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SIMULATION AND OPTIMIZATION FOR REAL OPTIONS VALUATION

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ABSTRACT

Real options valuation (ROV) considers the managerial flexibility to make ongoing decisions regarding implementation of investment projects and deployment of real assets. This paper introduces a simulation-optimization approach to valuing real investment options based on a model containing several decision variables and realistic stochastic inputs. Using this approach, the value of a portfolio of real investment projects is determined by maximizing the mean discounted cash flows calculated by the model over many combinations of the decision variables. This yields an optimal decision rule that significantly increases the value extracted from the investment projects in comparison to arbitrary decision rules.

1 INTRODUCTION

Discounted cash flow approaches, such as net present value (NPV), have traditionally been the preferred methods for evaluating investments in real assets. Recently, real options valuation (ROV) emerged as an alternative to simplistic discounted cash flow methods. ROV values the managerial flexibility to make ongoing decisions regarding implementation of investment projects and deployment of real assets.

ROV extends valuation models used to price financial options and applies them to investments in real assets. Black and Scholes (1973) developed a model to value financial options that focuses on factors affecting the value of the underlying financial asset over time, which is assumed to follow a geometric Brownian motion stochastic process. Several ROV methods have been implemented that rely on similar assumptions to those made in the Black-Scholes model. These include lattice models and dynamic programming methods, both of which are are based on a simple representation of the evolution of the value of the underlying asset.

Previous ROV approaches using simulation have maintained the requirement of market replication assumed by other real options valuation methods. These approaches assume that the risk of the project underlying the real option can be duplicated by assets in financial markets. Additionally, an implicit assumption of financial options pricing methods is that the value of the underlying asset is known at the time the exercise decision is made. For instance, a European put option on a traded stock should always be exercised if the market price is less than the exercise price. As a result, financial option pricing methods are most concerned with providing a value for the option so an investor can determine whether to invest in the option. In contrast, the optimal decision rule for exercising real options is not always as apparent as for financial options.

Real asset investment decisions also differ from financial option exercise decisions because the optimal decision rule is not based on the observable market price of an underlying asset. The decision rule for a real asset investment may be based on observation of project performance indicators in the periods leading up to the exercise date; for instance, a decision to expand a product line may be based on the market demand for a similar product during the past year. In some instances, the decision rule might be based on an updated forecast of expected future performance. Neither performance indicators or updated estimates, however, provide perfect information, so a value placed on a real option by a method that assumes the underlying asset value is observable is an upper bound.

Our approach relaxes the assumptions of market replication and perfect knowledge of the project value. We develop a model containing decision variables and stochastic assumptions, then maximize the mean discounted cash flows calculated by the model over many combinations of the decision variables. The decision variables are used to determine a decision rule which is stated in terms of observable stochastic variables. Thus, the output of the model

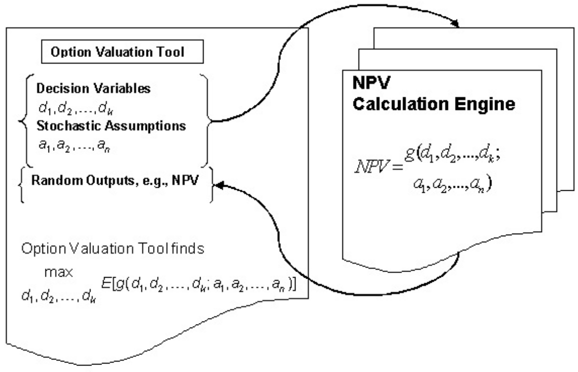

Figure 1: Graphical Depiction of the Links Between the Option Valuation Tool and the NPV Calculation Engine

is both a value of the real option(s) and an optimal decision rule.

The remainder of this paper is organized as follows. Section 2 outlines the simulation-optimization approach. Section 3 describes the problem and the cash flow model. Section 4 explains the methods used to simulate first-order autoregressive, AR(1), processes. Section 5 illustrates the simulation-optimization approach with an example problem. Section 6 provides a summary and conclusions.

2 A SIMULATION-OPTIMIZATION APPROACH

2.1 Overview

The simulation-optimization method proposed relies on an "NPV Calculation Engine" to determine the value of potential investment projects. The assumptions used in the NPV Calculation Engine are classified as follows:

1. Decision variables — these assumptions are those under the control of the decision makers and can be adjusted to increase project value as required.
2. Stochastic assumptions — these assumptions are random variables with known or estimated probability distributions.
3. Deterministic assumptions — these assumptions are based on established benchmarks.

The second component of the simulation-optimization model is an "Option Valuation Tool" which interacts with the NPV Calculation Engine by selecting different combinations of the decision variables and generating random simulation trials using the stochastic assumptions. The Option Valuation Tool tracks the mean net present value of investment projects for each combination of the decision variables to determine the optimal decision rule. Figure 1 provides a graphical depiction of the simulation-optimization model.

2.2 Notation

This section defines variables that will be used throughout the remainder of the paper.

The K potential investment projects are denoted by subscripts on variables, $k = 1, \dots, K$. Subscripts $t = 1, \dots, T$ are used to denote the value of variables in a specific time period. Two sets of decision variables are required:

α_k = Intercept parameter of the linear cash flow threshold for project k

β_k = Slope parameter of the linear cash flow threshold for project k.

Two stochastic time series assumptions are used:

B_t = Customer base in period t

R_t = Unit revenue in period t.

Deterministic assumptions are defined as follows:

C_t = Unit variable cost in period t

I_{kt} = Indicator variable representing availability of project k in period t

N_{kj} = Indicator variable representing availability of project k on project j

F_k = Investment required to initiate project k

c_{kt} = Unit variable cost decrease provided by project k (if active) in period t (stated as a percentage of C_t)

r_{kt} = Unit revenue increase provided by project k (if active) in period t (stated as a percentage of R_t).

The remaining variables are deterministic, given a specific instantiation of the decision variables and stochastic assumptions:

Y_{kt} = Linear cash flow decision threshold for project k in period t

D_{kt} = Indicator variable representing comparison of baseline cash flows in period $t-1$ to a specified linear threshold for project k

A_{kt} = Indicator variable representing activation of project k in period t

G_{kt} = Indicator variable representing timing of fixed investment cost made for project k in period t.

3 PROBLEM DESCRIPTION

This paper addresses the general problem of making investment decisions to facilitate ongoing operations of a business. An embedded base of assets provides service to an established base of customers at a given level of unit revenue per period. The number of customers in this base, B_t, and the amount of the unit revenue per period, R_t, both follow stochastic processes. In this paper, we assume both follow AR(1) processes. Additionally, the customer base and unit revenue processes are cross-correlated. The unit variable cost, C_t, is deterministic.

The ROV methodology proposed here is not limited to use with AR(1) processes. Indeed, one of our method's advantages is that it can be used with virtually any stochastic inputs or processes selected by the analyst. We have chosen to use cross-correlated AR(1) processes in this paper so that we can study the behavior of the investment value due to systematic changes in the autocorrelation and cross-correlation parameters.

The company must make decisions on potential enhancements and upgrades to the embedded base of assets, each considered as a separate investment project. Although the projects are separate, there are logical interdependencies between the potential investment projects.

The embedded base of assets and the investment projects—if pursued—provide cash flows for a specified time horizon of T periods. The project availability variable, I_{kt}, equals 1 if project k is available in period t and 0 otherwise. The project interdependency variable, N_{kj}, equals 1 if project k is logically dependent on project j (or if $k = j$), and 0 otherwise.

Project k is said to be logically dependent on project j if some functional aspect of project k is necessary for successful implementation of project j. For instance, a telecommunications company that installs a voice-over-packet call server (project k) has the future option to install a multimedia call server (project j) that provides videophone service only if project k is undertaken. Thus, the installation of the voice-over-packet call server provides the telecommunications company with the right, but not the obligation, to provide videophone service later. This is an example of a real option that can be valued with simulation and optimization.

Decisions on whether to invest in projects are made based on observations of baseline cash flows. The value of the linear cash flow decision threshold parameter for project k in period t is

$$Y_{kt} = \alpha_k I_{kt} + \beta_k \left(\left(\sum_{i=1}^{t} I_{ki} \right) - 1 \right).$$

Values of $D_{k1} = 0$, for $k = 1, \ldots, K$. When $t \neq 1$, values of D_{kt} equal 1 when $B_{t-1}(R_{t-1} - C_{t-1}) \geq Y_{kt}$ and 0 otherwise.

For project k at period $t = 1$, the project activation variable $A_{k1} = D_{k1} I_{k1}$. For $t > 1$: if $A_{k,t-1} \neq 1$, then

$$A_{kt} = \prod_{\substack{j \\ N_{kj} \neq 0}} (I_{jt} D_{jt});$$

else if $A_{k,t-1} = 1$, then $A_{kt} = 1$. The project activation variable represents the assumption that for project k to be active, all projects on which project k is logically dependent must be activated, which requires one of two conditions to be met:

1. Cash flows in period $t-1$ exceed the linear cash flow threshold for project k and each project on which project k is dependent, or
2. The project was activated in an earlier period.

In this model, projects are assumed to remain active once they are initiated. However, the option to abandon active projects can also be valued readily with our approach.

If indicator variable G_{kt} is assigned a value of 1, F_k is expended in period t. Values of G_{kt} are assigned as follows:

$$G_{kt} = \begin{cases} 1 & \text{if } \sum_{i=1}^{t} A_{ki} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let ICF_t be the incremental cash flows from the investment projects. Incremental cash flows include variable cost savings and/or additional unit revenue and are calculated as

$$ICF_t = \sum_{k=1}^{K} (B_t A_{kt} (c_{kt} C_t + r_{kt} R_t) - G_{kt} F_k),$$

for $t = 1, \ldots, T$. These incremental cash flows are discounted to determine the net present value of the portfolio of projects using the formula

$$\text{NPV} = \sum_{t=1}^{T} (ICF_t) e^{-rt},$$

where r is the appropriate discount rate.

4 SIMULATING CROSS-CORRELATED AR(1) PROCESSES

To study the behavior of the investment value due to systematic changes in the autocorrelation and cross-correlation parameters, we assume that B_t and R_t follow AR(1) processes.

Define Z_{Rt} as the AR(1) process for the deviation of unit revenue, R_t, from its long-run average of μ_R in year t, and Z_{Bt} as the AR(1) process for the deviation of customer base, B_t, from its long-run average of μ_B in year t. These processes evolve as

$$Z_{Rt} = \phi_R Z_{R,t-1} + a_{Rt}$$

and

$$Z_{Bt} = \phi_B Z_{B,t-1} + a_{Bt},$$

where a_{Rt} and a_{Bt} are defined as Gaussian white noise terms, each having mean zero and having variances equal to σ_R^2 and σ_B^2, respectively. Unit revenue and customer base for year t can be represented as follows:

$$R_t = \mu_R + Z_{Rt}$$

$$B_t = \mu_B + Z_{Bt}.$$

An AR(1) process can also be represented as an infinite series in terms of all past white noise terms as

$$Z_{Rt} = a_{Rt} + \phi_R a_{R,t-1} + \phi_R^2 a_{R,t-2} + \ldots$$

and

$$Z_{Bt} = a_{Bt} + \phi_B a_{B,t-1} + \phi_B^2 a_{B,t-2} + \ldots.$$

From these representations, it is easy to observe that that $E[Z_{Rt}]$=$E[Z_{Bt}] = 0$. The variance of the unit revenue process is calculated as

$$\text{Var}[Z_{Rt}] \;=\; \text{Var}[a_{Rt} + \phi_R a_{R,t-1} + \phi_R^2 a_{R,t-2} + \ldots].$$

Since the white noise terms are independent and identically distributed, the covariance terms all equal zero. Thus, the variance of the unit revenue process can be rewritten as

$$\begin{aligned}
\text{Var}[Z_{Rt}] &= \text{Var}[a_{Rt}] + \phi_R^2 \text{Var}[a_{R,t-1}] \\
&\quad + \phi_R^4 \text{Var}[a_{R,t-2}] + \ldots \\
\text{Var}[Z_{Rt}] &= \sigma_R^2 + \phi_R^2 \sigma_R^2 + \phi_R^4 \sigma_R^2 + \ldots \\
\text{Var}[Z_{Rt}] &= \text{Var}[a_{Rt}] + \phi_R^2 \text{Var}[a_{R,t-1}] \\
&\quad + \phi_R^4 \text{Var}[a_{R,t-2}] + \ldots \\
\text{Var}[Z_{Rt}] &= \sigma_R^2 + \phi_R^2 \sigma_R^2 + \phi_R^4 \sigma_R^2 + \ldots
\end{aligned}$$

$$\text{Var}[Z_{Rt}] = \sigma_R^2/(1 - \phi_R^2).$$

The variance in the deviation from the long-run average for customer base can be calculated similarly as

$$\text{Var}[Z_{Bt}] = \sigma_B^2/(1 - \phi_B^2).$$

Note the covariance between two terms in the time series for deviation from long-run unit revenue average ℓ years apart as $\gamma_{R\ell}$. The covariance for the unit revenue process can be calculated as

$$\begin{aligned}
\gamma_{R\ell} &= E[Z_{Rt} Z_{R,t-\ell}] \\
\gamma_{R\ell} &= E[Z_{R,t-\ell}(\phi_R Z_{R,t-1} + a_{Rt})] - E[Z_{Rt}]E[Z_{R,t-\ell}] \\
\gamma_{R\ell} &= E[Z_{R,t-\ell}\phi_R Z_{R,t-1} + Z_{R,t-\ell} a_{Rt}] \\
\gamma_{R\ell} &= E[Z_{R,t-\ell}\phi_R Z_{R,t-1}] + E[Z_{R,t-\ell} a_{Rt}].
\end{aligned}$$

The last term on the right hand side is zero, because this term can be re-written as a sumproduct of the independent and uncorrelated white noise terms. Thus, the expression for the covariance of the unit revenue process for terms with lag ℓ can be written as

$$\gamma_{R\ell} = \phi_R \gamma_{R,\ell-1}.$$

For $\ell = 0$, $\gamma_{R0} = \sigma_R^2$. The general covariance term for the customer base process can be found similarly.

The correlation between two terms with lag ℓ is defined as

$$\rho_{R\ell} = \frac{\gamma_{R\ell}}{\gamma_{R0}} = \frac{\gamma_{R\ell}}{\sigma_R^2} = \phi_R^\ell.$$

Thus, the term ϕ_R in the AR(1) process is equal to ρ_{R1}. The correlation terms in the customer base deviation process are defined similarly.

To simulate the AR(1) processes for unit revenue and customer base, initial terms are generated randomly as

$$Z_{R0} \sim N[0, \sigma_R^2/(1 - \phi_R^2)]$$

and

$$Z_{B0} \sim N[0, \sigma_B^2/(1 - \phi_B^2)].$$

Standard normal random variables, $X_{R1}, \ldots, X_{RT}$ and $X_{B1}, \ldots, X_{BT}$ are drawn for each period in the time horizon. These standard normal random variables are transformed into cross-correlated random variables, Y_{Rt} and Y_{Bt}, each with a standard deviation equal to the underlying white noise variable, as

$$Y_{Rt} = X_{Rt}\sigma_R$$

and

$$Y_{Bt} = \left(X_{Bt}\rho_C + \sqrt{1 - \rho_C^2}\right)\sigma_B$$

where ρ_C is the cross-correlation between the price and demand processes.

When the AR(1) processes are cross-correlated, the equations used to calculate the values of the processes in the remaining time horizon are

$$Z_{Rt} = \phi_R Z_{R,t-1} + Y_{Rt},$$

$$Z_{Bt} = \phi_B Z_{B,t-1} + Y_{Bt}.$$

The values for price and demand are then calculated by adding the μ_R and μ_B to Z_{Rt} and Z_{Bt}.

5 EXAMPLE PROBLEM AND RESULTS

5.1 Problem Inputs and Deterministic Approximation

For this problem, the time horizon is five annual periods ($t = 1, \ldots, 5$) and there are three available enhancement projects ($K = 3$). Other parameters are listed in Table 1.

The total expected value of cash flows from all three projects in this example—assuming all projects are exercised as soon as available—is calculated (in millions) as follows:

$$\begin{aligned}
ICF_1 &= \$0 \\
ICF_2 &= B_2 c_{12} C_2 - F_1 \\
&= (3.0)(50\%)(\$175) - \$975 \\
&= -\$712.50 \\
ICF_3 &= B_3(c_{13}C_3 + r_{23}R_3) - F_2 \\
&= (3.0)(50\%)(\$175) \\
&\quad +(3.0)(25\%)(\$450) - \$925 \\
&= -\$325 \\
ICF_4 &= B_4((c_{14} + c_{34})C_4 + r_{24}R_4) - F_3 \\
&\quad = (3.0)((50\% + 25\%)(\$175) \\
&\quad +(25\%)(\$450)) - \$245 \\
&= \$486.25 \\
ICF_5 &= B_5((c_{15} + c_{35})C_5 + r_{25}R_5) \\
&\quad = (3.0)((50\% + 25\%)(\$175) \\
&\quad +(25\%)(\$450)) \\
&= \$731.25.
\end{aligned}$$

A single discount rate, $r = 7\%$, is used in this problem. The calculation of expected incremental cash flows assumes that the customer base and unit revenue processes are equal to their long-run averages of μ_B and μ_R in each year.

The following formula is used to determine the net present value of the expected incremental cash flows and the required investment cash flows,

$$\text{NPV} = \sum_{t=1}^{5} (ICF_t)e^{-rt} = -\$0.05 \text{ million.}$$

Incremental cash flows for Projects 1, 2, and 3 (in millions) are $-\$23.27$, \$16.69, and \$6.52, respectively. While Projects 2 and 3 have positive net present values, only Projects 1 and 2 can be exercised individually, as Project 3 is dependent on Projects 1 and 2.

5.2 Simulation Results

To better understand the results of the deterministic approximation, 50,000 different sample paths of the customer base and unit revenue processes were simulated. Incremental cash flows were calculated based on these sample paths using the assumption that all projects are exercised as soon as available. Given the general formulation from Section 3, this assumption is implemented by setting arbitrarily low values of α_k and β_k for all projects. Correlation assumptions are set at $\phi_R = \phi_B = 0.45$ and $\rho_C = -0.45$. Figure 2 shows a frequency distribution of the total net present value of incremental cash flows for the 50,000 trials.

Table 1: Parameters for the Example Problem

Parameter	Value(s)
Project availability	$I_{12} = \cdots = I_{15} = I_{23} = \cdots = I_{25} = I_{34} = I_{35} = 1$; all other values $I_{kt} = 0$
Revenue increases	$r_{1t} = r_{3t} = 0\%$ and $r_{2t} = 25\%$ for $t = 1, \ldots, 5$
Variable cost decreases	$c_{1t} = 50\%$, $c_{2t} = 0\%$, and $c_{3t} = 25\%$ for $t = 1, \ldots, 5$
Unit variable cost	$C_t = \$175$ for $t = 1, \ldots, 5$
Long-run average unit revenue	$\mu_R = \$450$
Long-run average customer base	$\mu_B = 3$ million
Variance of unit revenue process	$\sigma_R^2 = \$900$
Variance of customer base process	$\sigma_B^2 = 0.06$ million
Required investment costs	$F_1 = \$975$, $F_2 = \$925$, and $F_3 = \$245$

The mean of the frequency distribution is $-\$2.22$ million and a 95% confidence interval for the mean is from $-\$3.18$ million to $-\$1.28$ million. In this distribution, 95% of the observations of total net present value of incremental cash flows are between $-\$212.59$ million to \$207.91 million. Since cross-correlation exists in the AR(1) processes, the mean of the frequency distribution does not equal the mean net present value from the deterministic approximation.

The solution above is based on an arbitrary decision rule to invest in all projects regardless of prior cash flows. Other arbitrary decision rules could be used to produce different solutions; for instance, we could establish a decision rule that dictates investing in a project if cash flows in the prior year are greater than or equal to the expected value of \$825. This decision rule is implemented in the model by setting $\alpha_k = \$825$ and $\beta_k = \$0$ for $k = 1, 2, 3$. Figure 3 shows

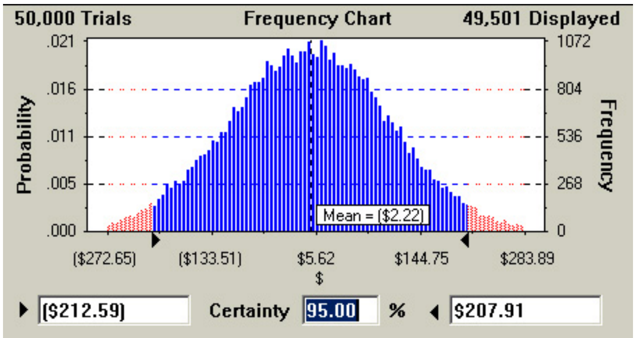

Figure 2: Frequency Distribution of Simulation Results for the Example Problem with all Projects Exercised as Soon as Available

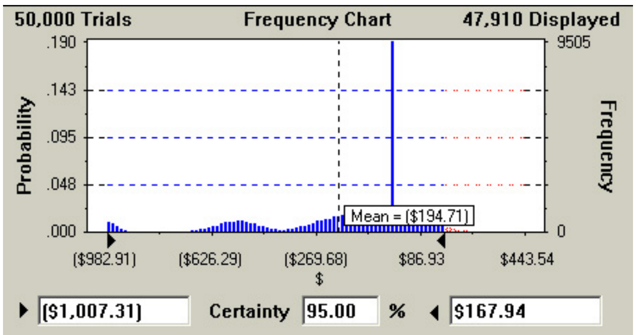

Figure 3: Frequency Distribution of Simulation Results for the Example Problem Where all Projects are Exercised when Prior Year Cash Flows are Greater than the Expected Value

the frequency distribution for 50,000 simulation trials using this decision rule and the same correlation assumptions as the prior example.

The mean of the frequency distribution is −\$194.71 million and a 95% confidence interval for the mean is from −\$197.38 million to −\$192.04 million. In this distribution, 95% of the observations of total net present value of incremental cash flows are between −\$1007.31 million to \$167.94 million.

Given the correlation assumptions for the customer base and unit revenue processes, it is difficult to determine a decision rule for exercising the enhancement projects arbitrarily, as illustrated by this example. The next section discusses the selection of an approximately optimal decision rule for exercising investment projects based on the information revealed by the realizations of the customer base and unit revenue processes.

5.3 Simulation-Optimization Results

The solutions from the previous section use arbitrary decision rules to make investment decisions regarding the three projects. To improve the effectiveness of the investment decisions, a method that analyzes multiple potential decision rules is required. The decision rules are based on observable information, namely the realizations of the customer base and unit revenue processes. To simplify the process of determining an optimal decision rule, we will consider a rule based on cash flows from the embedded base of assets, since this variable depends on both the customer base and unit revenue processes.

Using the same simulation model, the optimal decision rule is determined by considering many possible combinations of the decision variables (the intercept and slope parameters, α_k and β_k) for the three projects. Using Crystal Ball along with OptQuest software, which employs a scatter search algorithm to select decision variable scenarios, an approximately optimal solution can be obtained without testing a complete enumeration of the possible combinations of the decision variables (for more information on the scatter search algorithm, see Glover *et al.* (1996)).

For this example, possible values of the slope parameters, β_1, β_2, and β_3, ranging from −\$2000 million to \$2000 million in increments of \$250 million, and possible values of the intercept parameters α_1, α_2, and α_3 from \$0 to \$1000 million in increments of \$250 million are considered. Using smaller increments for the decision variables might provide a more precise decision rule, but will require more time to obtain a solution. Since the decision rule is in terms of estimated, future cash flows, an approximately optimal decision rule in \$250 million increments is satisfactory for demonstration purposes.

Using the scatter search algorithm, 50,000 (out of a possible 3,581,577) distinct combinations of decision variable values are considered, with 500 trials used on each simulation run. The combination that produces the largest mean net present value of incremental cash flows is considered the approximately optimal decision rule. Using the assumptions that $\phi_B = \phi_R = 0.45$ and $\rho_C = -0.45$, the approximately optimal decision rule includes the following values of the decision variables (in millions),

$$\alpha_1 = \$1250, \alpha_2 = \$750, \alpha_3 = \$0,$$
$$\beta_1 = \$0, \beta_2 = \$250, \beta_3 = \$1250.$$

Given the values in this example for I_{kt}, the decision rule can be interpreted as follows. Project 1 will be exercised in year 2, 3, or 4 if cash flows in the prior year are \$1250 million or greater. Project 2 will be exercised in year 3 if year 2 cash flows are \$750 million or greater, in year 4 if year 3 cash flows are \$1000 million or greater, or in year 5 if year 4 cash flows are \$1250 million or greater. Project

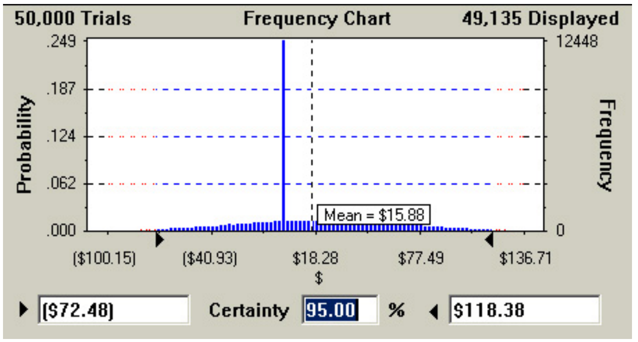

Figure 4: Frequency Distribution of Simulation Results for the Example Problem Using the Optimal Solution from the Optimization Routine

3 will be exercised in year 4 if year 3 cash flows are $0 or greater, or in year 5 if year 4 cash flows are $1250 million or greater.

The largest mean net present value of incremental cash flows was obtained on the 409th distinct combination of decision variable values selected by the optimization routine. The mean net present value of incremental cash flows for the 500 trials in the simulation-optimization routine is $16.83 million. To analyze the risk associated with following this strategy, a longer simulation run of 50,000 trials is performed with the approximately optimal decision values. The mean net present value of incremental cash flows for this simulation run is $15.88 million (see Figure 4), with a 95% confidence interval for the mean running from $15.47 million to $16.29 million. In this simulation, 95% of the 50,000 observations fell between −$72.48 million and $118.38 million. Approximately 25% of the simulation trials produced $0 in incremental net present value because the decision rules for exercising Projects 1 and 2 are not satisfied.

The optimal decision rule changes significantly if the correlation inputs to the model are adjusted. To test the effect of changing these inputs, combinations of inputs were tested for two levels of each parameter. Customer base autocorrelation (ϕ_B) and unit revenue autocorrelation (ϕ_R) values of 0.45 and 0.90, along with cross-correlation between customer base and unit revenue (ρ_C) values of −0.90 and −0.45. This created an experiment with 8 different combinations of inputs.

In this extended experiment, the optimization routine is allowed to run for 180 minutes using 500 simulation trials for each combination. This is adequate time to test approximately 10,000 distinct combinations of decision variables values when running the routine on a workstation with a 1.0 GHz Pentium III processer. On average, the scatter search algorithm identified the largest mean net present value on the 219th distinct combination of decision variable values after an average run time of 4.75 minutes. After using the optimization routine to obtain an optimal decision rule for each of the eight scenarios, this rule was used in a longer simulation run of 50,000 trials for each scenario. Table 2 lists the correlation assumptions for each scenario and the mean net present value for 50,000 simulation trials, Table 3 lists the optimal values of the decision variables for each correlation scenario, and Table 4 lists standard errors and risk analysis statistics.

Table 2: Simulation-Optimization NPV Results for Various Correlation Scenarios (Mean NPV in Millions of $)

Scenario	ρ_C	ϕ_B	ϕ_R	Mean NPV
1	−0.90	0.45	0.45	$12.07
2	−0.90	0.45	0.90	$39.19
3	−0.90	0.90	0.45	$63.01
4	−0.90	0.90	0.90	$65.10
5	−0.45	0.45	0.45	$15.88
6	−0.45	0.45	0.90	$43.47
7	−0.45	0.90	0.45	$69.11
8	−0.45	0.90	0.90	$76.53

Table 3: Simulation-Optimization Optimal Decision Rules for Various Correlation Scenarios (Billions of $)

Scen.	α_1	β_1	α_2	β_2	α_3	β_3
1	2.00	1.25	0.50	−2.00	0.00	1.25
2	2.00	2.00	0.75	2.00	2.00	2.00
3	0.75	0.25	0.75	0.50	1.25	0.50
4	1.00	0.50	0.75	0.25	1.00	0.00
5	1.25	0.00	0.75	0.25	0.00	1.25
6	1.50	1.00	0.75	0.50	1.25	0.50
7	0.75	0.25	0.75	0.75	0.50	0.00
8	1.00	0.25	0.75	0.50	1.00	0.00

The highest mean net present value occurs in Scenario 8 when the customer base and unit revenue processes are highly autocorrelated ($\phi_B = \phi_R = 0.90$) and have lower cross-correlation ($\rho_C = -0.45$), with a maximum mean NPV of $76.53. When the stochastic processes are simulated with these correlation assumptions, the approximately optimal decision rule is determined by the following values of the decision variables (in millions),

$$\alpha_1 = \$1000, \alpha_2 = \$750, \alpha_3 = \$1000,$$
$$\beta_1 = \$250, \beta_2 = \$500, \beta_3 = \$0.$$

Note that the assumptions used to simulate the stochastic processes can significantly change the mix of projects that are most likely to be implemented. For instance, given that $\phi_R = 0.45$ and $\rho_C = -0.45$, Project 3 is more likely to be implemented when $\phi_B = 0.90$ in Scenario 5, than when $\phi_B = 0.45$ in Scenario 7.

Table 4: Statistics for 50,000 Simulation Trials for Each Correlation Scenario with Optimal Decision Variable Values Listed in Table 3 (Millions of $, Except % > $0)

		95% Confidence Interval		95% Certainty Interval		% Trials > $0
Scen.	S.E.	Min.	Max.	Min.	Max.	NPV
1	0.11	11.9	12.3	−39	61	69 %
2	0.27	38.7	39.7	−39	188	89 %
3	0.82	61.4	64.6	−260	499	73 %
4	0.62	63.9	66.3	−82	479	45 %
5	0.21	15.5	16.3	−73	118	74 %
6	0.34	42.8	44.1	−72	233	86 %
7	0.85	67.4	70.8	−246	535	47 %
8	0.73	75.1	78.0	−114	545	86 %

6 SUMMARY AND CONCLUSIONS

This paper introduced a simulation-optimization approach to valuing a portfolio of real investment projects when uncertainty is characterized by stochastic time series assumptions. The net present values of the potential investment projects are functions of decision variables and stochastic assumptions. By calculating the net present value over many combinations of decision variables, an approximately optimal decision rule based on observable criteria is established. We demonstrated that selecting an approximately optimal decision rule using the simulation-optimization approach provides significant improvement over arbitrary decision rules. Also, we established that the correlation assumptions in the underlying stochastic assumptions affect the decision rules and net present value of the portfolio of projects.

Extensive future research will be required to adapt this approach to a broader range of investment problems. More complicated net present value functions and stochastic time series assumptions may be required to model more complex portfolios of investment projects. This paper has not addressed the issue of assigning discount rates on a project-by-project basis. Our conjecture is that the approximately optimal decision rules are largely insensitive to perturbations in the discount rate, but this remains to be determined by additional research.

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