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SPORTS ANALOGY FOR MODELLING OF COMBAT IN THE AIR DOMAIN

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ABSTRACT

Aggregated models of Air Warfare invariably rely on a user input value for probability of success (kill) or 'exchange ratio' in Air-to-Air Combat. There is limited historical data available to validate these parameters for engagements between non-peer opponents. This paper explores the potential for gaining insights to non-peer Air-to-Air outcomes from the world of sport, and examines the results from Association Football competitions in England.

1 INTRODUCTION

Most discrete-event simulation models have some stochastic elements that are intended to mimic the probabilistic nature of the system being considered. A close match between the input model and the true underlying probabilistic mechanism associated with the system is desirable for successful modelling to be undertaken (Leemis 2004). Most simulation texts contain descriptions of input modelling techniques and procedures which can be used to select the most appropriate distributions to be used within the simulation (e.g. Law and Kelton 2000).

Ideally, sufficient observed data will be available for the actual system being modelled to validate the input distribution being used within the model to simulate the system. In some circumstances however there may be very little actual data available for the actual system, in which case standardised theoretical distributions such as Exponential, Normal or Poisson are usually used.

In the military domain, most simulation models at the Operational or Campaign level model combat at a highly aggregated level. Relatively simple simulations will tend to utilise a single 'attrition' value for a force and utilise a model such as Lanchester's Equation (Lanchester 1916) to derive combat outcomes. Larger more sophisticated simulations such as JWARS (Stone 2001) or JTLS (Bowers 2003) will represent engagements between different entities in much more detail, but will still tend to rely on an input variable for 'probability of kill' or 'exchange ratio' to be used in evaluating the outcome of combat engagements.

Empirical evidence suggests that aggregation modelling for the Land Campaign is a useful tool for understanding some of the dynamics of large engagement and for assessing potential outcomes of ground combat (Davis 1995).

'Probability of kill' and subsequent exchange-ratios for engagements between platforms and weapon systems can be obtained from detailed sensor and weapon modelling and by observing synthetic and live Exercises. Data derived from modelling and from observing Exercises will however have artificialities, as these methods are unlikely to represent the true 'fog of war' and the exact doctrine, expertise and morale which would be exhibited by a potential adversary.

Historical analysis has been used extensively for estimating attrition figures for the Land Campaign, and for developing empirical evidence for understanding some of the human factors in combat and developing estimates for the level of combat degradation occurring in real combat situation (Roland 1987, 1991). This work has been extended to assess the impact of human factors issues, such as shock, surprise, breakthrough and manoeuvre, on the outcome of the Land Campaign (Roland 1996).

While Air Combat (both Air-to-Air and Surface-to-Air engagements) could be modelled as aggregations of individual stochastic duals, the outcome of modern Air Combat relies heavily on aircrew situational awareness and the level of support provided by other support assets. Consequently, higher level aggregations that principally rely on data from equipment performance may provide fewer insights into the Air Campaign than similar level modelling may achieve for the Land Campaign.

When choosing input parameters for use within Campaign level simulations, there have been relatively few recent real-world historical incidents of Air-to-Air combat with which to validate a model. While the factors which influence the outcome of Air-to-Air engagements have been examined (Spink 1988) the historical analysis has relied principally on data from World War II and Korea.

Recent military Operations in Iraq (1992), Bosnia, Kosovo and Iraq (2003) have seen few Air-to-Air engagements (Kenney 1993, Lambeth 2001). While the engagements were few in number, the Iraqi and Serbian Air Forces suffered high relative attrition in those engagements which did occur. While the Iraqi and Serbian Air Forces had capable air platforms in their inventory, in comparison to Coalition Forces they had limited training, a lack of support assets and a limited doctrine. Consequently this placed the Iraqi and Serbian Air Forces at a considerable disadvantage in a combat situation.

Maintaining the combat edge for modern Air Campaign will take considerable political commitment and financial investment. In practice, few nations will be able to maintain this commitment resulting in the creation of a number of 'tiers' of Air Combat capability. Consequently, it is increasingly likely that future conflicts may be between non-peer nations, while the assets may be similar, their effective capabilities may be very different.

One of the challenges for the higher-level Air Campaign modeller is therefore to be able to derive a relationship between 'exchange-ratio' and relative peer group capabilities. Methods currently used include nomographs of platform performance and single static score values.

Nomographs comprise columns representing platform capabilities. The relative exchange ratio is then obtained by reading the value from a logarithmic scale. For example, in Figure 1, the exchange ratio between 'BlueSystem' and 'RedSystem1' would be 1 to 1 due to the parity of the column values. An engagement between 'BlueSystem' and 'RedSystem2' would result in an exchange ratio of around 2.5 to 1 in favour of Blue. Similarly, an engagement between 'BlueSystem' and 'RedSystem3' would result in an exchange ratio of around 2.5 to 1 in favour of Red.

Static scores allocate a single value to each platform type. Air static score methods are often based on parameters such as wing area, combat weight and thrust (Campbell 2005). The static score value is then normalized and used to generate an exchange ratio between two dissimilar aircraft types. The exchange ratio is typically calculated as $10^{(a-b)}$ where a and b are the static scores for the two engaging platforms. For example, if system 'a' has a normalized score of 1.4 and system 'b' has a normalized score of 1.2, then the exchange ratio would be obtained as 1.58 to 1 in favour of system 'a'.

While intuitively reasonable, there is very limited historical data available to validate these types of relationships. Although there is little historical data for air-to-air combat, there may however be data available from other fields which may exhibit similar characteristics.

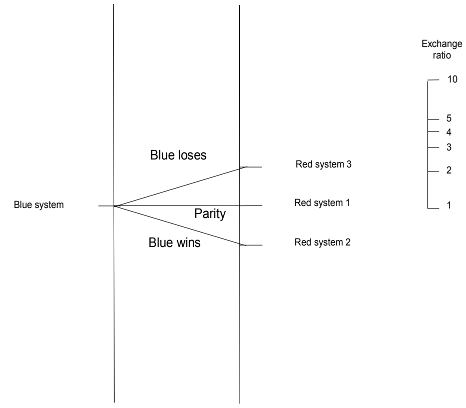

Figure 1: Example Nomograph

2 ASSOCIATION FOOTBALL IN ENGLAND

Most professional sports are designed to enable individuals or teams of broadly similar abilities to compete against each other. Consequently there are generally few opportunities to collect meaningful statistics from competitions involving non-peer competitors.

Association Football (soccer) in England has a hierarchical league structure with teams being promoted and relegated between leagues depending on their end of season positions. The process of promotion and relegation ensure that the best teams play in the higher divisions of the league structure. This is reinforced by the fact that teams in the higher divisions of the league attract the most revenue through television rights, sponsorship and gate receipts, and consequently can afford to pay the largest salaries to attract the top players.

The Football Association (FA) Cup dates back to 1871 and still maintains a central position in English soccer to this day. The FA Cup can be entered by any soccer team affiliated to the English FA. This 'open' competition allows the potential (however remote) for amateur players representing small towns and villages around the country to have the opportunity to play against the top international stars. The popularity of the FA Cup is such that typically over 600 clubs enter the competition each year (624 entered in 2003).

The competition is played on a 'knock-out' basis with the winning team from each match progressing to the next round. Matches are chosen by selecting two teams at random with the team selected first being designated the 'home' team and the team selected second being designated the 'away' team. The home team has the honour of hosting the match at their stadium. In the event of the match ending in a tie, the match is replayed at the away team's stadium. If this replay also ends in a tie, the match is continued to extra-time (over-time) and if the match is still tied, the winner is decided on a penalty 'shoot-out'.

The competition is arranged in seeded stages with the top professional teams entering the competition at the latter stages. The 44 teams from the top two professional leagues join the competition at the 'round of 64' stage, with the 48 teams from the 3rd and 4th leagues of the professional hierarchy joining two rounds earlier.

3 STUDY AIM AND ANALYSIS

The structure of the FA Cup competition therefore allows an examination of performance between team of different, but know peer groups. The purpose of this analysis is to investigate the different peer group performances and to assess if there are any structural relationships which may have applicability for addressing the analogous problem in the Air Combat domain.

By collating individual match results from the FA Cup, probability tables can be constructed for team performance in inter-league and intra-league matches. Taking matches from 1960 to 2003 (Collett 2003) there were 5729 matches involving at least one of the 92 top professional league teams. Table 1 illustrates the number of matches played between teams of each division during this period.

Table 1: Total Number of Matches Played

Division of team involved	Number of Matches Played
1st v 1st	815
1st v 2nd	981
1st v 3rd	382
1st v 4th	159
1st v others	72
2nd v 2nd	369
2nd v 3rd	309
2nd v 4th	189
2nd v others	69
3rd v 3rd	436
3rd v 4th	816
3rd v others	824
4th v 4th	331
4th v others	695

Table 2 illustrates the probability of a win given a 'home' tie. The convention used within the Table is that '1st v 3rd' implies that the team from Division1 is the 'home' team and the team from Division3 is the 'away' team. For example, the Table shows that a team from Division2 who play at home to a team from Divison1 have a probability of 0.297 of a win.

Table 3 illustrates the probability of a draw given a 'home' tie, and Table 4 illustrates the probability of a defeat given a 'home' tie. It should be noted that the summation of win, draw and lose probabilities for each cell will be unity.

Table 2: Probability of Win

Division of teams involved	Probability of win	Division of teams involved	Probability of win
1st v 1st	0.451	1st v 1st	0.451
1st v 2nd	0.667	2nd v 1st	0.297
1st v 3rd	0.740	3rd v 1st	0.235
1st v 4th	0.768	4th v 1st	0.143
1st v others	0.725	others v 1st	0.071
2nd v 2nd	0.457	2nd v 2nd	0.457
2nd v 3rd	0.609	3rd v 2nd	0.404
2nd v 4th	0.680	4th v 2nd	0.313
2nd v others	0.756	others v 2nd	0.080
3rd v 3rd	0.495	3rd v 3rd	0.495
3rd v 4th	0.620	4th v 3rd	0.391
3rd v others	0.743	others v 3rd	0.256
4th v 4th	0.495	4th v 4th	0.495
4th v others	0.667	others v 4th	0.245

The Tables show intuitively reasonable results. Being drawn at 'home' (and hence playing on the team's home ground in front of the teams own supporter's) is generally an advantage, and playing against teams from a lower division generally result in a greater probability of success. For intra-division games the split of win/draw/lose are broadly similar, although the probability of a 'result' (ie a game not ending in a draw) increases lower down the league hierarchy.

Table 3: Probability of Draw

Division of teams involved	Probability of draw	Division of teams involved	Probability of draw
1st v 1st	0.283	1st v 1st	0.283
1st v 2nd	0.210	2nd v 1st	0.265
1st v 3rd	0.200	3rd v 1st	0.335
1st v 4th	0.171	4th v 1st	0.169
1st v others	0.225	others v 1st	0.286
2nd v 2nd	0.251	2nd v 2nd	0.251
2nd v 3rd	0.173	3rd v 2nd	0.281
2nd v 4th	0.200	4th v 2nd	0.337
2nd v others	0.146	others v 2nd	0.280
3rd v 3rd	0.243	3rd v 3rd	0.243
3rd v 4th	0.223	4th v 3rd	0.285
3rd v others	0.173	others v 3rd	0.276
4th v 4th	0.205	4th v 4th	0.205
4th v others	0.176	others v 4th	0.307

Table 4: Probability of Loss

Division of teams involved	Probability of loss	Division of teams involved	Probability of loss
1st v 1st	0.265	1st v 1st	0.265
1st v 2nd	0.123	2nd v 1st	0.438
1st v 3rd	0.060	3rd v 1st	0.429
1st v 4th	0.061	4th v 1st	0.688
1st v others	0.050	others v 1st	0.643
2nd v 2nd	0.292	2nd v 2nd	0.292
2nd v 3rd	0.218	3rd v 2nd	0.315
2nd v 4th	0.120	4th v 2nd	0.349
2nd v others	0.097	others v 2nd	0.640
3rd v 3rd	0.262	3rd v 3rd	0.262
3rd v 4th	0.157	4th v 3rd	0.324
3rd v others	0.084	others v 3rd	0.468
4th v 4th	0.300	4th v 4th	0.300
4th v others	0.156	others v 4th	0.447

Table 5 illustrates the overall result (ie taking into account the replayed result from drawn matches) given a home match in the first match. In this case there are no significant differences in the proportion of wins between intra-division matches for different divisions.

Table 5: Overall Probability of Win

Division of teams involved	Probability of overall win	Division of teams involved	Probability of overall win
1st v 1st	0.566	1st v 1st	0.566
1st v 2nd	0.787	2nd v 1st	0.357
1st v 3rd	0.859	3rd v 1st	0.289
1st v 4th	0.900	4th v 1st	0.168
1st v others	0.902	others v 1st	0.118
2nd v 2nd	0.562	2nd v 2nd	0.562
2nd v 3rd	0.688	3rd v 2nd	0.490
2nd v 4th	0.784	4th v 2nd	0.387
2nd v others	0.870	others v 2nd	0.128
3rd v 3rd	0.588	3rd v 3rd	0.588
3rd v 4th	0.724	4th v 3rd	0.468
3rd v others	0.847	others v 3rd	0.303
4th v 4th	0.578	4th v 4th	0.578
4th v others	0.774	others v 4th	0.320

4 MODEL VALIDATION

Given the probabilities of Table 5 it is easy to construct a model to simulate the overall FA Cup competition. For this study a simple spreadsheet model was constructed using MicroSoft EXCEL. To test the face validity of the simulation, multiple replications of the model were executed to ascertain the probabilities of teams outside the top division reaching the final and winning the competition outright. The simulation showed teams from the second division reaching the final around 18% of occasions and actually winning the competition on around 7% of occasions. The simulation showed teams from outside the top two divisions reaching the final on less that 2% of occasions and actually winning the competition on less than 1% of occasions. Over the past 50 years, 7 teams from the 2nd Division have reached the final, with 4 of them going on to win the competition, and no team from outside the top 2 divisions has been beyond the semi-final stages. Consequently, the model would appear to give a reasonable representation for the performance of teams from lower divisions throughout the competition.

5 DISCUSSIONS

Table 6 illustrates the overall results in an exchange ratio format, combining the results for both 'home' and 'away' fixtures. It should be noted that intra-divisional matches invariably have an exchange ratio of 1-to-1.

Table 6: Probability of a Win Expressed as an Exchange Ratio

Division of teams involved	Overall exchange ratio
1st v 1st	1 to 1
1st v 2nd	1 to 2.51
1st v 3rd	1 to 3.66
1st v 4th	1 to 6.48
1st v others	1 to 8.26
2nd v 2nd	1 to 1
2nd v 3rd	1 to 1.49
2nd v 4th	1 to 2.31
2nd v others	1 to 6.76
3rd v 3rd	1 to 1
3rd v 4th	1 to 1.69
3rd v others	1 to 3.39
4th v 4th	1 to 1
4th v others	1 to 2.66

Figure 2 illustrates the exchange ratios plotted on a logarithmic scale. If these lines were to be parallel and equally spaced then there would be a simple relationship for matches between the divisions given by;

$P_{ac} = P_{ab}P_{bc}/((1-P_{ab})(1-P_{bc}))$ where P_{xy} is the probability of a team from division x beating a team from division y.

While the relationships works well at times, matches between teams from the higher divisions and teams from outside the top four divisions have a lower exchange ratio than would be expected from the linear relationship. Analysis from other environments suggests that for complex

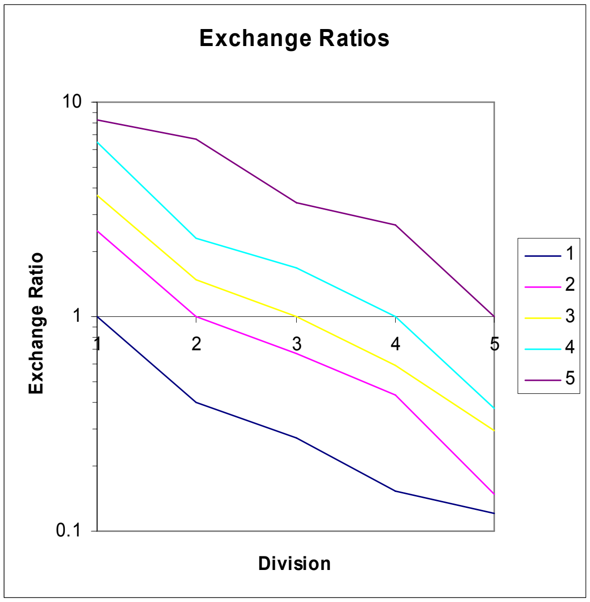

Figure 2: Exchange Ratio for Each Division

systems, power law relationships or scaling distributions may be more common than the more traditional statistical distributions (Willinger, Alderson, Doyle and Li 2004). Whilst fitting a power relationship may produce a better fit for the data, there may be a simpler explanation for these observations.

By the time 1st and 2nd Division clubs enter the competition, teams from outside the top four divisions will have had to have won two rounds in the main competition and a number of qualifying matches to have reached this stage. As a result, matches between teams from the top two Divisions and teams from outside the top four Divisions are rare, on average providing less than two matches a year. The teams from outside the top four divisions who progress to this section of the competition are therefore more likely to be 'better than average' non-divisional teams.

The largest exchange ratio is obtained for matches between teams from the 1st division and teams from outside the main league structure. The exchange ratio for these encounters is around 8.3 to 1. This is significantly lower however than the 33 to 1 exchange ratio observed during air-to-air engagements during the first Gulf War (Kenney 1993).

6 CONCLUSION

This study suggests that a linear relationship for association football matches between teams from different divisions may be a reasonable representation of likely 'exchange-ratio'. The relationship is however poor for matches between teams from the top divisions and teams from outside the main league structure. This is probably due both to the structure of the competition and the diversity in quality of teams from outside the major leagues.

The study illustrates however that results from professional sport can be used to develop exchange ratios in a similar way to those developed for use in air-to-air combat modeling. The results from English Association Football however do not produce exchange ratios as large as those observed during recent air-to-air combat engagements between non-peer air forces.

Consequently, the study would suggest a number of possible future avenues. Firstly, examining the potential to provide greater granularity to a team's ability, by subdividing teams in each division based on their relative leaguer position. Secondly, considering other sports that could be examined to ascertain the existence of enduring factors in the dynamics of non-peer engagements, particularly for sports which have the potential to derive large exchange ratios.

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