Proceedings of the 2002 Winter Simulation Conference
E. Yücesan, C.-H. Chen, J. L. Snowdon, and J. M. Charnes, eds.

CREDIT RISK MODELING FOR CATASTROPHIC EVENTS

Tarja Joro

University of Alberta School of Business
3-40F Business Building
Edmonton, Alberta T6G 2R6, CANADA

Paul Na

Bayerische Landesbank
New York Branch
560 Lexington Avenue
New York, NY 10022, U.S.A.

ABSTRACT

Estimating default probabilities of companies is one of the fundamental tasks in credit risk models and lending decision-making. One area of particular interest is how the companies' asset value behaves in the presence of unforeseen external shocks or catastrophes. On one hand, we want the default probabilities to address the likelihood of catastrophes correctly, and on the other hand, we want to be able to perform what-if analysis to investigate the possible consequences of catastrophes. This study proposes a framework to perform such what-if analysis in the jump diffusion framework.

1 INTRODUCTION

It can be argued that the two central questions in credit risk modeling are (1) "What is the probability of a default?" and (2) "How likely will the company default given a substantial, unforeseen shock?" There are two broad modeling approaches that aim on answering the first question, namely structural approach (see e.g. Merton 1974, Merton 1976, Black and Cox 1976, Longstaff and Schwartz 1995), and reduced form approach (see e.g. Duffie and Singleton 1995, Jarrow, Lando, and Turnbull 1994, Jarrow and Turnbull 1995, Madan and Unal 1994). Whereas both approaches aim on answering the same question, they make quite different assumptions. The structural approach assumes that all defaults happen gradually, whereas the reduced form approach assumes that all bankruptcies are sudden. We discuss the differences of these models in Section 2. The second question can be answered by performing what if analysis with various shock scenarios.

The philosophies behind the structural and reduced form approaches can be combined by using a jump diffusion model that allows both gradual and sudden defaults (see e.g. Merton 1976, Ahn and Thompson 1988, Zhou 1997, 2001, Kau and Keenan 1999, and Kou 2001). The aim of this paper is to show how catastrophic events can be modeled and how what-if analysis can be carried out in the jump diffusion framework. The proposed approach will cope with the difficulties associated to estimating the probabilities of catastrophic events by using a parametric approach.

The paper is organized as follows. Section two discusses modeling the company's asset value, and Section three presents the proposed approach. A computational example is provided in Section four, and Section five concludes the paper.

2 MODELING FIRM'S ASSET VALUE

In credit risk evaluation, predicting the default probability (the probability of bankruptcy) is one of the central tasks. As first introduced by Merton (1974), this can be done by estimating the probability of asset value being lower than the debt.

Traditionally, there have been two approaches in how the process of defaults should be modeled. Zhou (1997) characterizes the two approaches as follows:

1. *Structural approach* proposes that the evolution of firm's asset value follows a diffusion process, as proposed by Merton (1974) (see also Black and Cox 1976 and Longstaff and Schwartz 1995). According to structural approach, firms never default by surprise.
2. *Reduced-form approach* (see Duffie and Singleton 1995, Jarrow, Lando, and Turnbull 1994, Jarrow and Turnbull 1995 and Madan and Unal 1994) assumes that there is no relation between the firm value and default. Default is seen as an unpredictable Poisson event involving a sudden loss in market value. Thus, according to reduced-form approach, firms never default gradually.

However, as Zhou (1997) argues, in reality, both ways are possible: firms can default either gradually or by surprise due to an unforeseen external shocks.

Like in Merton (1974), the simulation of a company's asset value in future is usually based on the assumption that the asset value, *V*, follows a Geometric Brownian motion

$$dV = \mu V dt + \sigma V dW, \quad (1)$$

where $\mu \in \Re$ is a constant growth rate of the asset value, $\sigma > 0$ is a constant volatility coefficient, and W is a standard Brownian motion (Wiener process) defined on a filtered probability space (Ω, F, P). Equation (1) is also known as a diffusion process. One underlying assumption in modeling the asset value with a diffusion process is that the asset value is continuous. However, in reality, this is not the case. In market data, it is possible to point out jumps in the value of company's stock and consequently the asset value also contains jumps that make the path discontinuous. Quite often, these jumps are associated with actions taken by the central banks, significant economic events, etc. In order to accommodate such discontinuities, many studies such as Merton (1976), Ahn and Thompson (1988), Zhou (1997, 2001), Kau and Keenan (1999), and Kou (2001) proposed a jump-diffusion process as a more accurate model for the stock price and asset value behavior

$$dV = \mu_A V dt + \sigma V dW + (J-1) V dp, \quad (2)$$

where $\mu_A = \mu - \lambda E(J - 1)$ is the adjusted growth rate, J is a lognormally distributed jump term and dp is the Poisson process generating the jumps with the intensity of λ.

Jump diffusion models can be seen as an attempt to combine structural and reduced form approaches: the decrease in the value of the firm can result from diffusion process or from a Poisson distributed jump. The difference is that whereas in the pure reduced form approach the Poisson distributed term is the *default* itself, in jump diffusion model it is a *sudden decrease in asset value* that may or may not cause the firm to default. The idea in the jump diffusion model is that the jump term would produce all discontinuities in the asset value, ranging from small daily discontinuities to catastrophic drops.

Given that the parameters are correct, the jump process is capable of producing discontinuities ranging from small daily up and down movements to catastrophic drops such as Black Monday in October 1987, or 9/11 in September 2001. However, the practical difficulties in finding the right distribution for the jump size and frequency should not be underestimated.

3 CATASTROPHIC EVENTS IN JUMP DIFFUSION FRAMEWORK

To facilitate the modeling of catastrophic events in general, and what if analysis in particular, we propose an approach where the jumps are modeled with two independent Poisson processes:

$$dV = \mu_A V dt + \sigma V dW + (J_1 - 1) V dp_1 + J_2 V dp_2. \quad (3)$$

Here, we have two simultaneous jump diffusion processes dp_1 and dp_2. The first one is associated with the expected up and down jumps during normal markets, and the second one with catastrophic jumps. Note that the sum of Poisson processes is a Poisson process where the intensity is the sum of the intensities. Thus the combined arrival process of expected jumps and catastrophic jumps is Poisson as well.

Given the difficulty in estimating the size and frequency of catastrophic events, we will seek to parameterize the jump size J_2 and intensity λ_2 for the second jump process dp_2.

This can be seen as an alternative to the stress testing. Instead of using explicit stress scenarios, we propose examining various catastrophe sizes and frequencies. By varying the size (as measured by the decrease in the asset value) and the frequency of the catastrophes, we can examine how various catastrophes affect the default probability.

While the approach can be used in credit risk modeling in general, we think it will be particularly useful in lending decision-making. The aim of this approach is to change the question asked while modeling the loan decision: Instead of asking "What is the likelihood of this stress scenario to occur?" we propose the following thought process:

- "I am willing to accept a default probability of p."
- "It takes n catastrophic drops of m% in asset value during the loan period to increase the default probability over p."
- "Am I willing to take the risk?"

4 ILLUSTRATIVE SIMULATIONS

Assume that company's asset value is currently \$100, and it has \$50 as debt. Assume further that the annual growth rate is 20% with a standard deviation of 30%. To simplify the example, we are ignoring the first jump process of equation (3) in this example. We are considering lending money for a company for five years. The company's default probability from diffusion model (1) is 5.50%..

In order to see how the company survives from unexpected decreases in the asset value, we simulated the asset value under different stress conditions. We varied the arrival process of the catastrophes from 1 catastrophe in 5 years to 5 catastrophes in 5 years (1 catastrophe every year) and the size of the decrease in the asset value (the catastrophe size) from 5% to 50%. Table 1 reports the default probabilities from jump-diffusion simulation.

Note that in the simulation model, examining the asset value distribution at the end time T is not adequate, since some asset value paths may have dipped under the default threshold at a time t < T, but later on recovered. Figure 1 illustrated this: assume that the threshold is $50. The asset value drops below the threshold value at day #999, but recovers later.

Table 1. Jump-Diffusion Process (n = 1000)

Jump Size	Time Between Jumps 1 yr	2 yrs	3 yrs	4 yrs	5 yrs
-5%	11.34%	7.75%	7.15%	7%	6.40%
-10%	24.89%	11.85%	9.73%	9%	8.73%
-15%	40.65%	16.48%	12.99%	11%	10.86%
-20%	56.27%	26.00%	20.18%	15%	14.27%
-25%	69.07%	35.91%	26.32%	18%	16.87%
-30%	78.64%	45.31%	32.45%	23%	20.28%
-35%	86.38%	55.11%	39.64%	29%	26.02%
-40%	91.01%	63.26%	46.64%	35%	31.00%
-45%	93.17%	69.63%	53.33%	42%	34.31%
-50%	94.98%	73.51%	57.34%	47%	39.07%

Given that in banking business the commercial loan periods are relatively short, it might be enough to assume one catastrophe during the loan period. This can be modeled either by Poisson process, or by assuming a uniform distribution for the occurrence of the catastrophe over the loan period.

5 CONCLUSION

The aim of this paper is to present an approach that enables us to estimate default probabilities in the presence of catastrophic events. The approach extends the jump diffusion approach to estimate default probabilities as proposed in Zhou (1997, 2001) by adding a second jump process for catastrophic events. By manipulating the parameters of the catastrophic jump process we can perform stress testing and investigate how catastrophes of various sizes affect the default probabilities. This provides an alternative to the difficult task of estimating the probabilities of various catastrophic scenarios. The main goal of the approach is to aid the loan decision-making by showing what size of catastrophe is needed to increase the default probability enough to reject the loan.

ACKNOWLEDGMENTS

Tarja Joro acknowledges financial support from University of Alberta School of Business Canadian Utilities Fellowship. The views expressed in this paper are those of the authors and do not reflect the views of Bayerische Landesbank.

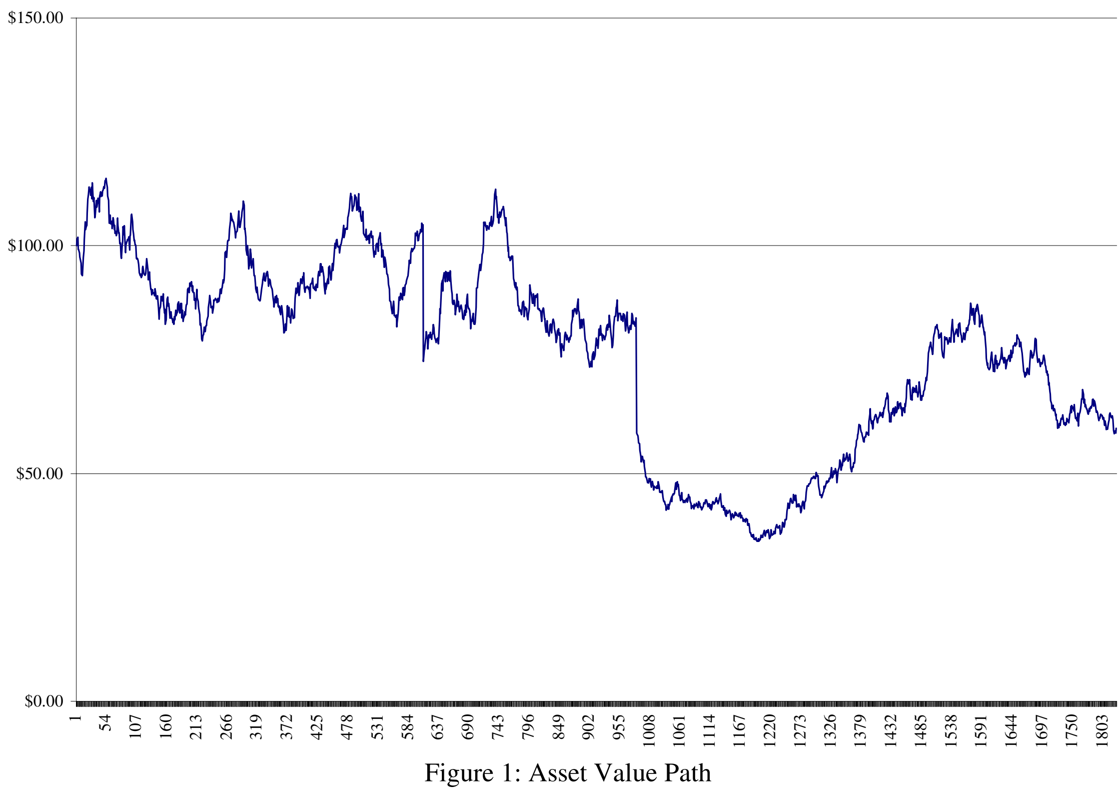

Figure 1: Asset Value Path

REFERENCES

Ahn, C. M. and H.E. Thompson, 1988, Jump-Diffusion Processes and the Term Structure of Interest Rates, *Journal of Finance*, 43, 155-174.

Black, F. and J.C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351-367.

Duffie, D. and K.J. Singleton, 1995, Modeling term structures of defaultable bonds, Working paper, Stanford University Business School.

Jarrow, R.A. D. Lando, and S. Turnbull, 1994, A Markov model for the term structure of credit risk spreads," Working paper, Cornell University.

Jarrow, R.A. and S. Turnbull, 1995, Pricing derivatives on financial securities, *Review of Financial Studies* 1, 427-445.

Kau, J.B. and D.C. Keenan, 1999, *Journal of Financial Services Research* 15, 87-102

Kou, S. G., 2001, A Jump Diffusion Model for Option Pricing with Three Properties: Leptokurtic Feature, Volatility Smile, and Analytical Tractability, working paper, Columbia University, New York.

Longstaff, F.A. and E.S. Schwartz, 1995, A simple approach to valuing risky and floating rate debt," *Journal of Finance* 50, 789-819.

Madan, D.B. and H. Unal, 1994, Pricing the risks of default, Working paper, The Wharton School of the University of Pennsylvania.

Merton, R. C., 1974, On the Pricing of Corporate Debt: the Risk Structure of Interest Rates, *Journal of Finance*, 29, 449-470.

Merton, R. C., 1976, Option Pricing when Underlying Stock Returns are Discontinuous, *Journal of Financial Economics*, 3, 125-144.

Zhou, C., 1997, *A Jump-Diffusion Approach to Modeling Credit Risk and Valuing Defaultable Securities*, Finance and Economics Discussion Series 1997-15 / Board of Governors of the Federal Reserve System (U.S.).

Zhou, C., 2001, The Term Structure of Credit Spreads with Jump Risk, *Journal of Banking and Finance*, 25, 2015-40.

AUTHOR BIOGRAPHIES

TARJA JORO is an Assistant Professor in Management Science at the University of Alberta School of Business in Edmonton, Canada. She received her Ph.D. from Helsinki School of Economics in 2000. Her research interests are in productivity and efficiency studies and financial engineering. Her email and web address are <tarja.joro@ualberta.ca> and <http://www.bus.ualberta.ca/tjoro>.

PAUL NA is a Vice President and Head of Risk Methodology and Systems at Bayerische Landesbank New York Branch. He received his Ph.D. from University of Georgia in 2000. His research interests are in credit risk modeling and mutual fund performance. His email is <pna@baylbny.com>.