrix}$	1.01%	1.70%	0.43%
$\begin{Bmatrix} \mu_{r_1} = 1 \\ \sigma_{r_1} = 0.05 \end{Bmatrix} \begin{Bmatrix} \mu_{r_2} = 1 \\ \sigma_{r_2} = 0.10 \end{Bmatrix} \begin{Bmatrix} \mu_{r_3} = 1 \\ \sigma_{r_3} = 0.15 \end{Bmatrix} \begin{Bmatrix} \mu_{a_1} = 0.7 \\ \sigma_{a_i} = 0.15 \end{Bmatrix}$	3.01%	3.66%	1.91%
$\begin{Bmatrix} \mu_{r_1} = 1 \\ \sigma_{r_1} = 0.10 \end{Bmatrix} \begin{Bmatrix} \mu_{r_2} = 1 \\ \sigma_{r_2} = 0.20 \end{Bmatrix} \begin{Bmatrix} \mu_{r_3} = 1 \\ \sigma_{r_3} = 0.30 \end{Bmatrix} \begin{Bmatrix} \mu_{a_1} = 0.9 \\ \sigma_{a_i} = 0.05 \end{Bmatrix}$	0.05%	0.34%	0.03%
$\begin{Bmatrix} \mu_{r_1} = 1 \\ \sigma_{r_1} = 0.10 \end{Bmatrix} \begin{Bmatrix} \mu_{r_2} = 1 \\ \sigma_{r_2} = 0.20 \end{Bmatrix} \begin{Bmatrix} \mu_{r_3} = 1 \\ \sigma_{r_3} = 0.30 \end{Bmatrix} \begin{Bmatrix} \mu_{a_1} = 0.8 \\ \sigma_{a_i} = 0.10 \end{Bmatrix}$	0.87%	1.24%	0.33%
$\begin{Bmatrix} \mu_{r_1} = 1 \\ \sigma_{r_1} = 0.10 \end{Bmatrix} \begin{Bmatrix} \mu_{r_2} = 1 \\ \sigma_{r_2} = 0.20 \end{Bmatrix} \begin{Bmatrix} \mu_{r_3} = 1 \\ \sigma_{r_3} = 0.30 \end{Bmatrix} \begin{Bmatrix} \mu_{a_1} = 0.7 \\ \sigma_{a_i} = 0.15 \end{Bmatrix}$	2.75%	3.00%	1.53%

Analyzing Table 4, it is seen that the absolute differences between the simulation results and the normal values tend to be higher at k equals 2. Again, k equals 3 has the smallest deviations and when the ratio between the uncertainties in the r_i and the uncertainties in the a_i increases, the deviations decrease.

5 CONCLUSION

Simulation models are very useful tools when dealing with safety stock models. They are useful not only to evaluate the proper safety stock to achieve a certain level of customer service, but also to validate and evaluate the adequacy of analytical expressions, which are easier to implement and more practical in many industrial problems.

The simulation models presented in this paper brought valuable information about the analytical expression. Depending on the target customer service and on the relative magnitude between the uncertainties considered, the analytical expression can result in significant differences from the theoretical customer service based on the assumption of normality.

Nevertheless, field research shows that in many real cases the uncertainty in forecast errors tends to have a higher magnitude than order fulfillment uncertainty, what makes the analytical model valid and adequate to many situations.

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