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INITIAL TRANSIENT PROBLEM FOR STEADY-STATE OUTPUT ANALYSIS

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ABSTRACT

This tutorial is concerned with providing an overview of the key issues that arise in consideration of the initial transient problem for steady-state simulations. In addition, we will discuss the related problem of construction of low-bias estimators.

1 INTRODUCTION

A central problem in the planning of steady-state simulations concerns the issue of how to deal with the presence of the so-called "initial transient". In particular, suppose that $X(t)$ is the state of the simulation at time t (so that X(t) includes full information on the physical state of the system at time t, as well as the full event schedule at that time). Assume that $X = (X(t) : t \geq 0)$ in an S-valued process that possesses a steady-state, in the sense that

$$X(t) \Rightarrow X(\infty) \tag{1}$$

as $t \to \infty$, where $\Rightarrow$ denotes weak convergence. For a given performance measure $f : S \to I\!\!R$ of interest (such as, for example, the function f that sends a simulation state $x \in S$ into the total workload $f(x)$ associated with state x), the goal of a steady-state simulation is to compute the steady-state expectation

$$\alpha = Ef(X(\infty)).$$

Since the distribution of $X(\infty)$ is unknown, one typically initializes the simulation of X via a distribution for $X(0)$ that is not characteristic of steady-state behavior. For example, production, inventory, and queueing simulations are often initialized so that at time $t = 0$, the content of the system (whether measured in units of inventory, workpieces, or customers) is zero.

As a consequence, the initial segment of the simulation is generally not representative of steady-state behavior. This non-representative initial portion of the simulation is called the "initial transient". A closely related concept is that of "initial bias." In particular, it is common (in the presence of assumption (1)) that a strong law of large numbers for $(f(X(t)) : t \geq 0)$ will hold, so that

$$\frac{1}{t}\int_0^t f(X(s))\,ds \to \alpha \quad \text{a.s.} \tag{2}$$

as $t \to \infty$. Given the law of large number (2), the time-average estimator $\alpha(t) = t^{-1}\int_0^t f(X(s))\,ds$ is the natural estimator for α based on having simulated X to time t. Because of the presence of the initial transient,

$$E\alpha(t) \neq \alpha.$$

The bias $E\alpha(t) - \alpha$ is known as "initial bias."

This advanced tutorial will be concerned with discussing the state-of-the-art for both the initial transient and initial bias problems. Among the key issues to be addressed are :

1. What are the types of simulations in which initial transient and initial bias are often particularly troublesome?
2. Are there any theoretical approximations that shed light on the duration of the initial transient?
3. Are there any statistical tools that can be used as initial transient diagnostic?
4. What is the role of "perfect simulation" (also known as "exact simulation") in this context?
5. Are there classes of simulations for which one can reliably compensate for the initial bias?
6. How does the presence of long-range dependence affect the design of steady-state simulations (in view of initial transient)?
7. What, if anything, changes in the setting of multiple parallel processors?

The references section below contains some papers and books that are representative of the substantial literature on these two important problems.

REFERENCES

Asmussen S., P. W. Glynn, and H. Thorisson. 1992. Stationarity detection in the initial transient problem. *ACM Transactions on Modeling and Computer Simulation* 2:2:130-157

Bratley P., B. L. Fox, and L. E. Schrage. 1987. *A Guide to Simulation*. New York: Springer-Verlag Inc.

Deutsch, S. J., J. E. Richards and C. Fernandez-Torres 1983. The initial transient problem—estimating transient and steady state performance. In *Proceedings of the 1983 Winter Simulation conference*, ed. S. Roberts, 345-348. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers.

Fishman, G. S. 1973. Bias Considerations in Simulation Experiments. *Operations Research* 20: 785-790

Fishman, G. C., L. W. Schruben, W. D. Kelton, and P. D. Welch . 1981. The initial transient in steady state simulation. In *Proceedings of the 1981 Winter Simulation conference*, ed. T. I. Ören, 113-120. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers.

Gafarian, A. V., C. J. Ancker Jr. and T. Morisaku. 1978. Evaluation of Commonly Used Rules for Detecting 'Steady State' in Computer Simulation. *Naval Research Logistics Quarterly* 25: 511-529

Law A. M., and W. D. Kelton. 1991. *Simulation Modeling and Analysis*. Boston: McGraw Hill.

Ockerman D. H., and D. Goldsman. 1997. The Impact of Transients on Simulation Variance Estimators. In *Proceedings of the 1997 Winter Simulation Conference*, ed. S. Andradóttir, K. J. Healy, D. H. Withers, and B. L. Nelson, 234-239. Piscataway, New Jersey: Institute of Electrical and Electronics Engineers.

Pawlikowski K. 1990. Steady-state simulation of queueing processes: survey of problems and solutions. *ACM Computing Surveys (CSUR)*, 22(2):123-170

Schruben, L.W. 1082. Detecting initialization bias in simulation output. *Operations Research* 30: 569-590.

Wilson, J. R., and A. A. B. Pritsker. 1978. A survey of research on the simulation startup problem. *Simulation* 31:55-58.

Wilson, J. R., and A. A.B. Pritsker. 1978. Evaluation of start-up policies in simulation experiments. *Simulation* 31:79-89.

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