

## THE APPLICATION OF EVALUATION METHOD BASED ON HMM FOR RESULTS VALIDITY OF COMPLEX SIMULATION SYSTEM

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### ABSTRACT

According to the characteristic of randomization and sequential logic in complex simulation systems, a new evaluation method based on hidden Markov model (HMM) is presented, which applies multivariate statistical theory to quantitatively evaluate the results validity of complex simulation system. By importing matrix of observed state vector, the method enhance the clarity of describing scenario and running states of simulation systems, and ultimately implement an exploring approach to quantitative analysis for the results validity. Furthermore, quantificational evaluation criterion of results validity is given and the critical algorithm adopted in the process of quantificational evaluation is discussed in detail.

### 1 INTRODUCTION

The complex simulation system is the abstract of real world. To determine whether a simulation system should be used in a given situation, its credibility should be established by evaluating fitness for the intended use. In simplest terms, verification, validation, and accreditation (VV&A) are three interrelated but distinct processes that gather and evaluate evidence to determine, based on the simulation's intended use, the simulation's capabilities, limitations, and performance relative to the real-world objects it simulate (DMSO 2000). The decision to use the simulation will depend on the simulation's capabilities and correctness, the accuracy of its results, and its usability in the specified application.

The purpose of validation activities is to assure development of correct and valid simulations and to provide simulation users with sufficient information to determine if the simulation can meet their needs (Chew 2000).

The results validation process is critical to V&V Programs (Caughlin 2000). Results validation is defined as the comparison of M&S predictions with the observations of reference system for the purpose of ensuring the fidelity

which is used as a measure of M&S performance (Balci 1998). Identification of the observations of reference system is defined by the intended use of the M&S and the behaviors recorded in the data and allowed by the M&S.

Briefly based on the identification and analysis of observations of reference system, the results validation procedure is to compare the prospective outputs in detail, and analyze the causes for differences in outputs.

Presently, the acquirement of observations of reference system is essential prerequisite for the quantitative analysis on the results validity of simulation system. But it is very difficult to carry out the analysis procedure when complex simulation system is applied in the military application domain because the complete data is difficult to get when much future weapon and unknown environment factors are included in simulation system (Law 2001).

This paper presents a new method to quantitatively describe the running states of complex simulation system and the simulation scenario, and to construct the output of reference system based on the observed output in simulation exercise. The method provides effective support to measure running precision of simulation system by adopting confidence interval to analyze the existing interval of truth value of simulation system with a given scenario. Considering the simulation system applied in military domain is multivariate system, we extend the confidence interval to simultaneous confidence intervals and detailed discuss the process of applying Roy-Bose multivariate method to calculate the simultaneous confidence intervals of mean difference between the output of simulation system and that of reference system.

### 2 DESCRIPTION OF SCENARIO AND THE RUNNING STATES OF SIMULATION SYSTEM

For the complex simulation system applied in military domain, scenario based on the application objective and requirements describes the battle background, assumed con-

ditions and the evolution process in exercise. Briefly, scenario includes all information about simulation exercise, such as the types of entities, and the initial condition, constraint condition and end condition, and so on. Figure 1 indicates the relationship between the entities composed of simulation system based on High Level Architecture (HLA) and the simulation scenario.

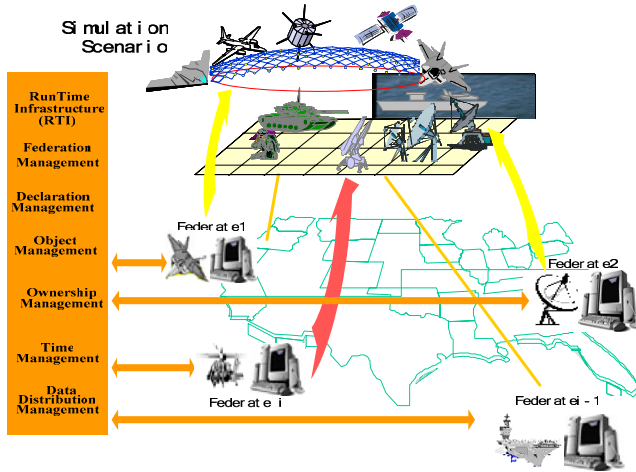


Figure 1: The Relationship Between Entities and Simulation Scenario

### 2.1 The Challenge for Quantitative Analysis on Results Validity

Although DMSO has developed a detailed data interchange format (DIF) to describe various elements of military operations in the simulation scenario, such as missions and tasks. As the primary criteria for results validity, scenario existing in text and graph table mode is impracticable to provide the effective support for quantitative analysis on results validity of complex simulation system.

Furthermore, in the military simulation system, it is impossible to get the output of reference system because of existing much future weapon and environment factor. Consequently, it is impractical to carry out the quantitative process of results validity (Li 2002).

During the process of simulation exercise, the entities and equipments sequentially update their running states within the required time according to the scenario. As known, the transition among running states during simulation exercise is driven by stochastic events which occur randomly. The characteristic is identical to the principal character of stochastic simulation system (Sanchez 2001). In order to quantify results validity, it is necessary to define an appropriate stochastic process with which to work. Thus, adopting the hidden Markov model (HMM) to quantitatively describe the scenario and running states of simulation exercise can visually reflect the characteristic while effectively supporting the quantitative analysis on results validity.

### 2.2 Description Method

As a doubly-stochastic process, HMM (Li 2001, Maillet 1999, Young 1988, Ephraim 1988, Hatzipantelis 1997) is derived from Markov chain. HMM applies Markov chain to represent the basic stochastic process and describes the statistical relationship between states and observed values by the other stochastic process. The HMM has the form

$$\lambda = (N, K, \pi, S, O). \tag{1}$$

Weapons and environments are abstracted as entities which are the elementary unit constituent parts of military simulation system. Thus, the running states of simulation system are comprised with a set of running states of all entities. By discretely describing the running states of entities, researchers can objectively embody the static characteristic of simulation system at any moment. As shown in figure 2, the transition between different states forms the dynamic characteristics of simulation system. Considering each state appearing in simulation scenario or simulation exercise satisfies strict sequential logic, equation (1) can be rewritten as (2)

$$\lambda' = (N, K, \pi', S', O'), \tag{2}$$

where  $\pi' = (1, 0, 0, \dots, 0)$ , and

$$S' = \begin{pmatrix} S_1 \\ S_2 \\ \vdots \\ S_{N-1} \\ S_N \end{pmatrix} = \begin{bmatrix} s_{11} & s_{12} & 0 & \dots & 0 & 0 \\ 0 & s_{22} & s_{23} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & s_{N-1,N-1} & s_{N-1,N} \\ 0 & 0 & \dots & \dots & 0 & s_{N,N} \end{bmatrix} \begin{cases} 0 \leq s_{ij} \leq 1 \\ s_{ij} = 0 (i \neq j, i \neq j-1) \\ \sum_{j=1}^{j=N} s_{ij} = 1 \end{cases} \tag{3}$$

$N$  and  $K$  respectively represent the number of states and the number of observed vectors confirmed by Subject Matter Experts (SMEs).

We apply Thile method to construct  $O'$  as correlation coefficient matrix (Hasegawa 2000), which represents discrepancy between observed vectors value of simulation system states and that of reference system. The detailed process of constructing  $O$  is discussed in section 3.1.

For military simulation system, the output of entities is the unique identification to describe the running states and characteristics of simulation exercise. Based on the hidden Markov model, we introduce the  $A^{(m)}$  as observed vector matrix to quantificationally represent the observed vector of simulation system as the output generated by the  $m^{th}$  exercise as Figure 2.

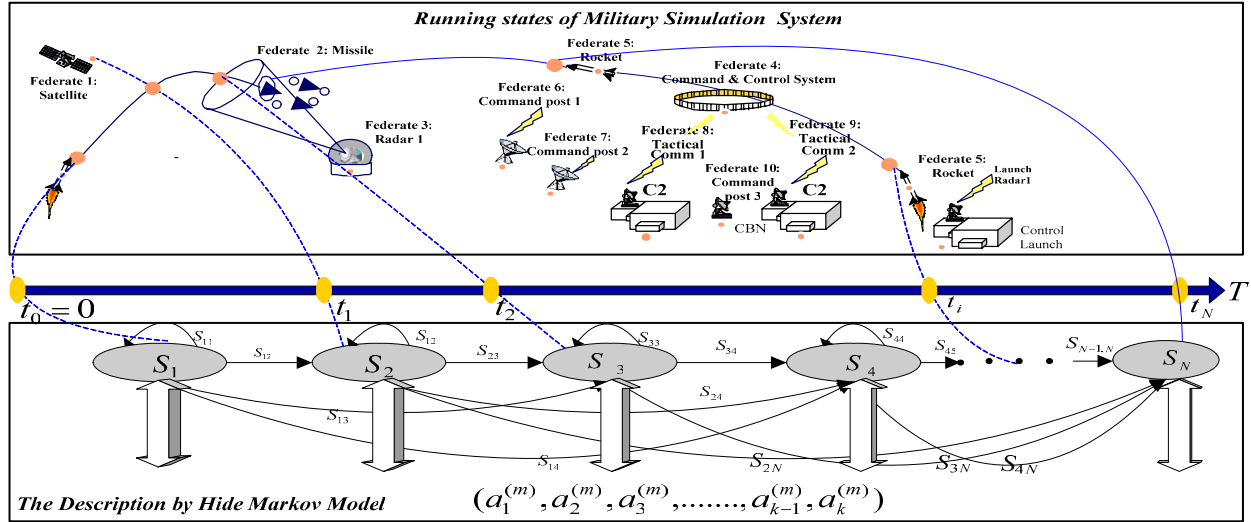


Figure 2: The Method of Describing States of Military Simulation System By HMM

It should be noted that as the abstract description mode, the running states including in simulation exercise can be embodied by the observed vector matrix as following,

$$A^{(m)} = (a_1^{(m)}, a_2^{(m)}, \dots, a_{k-1}^{(m)}, a_k^{(m)}) = (a_{ij}^{(m)})_{N \times K} = \begin{bmatrix} a_{11}^{(m)} & a_{12}^{(m)} & \dots & a_{1j}^{(m)} & \dots & a_{1k}^{(m)} \\ a_{21}^{(m)} & a_{22}^{(m)} & \dots & a_{2j}^{(m)} & \dots & a_{2k}^{(m)} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1}^{(m)} & a_{i2}^{(m)} & \dots & a_{ij}^{(m)} & \dots & a_{ik}^{(m)} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{N1}^{(m)} & a_{N2}^{(m)} & \dots & a_{Nj}^{(m)} & \dots & a_{Nk}^{(m)} \end{bmatrix}, \quad (4)$$

where  $i = 1, 2, \dots, N; j = 1, 2, \dots, K; m = 1, 2, \dots, M$ .

In  $A^{(m)}$ , the  $i^{th}$  row represents the observed vector defined with given scenario under  $S_i$  state, and the  $j^{th}$  column denotes the observed value of vector  $a_i^{(m)}$  under all states in simulation exercise.

It should be noted that  $N$  and  $K$  respectively specify the resolution of dividing scenario and the complete extent of representing the running states of simulation scenario or simulation exercise. By importing the  $A^{(m)}$  as observed vector matrix based on  $S'$  and  $O'$ , the simulation scenario and running states of simulation exercise are closely combined with, which provides the abstract mathematical basis to carry out quantitative analysis for results validity.

### 3 THE EVALUATION CRITERIA FOR RESULTS VALIDITY AND COMPUTING METHOD

In the simulation domain, researchers calculate accuracy of simulation by comparing the  $A^{(m)}$  to the observed vector values of reference system. The accuracy is used to vali-

date the results of simulation exercise with given scenario and expressed by acceptable range of accuracy.

The following sections will discuss the method to construct the observed state vector of reference system, and the algorithm adopted in the quantificational evaluation process.

#### 3.1 Constructing Model

Practically, it is impossible to get all the observed vector values of reference system used to be compared with (Lewis 1997). Fortunately, during the VV&A procedures previous to results validity, there must be many types of verify data. Although these verify data is insufficient to construct the observed state vector of reference system, we incorporated verify data as constituent parts of observed state vector of simulation system to solve the problem.

In order to solve above challenges discussed in 2.1, we defined  $B^{(m)}$  as observed vector of reference system which is derived from  $A^{(m)}$  incorporated with these verify data.

In order simply describe the construction process, we also apply  $B^{(m)}$  to represent the  $A^{(m)}$  incorporated with these verify data, where

$$B^{(m)} = (b_{ij}^{(m)})_{N \times K},$$

$$b_{ij}^{(m)} = \frac{1}{m} \sum_{l=1}^m a_{ij}^{(l)}, \quad (5)$$

where  $i = 1, 2, \dots, N; j = 1, 2, \dots, K; m = 1, 2, \dots, M$ .

As result, the mean value of  $B^{(m)}$  is

$$\overline{B^{(m)}} = \frac{1}{m} \sum_{l=1}^m \overline{B^{(l)}} = (\overline{B_1^{(m)}}, \overline{B_2^{(m)}}, \dots, \overline{B_j^{(m)}}, \dots, \overline{B_K^{(m)}}) = (\overline{b_{ij}^{(m)}})_{N \times K} = (\frac{1}{m} \sum_{l=1}^m (\frac{1}{q} \sum_{q=1}^l a_{ij}^{(q)}))_{N \times K}, \quad (6)$$

where  $i = 1, 2, \dots, N; j = 1, 2, \dots, K; m = 1, 2, \dots, M$ .

Thus, the  $O'$  defined in equation (2), will be constructed as

$$O' = (o'_{jk})_{N \times K} \begin{cases} 1 \leq j \leq N \\ 1 \leq k \leq K \end{cases},$$

and

$$o'_{jk} = \frac{1/m \sqrt{\sum_{l=1}^m (a_{jk}^{(l)} - \overline{b_{jk}^{(l)}})^2}}{1/m \sqrt{\sum_{l=1}^m (a_{jk}^{(l)})^2} + 1/m \sqrt{\sum_{l=1}^m (\overline{b_{jk}^{(l)}})^2}}, \quad (7)$$

where  $o'_{jk}$  ( $0 \leq o'_{jk} \leq 1$ ) is called incongruous coefficient which represents the coherence content of state observed vector  $a_k, k \in K$  and that of reference system.  $o'_{jk} = 0$  indicates that they are in full accord, otherwise;  $o'_{jk} = 1$  represents the discrepancy is maximum.

Integrating (3), (7) and (2), one obtains the HMM model for quantitative analysis on military simulation system with a given scenario.

### 3.2 The Evaluation Criterion for Results Validity

When researchers employ interval estimator to analyze the results validity, the focus is the discrepancy between the mean values of observed vector of simulation system and that of reference system. From (4) and (5), the mean value of observed vector of simulation system after the  $m^{th}$  exercise over is rewritten as

$$\overline{A^{(m)}} = \frac{1}{m} \sum_{l=1}^m A^{(l)} = (\overline{A_1^{(m)}}, \overline{A_2^{(m)}}, \dots, \overline{A_j^{(m)}}, \dots, \overline{A_K^{(m)}}) = (\overline{a_{ij}^{(m)}})_{N \times K} = (\frac{1}{m} \sum_{l=1}^m a_{ij}^{(l)})_{N \times K},$$

where  $i = 1, 2, \dots, N; j = 1, 2, \dots, K; m = 1, 2, \dots, M$ .

Similarly,  $B^{(m)}$  written as (6) represents the mean value of observed vector of reference system.

Considering the military simulation systems are always typical multivariate output system, thus, the accuracy of the  $j^{th}$  observed vector can be expressed by the confidence interval (*ci*) of  $A_j^{(m)}$  and  $B_j^{(m)}$ . Consequently, by integrating *ci* of all observed vectors, the accuracy of simulation system with the given scenario can be expressed by simultaneous confidence intervals (*s.ci*).

Consequently, when the accuracy requirement of the  $j^{th}$  vector and that of simulation system are expressed as  $\varepsilon_j$  and  $\varepsilon$  separately, the results of simulation system is validate if  $|\overline{A_j^{(m)}} - \overline{B_j^{(m)}}| \leq \varepsilon_j$  and  $|\overline{A^{(m)}} - \overline{B^{(m)}}| \leq \varepsilon$  are simultaneously satisfied.

### 3.3 The Accuracy of Simulation System

In the process of applying *ci* and *s.ci* to analyze the running accuracy of simulation system, the first is to determine the upper limit and lower limit of accuracy that are defined as the  $L$  and  $U$ .

$$L \leq |\overline{A^{(m)}} - \overline{B^{(m)}}| \leq U,$$

where  $L = (L_1, L_2, \dots, L_j, \dots, L_K), U = (U_1, U_2, \dots, U_j, \dots, U_K)$ .

As known, the military simulation systems always terminate under certain constraint condition, in order to ensure that the observed vectors of simulation system generated by simulation exercises are independent, we get the observed vector by adopting the repeated exercises method with same initial conditions.

There are many methods (Balci 1989) to determine the  $L$  and  $U$ , in order to simplify the computing process, we adopt the corresponding  $L_j, U_j$  determined by SMEs in terms of application objective and requirements of simulation system.

### 3.4 Statistical Calculation

Assuming  $(1-r_j)$  is the confidence level for  $(A_j^{(m)} - B_j^{(m)}) \in (l_j, u_j)$ , thus,  $(1-r)$  is the simultaneous confidence level for  $(A^{(m)} - B^{(m)}) \in (l, u)$ . Let  $x_j = 1 - r_j (j \in K)$ , and  $Y = 1 - r$ . Consequently, the  $Y$  is defined as:

$$Y = f(x_1, x_2, \dots, x_K).$$

In order to ensure the relationship between simultaneous confidence level of simulation system and the confidence level of observed vector is logical, supposing above equation satisfies

$$1 > \frac{\partial Y}{\partial (\forall x_j)} > 0,$$

when  $x_j$  is ascending.

Thus,  $[l, u]$  represent the simultaneous confidence interval of  $(\overline{A^{(m)}} - \overline{B^{(m)}})$ , and

$$l = (l_1, l_2, \dots, l_j, \dots, l_K); u = (u_1, u_2, \dots, u_j, \dots, u_K). \quad (8)$$

From equation (4) and (5), one obtains the sample variance of observed state vectors of simulation system

$$\begin{cases} S^{(j)2} = O(m=1) \\ S^{(m)2} = (S_1^{(m)2}, S_2^{(m)2}, S_3^{(m)2}, \dots, S_j^{(m)2}, \dots, S_K^{(m)2}) = \left( \frac{1}{m-1} \sum_{l=2}^m d_{ij}^{(l)} - \frac{1}{m} \sum_{l=2}^m d_{ij}^{(l)} \right)_{N \times K} \end{cases}, \quad (9)$$

and the sample variance of observed state vectors of reference system is

$$\begin{cases} \overline{S}^{(j)2} = O(m=1) \\ \overline{S}^{(m)2} = (\overline{S}_1^{(m)2}, \overline{S}_2^{(m)2}, \overline{S}_3^{(m)2}, \dots, \overline{S}_j^{(m)2}, \dots, \overline{S}_K^{(m)2}) = \left( \frac{1}{m-1} \sum_{l=2}^m \overline{d}_{ij}^{(l)} - \frac{1}{m} \sum_{l=2}^m \overline{d}_{ij}^{(l)} \right)_{N \times K} \end{cases}. \quad (10)$$

As discussed in section 3.3, the observed vectors of reference system constructed in terms of (4) are independent. Let it satisfy the normal distribution, by adopting Roy-Bose multivariable statistical method, one obtains the *c.i* expressed as equation (11) when confidence level is  $(1-r_j)$ .

$$(l_j, u_j) = \frac{\|\overline{A_j^{(m)}} - \overline{B_j^{(m)}}\|}{\sqrt{N}} \pm \sqrt{\frac{4(m-1)^2 \|\overline{S_j^{(m)2}} - \overline{S_j^{(m)2}}\|}{m\sqrt{N}}} T_{r_j, \tau, 2m-\tau-1}^2, \quad (11)$$

where  $T_{r_j, \tau, 2m-\tau-1}^2$  is  $r_j$  percent of the Hotelling  $T^2$  distribution when the degrees of freedom are  $\tau$  and  $2m-\tau-1$ . Integrating (11) and (8), one obtains the simultaneous confidence interval of simulation defined as  $[l, u]$ .

If  $[l, u] \subset [L, U]$ , the simulation exercise is validate, and its confidence level is  $100(1-r)\%$ ; If  $[l, u] \not\subset [L, U]$ , increasing the number of exercise, or increasing the corresponding  $r_j$  with acceptance domain and computing the  $[l, u]$  again. When  $m$  represented the number of exercises is sufficient enough or it is disallowance to increase  $r_j$  with acceptance domain, the exercise is invalidate.

#### 4 CONCLUSIONS

Considering the challenges and the state of the practice of military simulation systems, paper focus on practical validation from analyst's perspective in form of mathematical model and presents the effective and efficient evaluation

method based on the HMM for validating the behavior of military simulation systems. On the basis of importing matrix of observed state vector of simulation system, paper proposes a practical approach to establishing the HMM and construction principles for output of reference system. By applying interval estimator which extends the *c.i* of each observed state vector to the *s.c.i* of simulation system, paper proposes an approach to implementing the quantitative analysis for the results validity of military simulation systems.

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