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TUTORIAL ON PORTFOLIO CREDIT RISK MANAGEMENT

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ABSTRACT

The distribution of possible future losses for a portfolio of credit risky corporate assets, such as bonds or loans, shows strongly asymmetric behavior and a fat tail as the consequence of the limited upside of credit (the promised coupon payment) and substantial downside if the corporation defaults. Because of correlation, it is not possible to fully diversify away this fat tail. Detailed correlation models require Monte Carlo simulation to determine the loss distribution for a credit portfolio. This tutorial covers the basics of credit risk modeling including an overview of the credit markets, a summary of what data are available for defining and calibrating models, and a discussion of key modeling questions. Finally a detailed discussion of simulation methods used in calculation credit portfolio loss distribution and related credit risk measures is presented.

1 INTRODUCTION

Portfolios of bonds or loans generally show asymmetric returns, with limited upside and substantial but rare downside risk associated with multiple corporate defaults. Accurate assessment of how large potential losses on a credit portfolio may be plays a key role in the management of financial institutions with large credit portfolios. For example, a large bank with tens or hundreds of thousands of loans will often be required to have sufficient capital to withstand 99.9% of potential losses. Questions of to whom to lend and what rates to charge are often linked to the effect of the loan on the capital requirement and the associated return on capital.

This tutorial focuses on a discussion of the fundamental building blocks on which models for managing credit portfolio can be built. First a brief discussion of the credit markets and various institutions with interests in credit risk management is provided to give context and motivation for studying this topic. This is followed by a survey of the kinds of data available to analyze credit risk and to build and calibrate models. The next section presents a summary of various modeling questions, including how to model and measure default probability and how to describe its stochastic evolution. Finally, role of simulation in credit portfolio pricing and risk management is discussed and several simulation approaches are described.

Further information on the topic of modeling credit risk can be found in the books by Bluhm et. al. (2002), Duffie and Singleton (2003), and Schönbucher (2003). A collection of technical articles related to credit risk has also been compiled by Gordy (2003). For a more general discussion of simulation methods in finance see Glasserman (2003) and Jäckel (2002). Several articles of interested related to credit risk modeling can also be found on the Moody's KMV website including an overview of portfolio management of default risk by Kealhofer and Bohn (2001).

2 CREDIT MARKETS

Credit risk is the risk of losing contractually obligated cash flows promised by a corporation, financial institution, government, etc. (the counterparty) due to default on the debt obligation. Defaults are usually associated with a credit event such a bankruptcy or reorganization, although delinquency in payment may also be considered a credit event even if there is not a formal bankruptcy.

The most common corporate credit instruments are bonds and loans. Equity (e.g. common stock) is not considered a debt instrument because there are no contractually obligated payments, and it has only a residual claim on any remaining value of a firm after all the debt has been paid. There are a number of instruments, such as convertible bonds, preferred shares and equity default swaps which have properties of both debt and equity. Firms issues bonds or borrow from banks in the form of loans to finance projects without giving up ownership of the potential growth and revenue that those projects might produce. The field of corporate finance studies questions such as the optimal strategy for a firm to issue debt versus equity.

Investors in corporate debt seek stable cash flows and investments that hold their value. The predominant risk is that of default which can be diversified away by holding a large portfolio of bonds and loans across a range of industries, countries and sizes of firms. However, measuring the degree to which a portfolio has been diversified and the remaining risk requires sophisticated modeling.

Over the last decade there has been very rapid growth in the use of quantitative methods for assessing credit quality and credit portfolio risk. Simultaneously with the acceptance of these models has come growth in trading in the credit and credit derivatives markets. Traditionally corporate bonds, and especially loans, have been illiquid instruments with limited trading. Recently the markets have become more liquid and the pricing more reliable. In particular, the introduction of credit default swaps (CDS), a synthetic insurance contract written on a corporate name that pays if the firm has a credit event, has lead to a rapidly growing market as these contracts provide an easy method to hedge default risk. Other areas of growth include collateralized debt obligations (CDOs) that securitize cash flows of a portfolio of credit risky assets into investments of varying degrees of risk.

The institutions most interested in manage credit risk are commercial banks, investment banks, pension and mutual funds, asset managers and hedge funds. Commercial banks make loans to corporations, investment banks underwrite and sell bond issuance, and investors purchase the bonds and loan securitizations for their investment portfolios.

3 DATA

The ultimate key to any successful model for credit risk is availability of data to specify and calibrate the model. Traditionally there has been only very limited data widely available. This is because the debt markets have until recently not been very liquid, so that for many names market price data did not really exist, and details of the trades that were made were not disclosed. Default data was also very difficult to assemble. While banks have there own data about defaults within their own portfolio, these data were not widely shared. Also, banks often have legacy loan computer systems that are difficult to integrate, so the data management task has been one of great difficulty. Data on recovery on default loans and bonds has proved equally difficult to track.

Some key areas of data required to build sophisticated models are:

- Price data for bonds, loans, CDS, and equities
- Default data on which names have defaulted
- Recovery data
- Ratings data from the ratings agencies (Moody's, S&P and Fitch)
- Financial statement data including information about liabilities as well as industry and country of operation, etc.

4 MODELING ISSUES

The key modeling questions cover a range of issues form default probability to correlation of defaults to recovery. Because corporate behavior is so complex and the reasons for issuing debt and for default are so varied, models tend to rely on empirical data and economic intuition over mathematical processes. In some cases the mathematical model is chosen for its analytic tractability and is calibrated and adjusted to match empirical evidence. Several keys points to model are:

- The cumulative default probability of a firm as a function of time
- The stochastic process describing the evolution of credit quality of a firm through time
- The correlation of firms in the credit migration process
- The probability distribution of recoveries on defaulted names
- The relationship between the price of debt and the default probability as well as other relevant factors
- The relationship between price implied default probabilities (risk neutral) and actual default probabilities
- The relationships across the bond, loan, CDS and equity markets.

5 SIMULATION IN CREDIT RISK

For very large, homogenous portfolios, it is possible to derive a limiting distribution for the portfolio losses (Vasicek 1991). For portfolios that are smaller or are inhomogeneous with respect to position size, default probability, default correlation or recovery, it is usually necessary to use Monte Carlo simulation to determine the loss distribution unless substantial simplifying assumptions are made.

For large inhomogeneous portfolios, the standard Monte Carlo simulation of the underlying stochastic processes may require days of computation time to provide sufficiently accurate calculations of tail events and tail statistics. This can substantially limit the feasibility of stress testing portfolios around composition or parameters. Therefore, variance reduction methods that can reduce computation time from days to minutes or hours are of substantial interest. In the tutorial we consider methods of importance sampling, such as described by Glasserman and Li (2003) and Morokoff (2004) and control variates (Tchistiakov et al. 2004).

Other interesting simulation questions include the effects of the credit migration process and correlations on the joint default probabilities that have been described in Morokoff (2003).

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