

SEARCH THEORY, AGENT-BASED SIMULATION, AND U-BOATS IN THE BAY OF BISCAY

Lance Champagne
R. Greg Carl

Department of Operational Sciences
The Air Force Institute of Technology
Wright Patterson AFB, OH 45433, U.S.A.

Raymond Hill

Biomedical, Industrial & Human Factors Eng.
Wright State University
Dayton, OH 45435, U.S.A.

ABSTRACT

To date, most search theory study has focused either on analytical models of specific situations requiring rigid assumptions, or, as in the case of search and rescue, operational experiments aimed at obtaining detection probabilities for a variety of scenarios. Analytical search theory results provide bounds on empirical results. This research introduces an agent-based simulation approach to the subject of offensive search operations in combat. Generally, the value of a combat simulation is measured in terms of insights gained through experimentation. Agent-based simulation enables insights with regards to the emergent behavior of the individual combatants, groups of combatants, or the system as a whole. Emergent behavior for the purposes of this research is system behavior, not explicitly programmed, arising from local interactions between agents. Such behavior with respect to search effectiveness is investigated within the context of a historical case study involving offensive search.

1 INTRODUCTION

An agent is an entity that perceives and acts in its environment (Russell and Norvig 1995). An agent is further characterized as a physical or virtual entity having a partial representation of its environment and is capable of perceiving that environment and acting within it; agents interact with other agents, and have a set of internal tendencies or goals guiding their behavior in an attempt to satisfy their goals given their resources, abilities, and perceptions (i.e., autonomous; adapted from (Ferber 1999)). To some degree software agents are self-aware. Agents possess all the decision tools needed to determine their behavior based upon their perceptions of the model's environmental state.

One utility of using agent-based simulation is to identify emergent behaviors. When agents interact, certain effects not explicitly programmed arise providing insight into the situation the model represents. Recently, attention given to agent-based simulation has attempted to identify

adaptations likely to result from such agent interaction. Many disciplines are now interested in exploiting emergent behavior to gain insight into how cooperative and competitive agent behavior affects a real-world system (the whole versus the sum of the parts) and whether or not individual agent behavior has changed as a result.

This characteristic of ... analyzing the interaction systems that exist between agents is what distinguishes multi-agent systems from the more classical systemic approaches, in that preference is given to emergence, and action and interaction are considered as the motor elements in the structuring of a system taken as a whole.

Jacques Ferber, 1999

Combat agents are designed to purposefully compete with elements (i.e., other combat agents) that seek to prevent them from attaining their goals. The notion of such "antagonistic interaction" can certainly be analyzed as a possible "interaction system."

The U-boat war of WWII is amenable to both search theory and agent-based simulation. It pertains to offensive search since it involves the Allied search for hostile targets. Unlike other battles of WWII, the battle of the seas involving U-boats lasted the entire war. This means there is an abundance of actual data and analytical work with which to refer. Agent-based simulation presents a unique tool with which to analyze the U-boat war. With agent models, simulators have the ability to characterize detection devices of aircraft and U-boats with simple analytical expressions.

2 THE CONTEXT—U-BOATS IN WWII

Originally, Operations Research was defined as the "prediction of the effects of new weapons and tactics" (Waddington 1973). A chief concern for the allies in WW II was how to effectively counter the U-boat threat. U-boats were a particular problem to the British. Great Britain depended heavily on merchant shipping for supplies,

and the destruction of these supply ships was a primary U-boat mission.

To counter the U-boat threat, the Allied offensive reaction involved missions to patrol supply convoys, search the Bay of Biscay for U-boats, and bomb the captured ports used by the Germans; all of which the allies performed to varying degrees throughout the war. Wartime analysis was mainly motivated by the need to allocate air assets optimally to counter the serious U-boat threat.

Two items are also worth noting here. The first item is that for most of WWII, long-range sonar had not been fielded, leaving the radar-equipped allied aircraft with the ability to detect U-boats only while the U-boats were surfaced or at periscope depth with decks awash. The second item is that U-boat technology had not yet advanced to the point where a U-boat could remain submerged throughout a transit to open waters. This meant they had to surface en route through the Allied search zone, leaving U-boats most vulnerable in the Bay of Biscay either embarking from or returning to their ports on the west coast of France. The Bay of Biscay itself constituted 130,000 square miles of searchable area, and extended from the northern coast of Spain in the South to the coast of France in the East on to England and Ireland in the North.

U-boats imparted terrible losses to international shipping; Hitler himself stated, “U-boats will win the war” (Waddington 1973). The degree of concern held by the allies was perhaps best stated by British Prime Minister Sir Winston Churchill when he said, “the only thing that ever really frightened me during the war was the U-boat peril” (Churchill 1949).

Operational Research Section (ORS) insights, operational experience, and intelligence information produced a basic search patrol methodology applied to Bay of Biscay operations (HBMSO 1943). Aircraft on patrol flew to a specific bearing and covered a predefined area extending from the bearing for a fixed number of hours. The state of the weather, the number of hours of daylight, and the range of the aircraft regulated the duration of each patrol.

Historically, the allies used what is known as the barrier patrol search pattern. This pattern, resembling a bowtie, kept Allied aircraft largely at a 45-degree angle to the U-boat tracks. The allies believed that aircraft observers could see the wake of a U-boat easier than they could see the object itself and the 45-degree angle facilitated the observation process. Approaching the wake quartered-toward or quartered-away from the U-boat track was believed to maximize allied sighting distance. As a result, most search patterns ran either NW-SE or NE-SW across the assigned coverage area since they generally figured east-westerly transit routes for U-boats crossing the Bay. Operations Research analysts later determined that track spacing could be chosen arbitrarily without impacting search efficiency (though it was said that track spacing might have had an effect on the amount of search resources used (Koopman 1999), (OASG 1977)).

Analyses also revealed that U-boat distribution for 1942 and 1943 in the Bay could be modeled as a Poisson process (Waddington 1973).

Limitations of attack aircraft of the time meant aircraft that actually attacked a U-boat were not available for search effort elsewhere during that sortie. Aircraft flying search patrols almost always flew alone, and during a sortie in which an aircraft had sighted a U-boat and dropped weapons in hopes of damaging or sinking the craft, the aircraft would have maintained area presence to assess battle damage (HBMSO 1943).

3 SEARCH THEORY CONCEPTS

Search theory has always played an important role in military operations. Using agent-based simulation, we focus on applications of search theory using the Bay of Biscay as our scenario. McCue observes candidly that given recent advances in defense technology, “the operations of war are operations of search” (McCue 1990). An excellent survey of search theory literature is available in (Benkoski, et al. 1991). For this research, pertinent material addresses the following:

- Emphasize search planning, not search modeling;
- Take a tactical, not strategic, viewpoint;
- Assume search operations have uncertainty;
- Aim more at obtaining initial detections versus fusing multiple detections;
- Involve a moving target; and
- Involve a non-cooperative target.

3.1 Search Possesses Structure of its Own

Koopman (1999) describes the operation of search as “an organic whole having a structure of its own—more than the sum of its parts”. Search is relatively pervasive; search is used extensively for such things as mineral deposits, police operations, pattern recognition, disease or contamination, medical diagnostics, and markets (Koopman 1999). For example, though anti-submarine warfare (ASW) is conducted differently today than it was in WWII, search techniques used in historical ASW have potential application to these other areas (modern anti-submarine warfare tactics mainly involve surveillance around convoys by sonar-equipped warships, helicopters, and inshore mobile units; that is not to say offensive search operations similar to what were used in the Bay of Biscay would not recur, only that more modern examples of such tactics are not prevalent (OASG 1977)). This means that historical search examples are good illustrations because they are available, detailed (in many cases), and can be verified historically.

3.2 Current Uses of Search

A logical extension of offensive search operations involves drug interdiction. As opposed to flying specialized search patterns in order to ambush elusive, non-cooperative targets, authorities now emphasize the fusion of data elements including intelligence information, radar warning networks, military aircraft, and specialized interdiction police ground teams.

Broad area searches for arms control treaty violations are yet another example of offensive search operations. Since using aerial assets (i.e., aircraft or satellites) to search an entire country for illegal military equipment is not cost-effective, a suitable alternative might be to employ aerial surveillance based upon known search theory principles including prior information about where and for what to look. In 1991, a study sponsored by the United States Congress examined the conditions under which aerial monitoring would make a significant contribution to arms control verification (U. S. Congress 1991). Most of the quantitative analysis documented in the congressional report is based upon classical search theory concepts mentioned later in this paper.

More recent military operations involving airborne offensive search pattern analysis include hunting for mobile scud missile launchers, terrorist combat groups, and smugglers.

3.3 Foundations of Search Theory

We define the “target” as the object of interest while the “searcher” is the object concerned with finding the target. McCue introduces a mathematical foundation for search theory by stating that “instantaneous sighting probabilities form a sighting potential: potentials integrate to form a lateral range curve, whose integral is the sweep width” (McCue 1990).

The concept of a “lateral range curve” in reference to a specific sensor is a graph of the probability of detection (POD) against the perpendicular distance from the sensor relative track to the target (which is the same as the object’s distance from the sensor to the closest point of approach).

The “sweep width” is the area underneath the lateral range curve, and represents a measure of search effectiveness of a given sensor (Koopman 1999). The National Search and Rescue Manual gives sweep widths for a variety of sensors in a variety of environments (NSRC 2000) (These tables are periodically updated via simulation exercises with the respective sensors; see (Edwards, et al. 1980)). Underlying the “sweep width” is the definite range law. The basis of the definite range law is that no probability exists to detect targets outside the specified range, while targets within the specified range are detected with certainty. The [effective search] sweep rate is the mean number of targets detected per unit time (Koopman 1999).

Dr Bernard Koopman of the U.S. Navy’s Operations Evaluation Group derived the “inverse cube law,” or ICL, to characterize detection probabilities of most of the search devices used during the War. This law maintains that the probability of detecting a target is inversely proportional to the cube of the distance between searcher and target. Dr Koopman proved the ICL was a fair compromise between the extremes of random and exhaustive search.

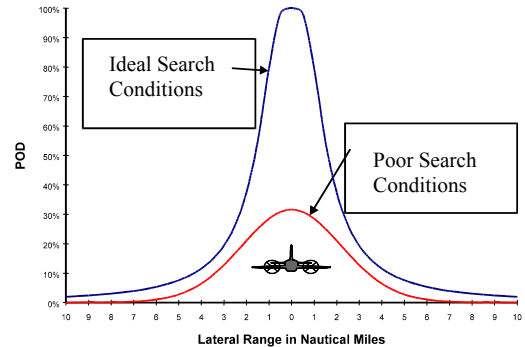


Figure 1: Sample Lateral Range Curves

For a continuously searching device, the ICL is characterized by an exponential cumulative distribution function that only depends on the sweep width, or effective range, of the device and the distance between searcher and target. This is important in an agent-based simulation, since aircraft and U-boat entities can possess this detection characteristic without placing constraints on their paths, speed, or general location.

The Search and Rescue (SAR) community still relies heavily on the ICL. The “first search curve” listed in the National Search and Rescue Manual, from which other curves are derived, is the ICL. The derivation of the ICL is characterized by four equations, a combination of infinite sums and integrals, that can be evaluated in closed form; Washburn states that the law is “therefore possibly holy” (Washburn 1989), and can be used even when the underlying assumptions are not directly verifiable. It is chiefly through the use of this law, and Koopman’s forestalling theorem (McCue 1990), that expressions for *a priori* detection probabilities can be derived for searchers and targets based upon realistic ranges of the equipment used and the distance between searcher and target (such as in a simulation).

3.4 Problems with Classical Approaches

There are problems associated with simulation models based just on the geometric analysis:

- Using the definite range law in practice can be unrealistic; search should have at least some degree of randomness;

- Real-world navigation is imperfect (especially when multiple searchers are involved); and
- Evading targets have unpredictable movement (Washburn 1989).

As the complexity of the search situation increases, so do the number of rigid assumptions needed to make the problem analytically tractable. For example, the target is constrained to move in a straight line or along an arc, or the searcher and the target must remain within the area to be searched for the duration of the analysis. Models based on detection-rates tend to be more robust than geometric models, as the events of detection in non-overlapping time intervals are assumed independent (Washburn 1989).

3.5 Computer Simulation and Search Theory

Operations Research analysts involved in search theory during WW II often used simulation. Some of these uses involved evaluating operational measures such as sweep rate, counter-measures, and probability of detection curves for a variety of search devices as a function of range to target (Morse and Kimball 1954). In his article on Koopman's life, Morse states that Kimball utilized simulation "to improve search procedures" (Morse 1982). In fact, one of the advantages of using computer simulation as opposed to other analytical models is that simulation allows one to relax mathematical assumptions required for those models.

4 MORE MODERN SEARCH THEORY?

A glimpse model involves models where target detection is attempted in discrete "snapshots" in time. For instance, a radar beam sweeping a circular area outside a search platform. Many glimpse models have been derived using geometric and stochastic methods. There appears to be a seemingly endless series of situational problems that must be addressed when employing analytical methods. The solution pattern for such problems has classically involved starting with a series of assumptions and deriving a sweep width based upon a distribution of effort (Washburn 1989).

Although analytical models tend to be complex, the real world is even more complex. A littoral environment (open ocean) is especially complex and variable in ways not easy to describe analytically.

It is therefore tempting to describe search capability through experiments where the searcher performs a specified maneuver in an attempt at detection, rather than trying to discover fundamental parameters of the environment and then reasoning deductively.

Alan Washburn, 1989

The above is the idea behind lateral range curves, and strongly suggest the utility of using simulation for investigating search.

4.1 Computer Simulation and Modern Search

The Computer-Assisted Search Planning System (CASP) is perhaps the best-known instance of computer simulation used in search theory. The United States Coast Guard introduced CASP in 1974 (Richardson and Discenza 1980). This system is based on Monte Carlo simulation and was devised specifically for search and rescue (i.e., target is stationary or subject to random drifts based on weather and littoral currents and is not trying to evade the searcher). CASP employs a probability map display overlaid on a search region. Within the search region each search grid square has some probability of target location. Its underlying structure is a Markov process with three-dimensional state space consisting of variables representing latitude, longitude, and search failure probability. CASP generates an initial probability distribution. This data are then updated taking into account wind and current information, as well as negative and false positive search results. Since CASP does not account for evading targets or for targets entering and leaving the search area, CASP is not applicable to the research effort described in this paper.

5 COMPUTER SIMULATION TO DETERMINE OPTIMAL SEARCH PATTERNS

The United States Coast Guard teaches personnel that choosing an appropriate search pattern involves many factors and the final choice is highly dependent upon the given scenario (Training Center Yorktown 2002). The National Search and Rescue Manual lists five search pattern types of interest to this effort. These are the Parallel, Creeping Line, Square, Sector, and Barrier Patrol patterns (NSRC 2000). These patterns are examined with respect to the number of U-boats sighted by patrol aircraft as a measure of search efficiency. Obviously, a valid measure of search efficiency must account for search resources used and target density in the search area. Although the U-boat model is stochastic on a day-to-day simulation basis, these two quantities are constant enough on the monthly level, given the same goals and abilities from replication to replication, to support our claim that comparing U-boat sightings is justified when comparing search efficiency ratings. If, as Washburn contends, such comparisons are analytically intractable, it stands to reason that this type of problem is a candidate for our agent-based simulation approach.

5.1 Search Patterns Defined

Each of the five search patterns from the National Search and Rescue Manual are next described. For each of the five patterns, a figure is provided depicting the pattern and the key assumptions are provided. Each figure includes a commence search pattern (CSP) point.

When the point of last contact with the target (datum) is not known with a high degree of certainty and the search area is large, either the parallel (Figure 2) or the creeping line (Figure 3) search is preferable. The parallel pattern is most desirable when the target is equally likely to occupy any part of the search area.

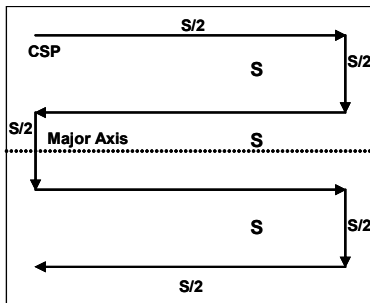


Figure 2: Parallel Search Pattern

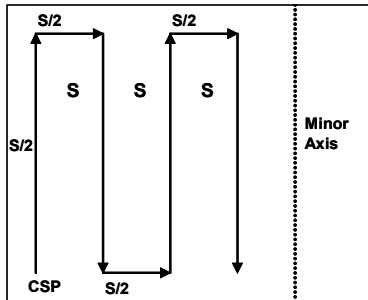


Figure 3: Creeping Line Search Pattern

The creeping line pattern, on the other hand, is typically employed when the target is more likely to be in one end of the search area than the other.

When the point of last contact is well known or established within close limits, the square (Figure 4) and sector (Figure 5) search patterns are preferable. The square pattern is used when uniform coverage of the search area is desired, while the sector search is used in scenarios where the target is difficult to detect.

Finally, when the target is fast moving or when a strong water current is present in the search area, the barrier patrol search pattern (Figure 6) is preferred.

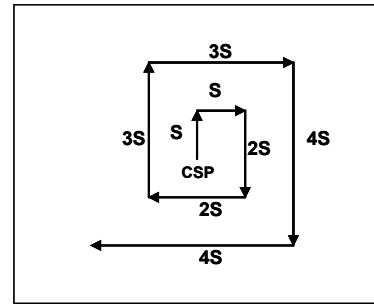


Figure 4: Square Search Pattern

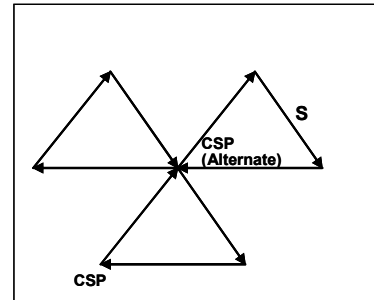


Figure 5: Sector Search Pattern

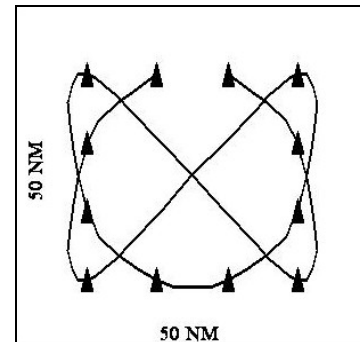


Figure 6: Barrier Patrol Search Pattern

5.2 Bay of Biscay Simulation Assumptions

The Bay of Biscay simulation was written in JAVA. Agent and simulation design data was compiled according to the following hierarchy: 1) historical fact as found directly from sources credited to allied and German participants; 2) published studies directly related to the offensive search in the bay; 3) data derived from raw numbers in one or more of the preceding sources; and 4) good judgment (operational expertise) when the three previous sources fail or contradict one another.

Within the simulation, “day” is defined as the time between nautical dawn and nautical dusk (i.e., the sun is above -12° with respect to the horizon). Detection sensors used by both aircraft and U-boats conform to the inverse cube law. Aircraft and U-boat agents are independent. For each iteration, a 12-month warm-up period was used fol-

lowed by six months of data collection. The six-month period of October, 1942, through March, 1943 was used in the scenario. This period involves homogeneous use of detection devices thus was the best period upon which to base our analysis.

Initially, 70 U-boats are in place, and replacements enter the Bay from the North Atlantic in numbers consistent with history (McCue 1990). The U-boats initialize uniformly distributed throughout the Bay, half heading to the North Atlantic, and half heading to their homeport. Each U-boat is assigned to one of five homeports on the west coast of France. The U-boats are distributed evenly among the ports. Each U-boat leaves port with 30 days of supplies and returns from operations in the North Atlantic with no supplies remaining. U-boats move at 10 knots surfaced and 2.5 knots submerged and must spend a minimum of 3 hours surfaced for each 100 nautical miles (NM) traveled to fully recharge their batteries. U-boats were diesel powered with limited battery capacity for submerged operations. Refueling at sea is implicitly accomplished by allowing a 0.25 probability of each U-boat agent extending time in the North Atlantic by 30 days. U-boats will submerge immediately upon detecting an aircraft.

The 40 Allied aircraft operate out of Plymouth, England, and will standoff from the coast of France to avoid enemy air patrols and escorts. There is no attrition due to accident or anti-aircraft defenses. Aircraft movement is 120 knots, and each aircraft will fly up to 70% of its fuel load, or until it has expended its munitions, whichever occurs first. An aircraft can detect a U-boat only when the U-boat is surfaced, and will attack the U-boat upon detection, expending its entire payload of munitions. Maintenance and weather cancellations occur before take-off only, and aircraft sortie take-off times are randomly scheduled to occur once in a 24-hour period while maintaining a minimum of 12-hours between landing and take-off for each aircraft.

For this effort, we ran two experiments. In the first experiment, aircraft search a 200 x 350 NM² area subdivided into 50 x 50 NM² grids. The grids are non-overlapping and at least one aircraft per day is assigned to search each grid. The search patterns were varied between those described previously. The simulation was run with 20 iterations per search pattern, and monthly statistics on the number of U-boats sighted by aircraft were collected.

In the second experiment, aircraft search the same 200 x 350 NM² area. This time, the total area is subdivided into 100 x 100 NM² grids. The grids are overlapping (new grid areas begin 50 NM to the right of each grid’s left-most side and 50 NM to the bottom of each grid’s top-side), and at least one aircraft per day is assigned to search each grid. The search patterns were again varied between those described previously. The simulation was run with 30 iterations per search pattern, and monthly statistics on the number of U-boats sighted by aircraft were again collected.

6 RESULTS

Simulation output was analyzed using the SAS JMP statistical software package. Figures 7 and 8 were generated by JMP.

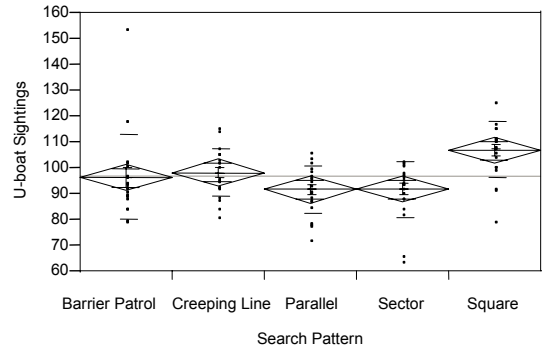


Figure 7: Sightings by Pattern – Non-Overlapping

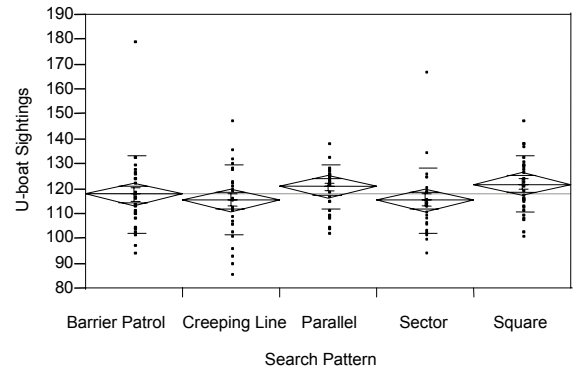


Figure 8: Sightings by Pattern – Overlapping

The simultaneous means comparison between search patterns are shown in Table 1. Letters in columns 2 and 3 signify statistical equivalence. Rows with common letters indicate no statistical difference between search methods. For instance, the square and barrier patrol patterns are equivalent (both have “A”), and barrier patrol and creeping line are equivalent (both have “B”). However, square and creeping line are not equivalent; they do not share a common letter.

Table 1: Means Comparison – Non-Overlapping

Search Pattern		Mean Sightings
Square	A	106.9
Barrier Patrol	A B	98.3
Creeping Line	B	96.4
Sector	B	91.9
Parallel	B	91.7

The most important value measure for our purposes is “U-boat sightings.” Figure 7 is a graphical representation of simultaneous confidence intervals for each search pat-

tern. Simulation output was analyzed at the $\alpha = 0.05$ level (i.e., a 95% confidence interval). The top three performing search patterns—square, barrier patrol, and creeping line—were re-run with 30 iterations per pattern. This data is summarized in Table 2. As in Table 1, rows with common letters indicate no statistical difference between search methods.

Table 2: Means Comparison – Non-Overlapping

Search Pattern		Mean Sightings
Square	A	105.9
Barrier Patrol	B	97.3
Creeping Line	B	91.4

Our results indicated the most useful pattern with regard to U-boats sighted is the square pattern. The same analysis was conducted in the overlapping case. Figure 8 shows a graphical representation of simultaneous confidence intervals for each search pattern. Simulation output was again analyzed at a confidence level of 95%.

The key insight from the overlapping search pattern experiment is that the pattern no longer holds as important a role. Although the square pattern appears better than the parallel in terms of sightings, the difference is not statistically significant (again based on these preliminary results). Table 3 summarizes the numerical results.

Table 3: Means Comparison – Overlapping

Search Pattern		Mean Sightings
Square	A	122.1
Barrier Patrol	A	121.0
Creeping Line	A	118.0
Sector	A	115.6
Parallel	A	115.6

7 CONCLUSIONS

With reference to the non-overlapping case, differing the search pattern impacts search efficiency. Though it was not our objective to rank the patterns, it is worth noting that the square pattern produced the most U-boat sightings. The creeping line pattern came in second, though was only statistically equivalent in the experiment with 20 iterations. This result may be due to the fact that these two patterns, more so than the others tested, cause aircraft to spend most of their time on a 90-degree approach to the target tracks. The fact that differences in the number of sightings exist due to simply varying the search pattern used is counterintuitive. The assumptions the SAR community typically makes when selecting patterns has been relaxed; for example, aircraft have no information about where U-boats might be located other than assuming general east-westerly headings. Also, the same number of aircraft is searching the same area with no programmed advantage in terms of detection.

In the overlapping case, differing the search pattern does not impact search efficiency. This is an intuitive result; even if differing search patterns in the non-overlapping case did produce significant sighting differences, one could assume that such differences would be overcome by the overlapping nature of the routes in this scenario. What is counterintuitive is the higher mean numbers of sightings than for the non-overlapping case. The overlapping case represents a less efficient search strategy, since the same regions are covered with more resources a lot more often (that is, coverage is duplicated). This may be caused by the fact that U-boats do not stray from their east-west transit strategies, thus allowing more aircraft access to their actual routes. Since the aircraft have no knowledge of U-boat locations during their search, this emphasizes the point that more efficient search methodologies are not necessarily “top performers” with reference to value measures of interest to the searcher.

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AUTHOR BIOGRAPHIES

LANCE CHAMPAGE is a Major in the United States Air Force and a Ph.D. student within the Department of Operational Sciences, Air Force Institute of Technology. He has a B.S. degree in Mathematics and Biomedical Engineering from Tulane and an M.S. in Operational Sciences from AFIT. His research interests include agent-based modeling and verification and validation methodology. His email address is lance.champagne@afit.edu.

R. GREG CARL is a Captain in the United States Air Force and a M.S. student within the Department of Operational Sciences, Air Force Institute of Technology. He has a B.S. degree in Mathematics. His research interests include agent-based simulation. His email address is ronald.carl@afit.edu.

RAYMOND HILL is an Associate Professor of Industrial and Human Factors Engineering with the Department of Biomedical, Industrial, & Human Factors Engineering of Wright State University where he runs the Advanced Modeling, Optimization, & Systems Laboratory. His Ph.D. is from The Ohio State University. His research interests include agent-based modeling, applied simulation modeling, and applied optimization modeling. His email address is ray.hill@wright.edu.

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