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UPSTREAM DEMAND PROJECTION AND PERFORMANCE MAPPING IN SUPPLY CHAINS

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ABSTRACT

This paper deals with modeling steady-state behavior of a single-product, pull-type, serial supply chain, frequently encountered in the automotive industries. The proposed analytical method enables projection of the end-customer demand information to upstream of the supply chain and estimate demand forecast at the individual tier levels. The supply chain performance assessment is based on the Due-Time Performance metric (DTP - probability to ship a required product/parts volume in a fixed time interval) under the assumption of customer demand following a discrete time Markov process, a special case for correlated demands. A numerical case study demonstrates the use of the DTP measure for a two-tier supply chain. The analytical results (verified by simulations) quantify important relationships in the supply chain, involving reliabilities of machines/stations, capacities of the buffers, demands correlation, and the due times and will find use in performance assessment, optimization and design of supply chains.

1 INTRODUCTION

In today's global economies, customer-driven world markets experience high demand fluctuations, turbulence of which is often amplified by arrival of new products and new technologies. To stay competitive under these conditions the entire supply chain needs capabilities for quick adjustment of production capacity and functionality (Asl and Ulsoy 2002). Understanding how market dynamics is carried over from the OEMs to different tier-levels in the supply chain has a fundamental importance for both the OEMs and the supplier companies in developing new production and operation strategies towards improving local, as well as overall supply chain performance.

Supply chains are studied at different levels of abstractions and different time scales of operations and decision making (Ganeshan, Jack *et al*. 1998; Shapiro 2001). Figure 1 represents one generic pull type, multi layered and multi-tier supply chain abstraction. The OEM receives intermediate parts for final assembly from its supplier base and delivers the finished product to meet the end-customer demands via network of distributors and retailers. Both the OEM and the supply base have unreliable capacitated production systems. The end aggregated random correlated demands for the finished product is seen only by the OEM. The end-customer demands first consume finished products from the inventory and the vacancy thus generated creates a pull for the entire supply chain. However, the production system variability at different tiers, end demand fluctuations and different time scales and modes of information sharing creates disparity in meeting the end-customer demands in committed lead times (Cachon and Fisher 2000). A well functioning supply chain has to continuously satisfy the end-customer demands in committed due times irrespective of any uncertainties in its supply, production and/or demand base without building excessive inventories.

This paper deals with the following questions: A) How to evaluate and quantify the end-customer demand satisfaction for a random correlated (or dynamic) demand driven supply chain, and B) How to decompose or isolate different tier supplier's capabilities in terms of meeting the downstream orders that flow from the end-customer orders.

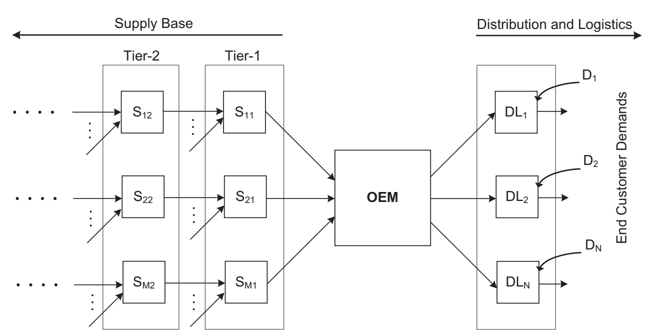

Figure 1: Generic Pull Type Supply Chain Network

Due-Time Performance (DTP) is the used (solution to problem A) end-customer demand satisfaction metric and is defined as the probability to ship to the customer a required number of parts during a fixed time interval (Jacobs and Meerkov 1995; Li and Meerkov 2001). The approach followed for solution to problem B is first projecting the end-customer demand model to independent upstream tier levels and then calculating DTP against the projected demand model using solution to problem A. This isolation of the DTP measure to individual upstream suppliers also helps in the root cause analysis and bottleneck identification for low end-customer demand satisfaction in the network. Such a measure (that is spatially characterized, flows from the end-customer demand model and accounts for all modeled uncertainties both in the upstream and downstream production sides) empowers each tier-level supplier to exert independent control on its own capacity and improve command on its upstream supply base due to improved visibility of its contribution to the overall supply chain performance in an otherwise OEM dominated supply chain.

This work focuses on a single product, pull type serial supply chain. The purpose and the contribution of this paper is in providing the analytical method to project the correlated end-customer demand forecast to the upstream suppliers in midst of all production uncertainties and isolating the performance of individual entities in the supply chain. It parameterizes and quantifies some significant cause-effect relationships in complex demand driven supply chains for better steady state (strategic) decision making.

2 PROBLEM FORMULATION

2.1 Supply Chain Model

This paper concentrates on a single level serial multi-tier supply chain structure as shown in Figure 2, a simplification of the generic multi layered supply chain model of Figure 1. The suppliers and the final assembly plant are characterized by production systems with defined reliability models, and are represented by a serial arrangement of machines (circles) and buffers (rectangles). The entry point is fed with unlimited raw material supply while the end-of-line is subjected to customer demands. The detailed modeling assumptions on part and information flows are as follows:

[A1] Part type: The overall supply chain architecture is a *single part type, pull serial production line* formed by sub production systems S = {0,1,2 ….}, where 0 is the final Assembly plant, 1 is the Tier-1 Supplier and so on.

[A2] Machine/Stations: All machines follow the *Bernoulli reliability model* and work synchronously for equal fixed unit of time to process a part. In each time slot equal to the cycle time, the machine m_{Si} remains independently "up" (or "down") with a probability p_{Si} (or respectively 1- p_{Si}) equal to the isolated production rate of that machine.

[A3] Buffers: All buffers B_{Si}, including the Raw Material Buffer (RMB), Intermediate Finished Good Buffers (IFGB) and the Finished Good Buffer (FGB) has finite capacity N_{Si}; $0 < N_{Si} < \infty$. The buffer state is updated at the end of every cycle.

[A4] Machine and Buffer Interface: Each machine has information about immediate upstream buffer occupancy (empty or non-empty) and status of all machines and buffers downstream. A machine *starves* if the upstream buffer is empty at the beginning of the cycle, and is *blocked* if the downstream buffer is full and its immediate downstream machine is either down or blocked itself.

[A5] Demands: The end-customer demand follows an exogenous random *Markovian* model (D(i), P, T), in the time scale of epoch (1 epoch = T machine cycles time); characterized by J demand states $D=\{d_1, d_2 \cdots d_J\}$ with $2 \le J < \infty$, and transition probabilities $P_{nm} = \Pr\left(D(i)=d_n \mid D(i-1)=d_m\right)$, where $m,n \in \{1,2,\cdots J\}$ for transition from demand D(i-1) during (i-1)th epoch to D(i) during ith epoch. The demands remain constant during an *epoch,* also referred to as the due time period or shipment period. This can be considered as reduced discrete (quantized) dynamic model of real demand data.

[A6] Customer Demand Satisfaction: The end-customer demand satisfaction is measured in terms of *Due Time Performance*, more formally defined in Eqn. 2. At the beginning of epoch (i+1), parts are removed from the FGB in the amount of min(H(i), D(i+1)) and immediately sent for the shipment towards demand D(i+1). H(i) represents the FGB occupancy at the end of the epoch i, and it can take any values in the set {0,1 … N_{0M} }, where N_{0M} is the FGB capacity. If H(i) $\geq$ D(i+1), the shipment for epoch (i+1) is complete, and the parts then produced during the (i+1)th epoch will start getting accumulated in the FGB; otherwise, if H(i) < D(i+1), the system continues to produce parts and sending them directly for the shipment without accumulation until the balance demand (D(i+1) - H(i)) is met. If the shipment still remains incomplete by the end of that epoch, an incomplete shipment is sent to the customer. No backlog of the demand is allowed.

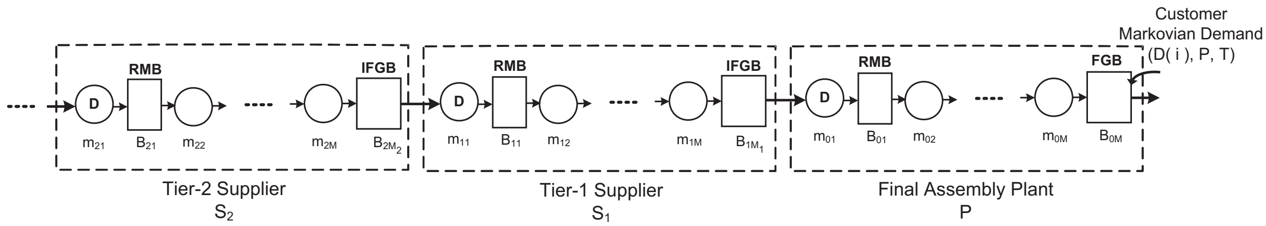

Figure 2: Serial Multi-Tier Pull Type Supply Chain Model

[A7] Part Transit: The finished part transfer from the assembly plant to the end-customer takes place in time scale of epochs (1 epoch = T cycles). The material flow from IFGB of any supplier plant to RMB of the consuming plant takes place in scale of cycle time through the distribution machinery D in the batch size of one unit. This difference in supply and demand time scales is fairly common in automotive and furniture industries, where the local suppliers refurnish OEM a few times a day on a recalculated need basis.

2.2 Objectives

The objectives of the presented efforts are described in terms of the serial production line without loss of any generality and will be later extended to the supply chain scenario in the numerical case study. The objectives are to:

1. Develop an analytical method to quantify the DTP index for an M machine serial line characterized by assumptions [A1-A6]. Since a close form solution for DTP calculation for this case is intractable, develop a converging estimation procedure (Section 3).
2. Develop an analytical Markovian model in unit of cycle times for the projected demand at the entry of the M machine serial line governed by assumptions [A1-A6] (Section 4).
3. Extend the projected Markovian demand model in unit of cycle times to higher time scales such as epoch times and quantify the variability statistics of the projected demands (Section 5.3).

3 DTP CALCULATIONS

This section briefly describes the analytical approach for DTP calculation for Figure 3 representing serial M-machine production line with FGB, subjected to Markovian demands. Further details including the proofs can be found in Bagdia (2004). Previous works (Li and Meerkov 2001; Li, Enginarlar *et al.* 2004) quantified DTP for constant and IID demand models.

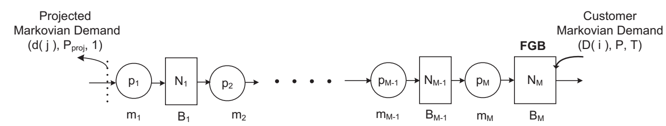

Figure 3: M Machine Serial Production-Inventory-Customer System

The production line in Figure 3 is governed under assumptions [A1-A6]. The overall system forms a stationary ergodic Markovian chain in the time scale of epoch and appropriately defined state space $S=\{(H,D); H\in(0,1\cdots N_M), D\in(d_1\cdots d_J)\}$, where H(i) and D(i) are respectively the FGB occupancy at the end of the epoch i, and the demand during epoch i. The ergodic Markovian characteristic ensures the existence of a steady state distribution and hence supports the calculation of the steady state DTP measure. The state dynamics are governed by Eqn. 1.

$$H(i) = H(i-1) + \bar{t}_i - D(i). \tag{1}$$

where $\bar{t}_i$ is the number of parts produced by the production system with FGB in epoch i during the steady state operation. The due time performance measure can then be defined by

$$DTP = \Pr\left(H(i-1) + \bar{t}_i \geq D(i)\right). \tag{2}$$

The DTP calculation process is broken down into two steps:

- Develop an analytical closed form solution for DTP calculation for a one machine FGB system.
- Develop a converging recursive algorithm to convert an M machine FGB system into an equivalent one machine FGB system and use DTP formula from step 1.

3.1 DTP of a One Machine FGB System

Consider a one machine FGB system characterized by Bernoulli machine reliability index p, FGB capacity N, and subjected to a Markovian demand model $(D(i), P, T)$. The following result remains valid for any value of demand states d_i. However under the no demand backlogging assumption, the result carries useful inferences with demand states that do not load the system more than its capacity, i.e., Load Factor $LF = \frac{\overline{D}}{PR \times T} \le 1$, where $\overline{D}$ is the average demand and PR is the Production Rate of the system. The DTP of this one machine FGB system is given by Eqn. 3.

$$\begin{aligned} DTP &= \sum_{l=0}^{N} \sum_{m=1}^{J} \sum_{n=1}^{J} P_B(d_n - l) P_{nm} z(l,m) \\ &:= DTP_1(p, N, T, D, P) \end{aligned} \tag{3}$$

where,

$$P_B(x) = \sum_{j=x}^{T} \binom{T}{j} p^j (1-p)^{(T-j)}; \quad 0 \le x \le T$$

is the cumulative mass function of the number of parts produced in an epoch without the FGB. The joint probability mass function of the FGB occupancy and demand in an epoch, represented as $z(l,m) = \Pr(H(i) = l, D(i) = d_m)$ can be calculated using the procedure developed in Bagdia (2004).

3.2 DTP of an M Machine FGB System

The behavior of a production line does not remain a stationary random process within an epoch, posing challenges for tractable close form DTP analysis for M machine line of
Figure 3. A recursive iteration originally adapted from (Li and Meerkov 2001) for estimating DTP of M machine FGB system is therefore developed as laid in Figure 4 and described briefly below (Bagdia 2004).

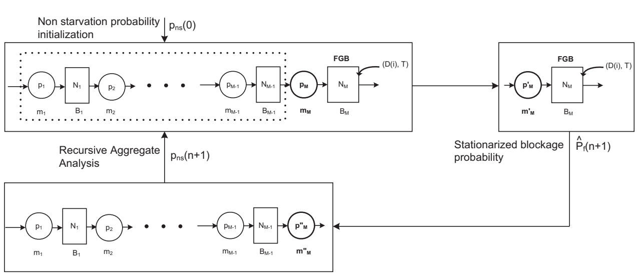

Figure 4: Recursive Procedure to Establish One Machine FGB System Equivalency

1. Assume that the probability of last machine mM not starving is known and denote its estimate by pns.
2. Modify the last machine's reliability index pM by multiplying it by pns and convert the line into an equivalent one machine FGB system with modified machine m'_M.
3. Estimate the stationarized probability of FGB blocking the equivalent one machine line (by remaining filled) in an epoch irrespective of the machine's working status as developed in Bagdia (2004). Denote it by $\hat{P}_f$.
4. Remove the FGB from the line by multiplying original last machine mM reliability index by non blocking probability $\left(1 - \hat{P}_f\right)$. This gives a new standard serial line without FGB with modified last machine m''_M.
5. Calculate the new pns for the transformed standard serial line using the recursive aggregate procedure (Jacobs and Meerkov 1995). Repeat steps 2-5 until a convergence is obtained.
6. Finally calculate the DTP for the convergent equivalent one machine FGB system using Equation 3.

4 UPSTREAM DEMAND PROJECTION

The order placed by the first machine m_1 of
Figure 3 subsequently becomes demand for the upstream raw material supplier in a supply chain scenario. This paper develops a close form, steady state statistical model of the projected end customer demand at the entry of the production line governed by assumptions [A1-A6]. The projected demand is modeled as a first order Markovian chain in unit of cycle times (same in which the material flow at the entry point takes place) with two demand states {1, 0}, respectively representing if an order is placed or not by the entry machine. The hypothesized model is verified through *discrete event simulations* of the production line.

This projection analysis builds over the previous framework developed for DTP calculations in section 3. First, the projection is studied for a one machine FGB system and later extended for an M machine FGB system by establishing equivalency to a one machine FGB system.

4.1 Demand Projection by One Machine FGB System

The one machine FGB system considered here is the same as discussed in section 3.1. Machine m_1, which is assumed to have unlimited raw material availability can process a part only if it is up and not blocked by the FGB. The blocking by FGB will henceforth imply that the FGB is filled irrespective of the machine being up or down (also referred to as "communication blockage"), and this probability $\hat{P}_f$ is calculated as discussed in Section 3.2. The order placing state is represented by state 1 of the projected

Markovian demand model, while state 0 is when the machine cannot process a part, it being either down or blocked by the FGB. The complete projected Markovian demand model in units of cycle times is shown in Figure 5.

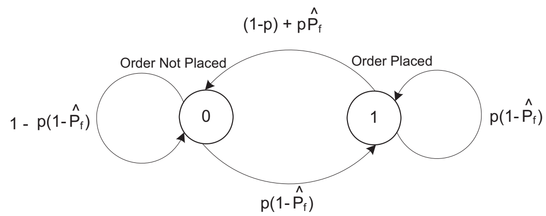

Figure 5: Projected Markovian Demand Model at the Entry Machine of a Production System subjected to Markovian End-Customer Demands

A simulation verification of the proposed projected demand model is made in Table 2. In each simulation run of the corresponding discrete event model, zero initial conditions for all buffers have been assumed. One run consisted of 10000 epochs (1 epoch = T time slots) and statistics were collected after an initial warm up period of 4500 epochs. P_{anl}^{proj} denotes the analytically estimated transition matrix of the projected Markovian demand model, while P_{sim}^{proj} represents the empirically calculated transition matrix from one simulation run's logged statistics. The percentage error matrix $\%err = 100 \times \left(\left| P_{sim}^{proj} - P_{anl}^{proj} \right| \right) / P_{sim}^{proj}$ shows percentage error less than 11% for all the four transition elements. The deviation results from the approximation induced in calculating FGB blocking probability $\hat{P}_f$.

4.2 Demand Projection by M Machine FGB System

The proposed analysis involves converting the M machine FGB line into an equivalent one machine FGB system using procedure developed in section 3.2 and then applying on it the one machine FGB projection results from section 4.1. The results from simulation verification are shown in Table 1. The less than 6.5% error in most of the calculated transition probabilities validates the Markovian projection in units of cycle times within the steady state simulation and $\hat{P}_f$ estimation accuracies. The projected model accuracy is better for multiple machine line than for a single machine FGB system. The existing gap still remains to be explored, but nevertheless the method should be useful for longer lines.

5 CASE STUDY

A numerical case study presented here demonstrates the use of the DTP measure for multi-tier supply chain example (see Figure 6). The supply chain under consideration consists of an assembly plant P, its supplier S_1 and it's supplier's supplier S_2 following assumptions [A1-A7]; machine reliabilities, buffer capacities and demand model are as indicated.

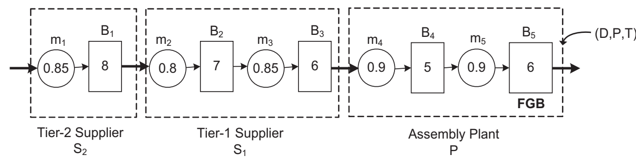

Figure 6: Two Tier Serial Supply Chain for Case Study

5.1 Effect of FGB Capacity and Customer Due Time on DTP

DTP is a function of all production, inventory and customer subsystem's parameters.

Figure 7 shows impact of FGB capacity and demand due time parameters on overall DTP of the Supply Chain in Figure 6, against following Markovian demand model: $D = [5 \quad 10 \quad 17]$;

$$P = \begin{bmatrix} 0.3 & 0.2 & 0.4 \\ 0.5 & 0.5 & 0.2 \\ 0.2 & 0.3 & 0.4 \end{bmatrix}$$

with average demand $\overline{D} = 10.6627$.

Observations DTP is a monotonic, non decreasing function of the FGB capacity, i.e. it only gets better with increase in FGB capacity, and gets saturated after a limiting value. This result can be used for optimal selection of the FGB capacity for a given due time. DTP shows similar non decreasing monotonicity with respect to the due time T. Higher allowable customer due times reduce the load (measured in terms of the Load Factor LF) on the system and increase the DTP. This can be used for negotiating due times with the customers to relax production system's load constraints and, in turn, guarantee confirmed delivery jointly with minimum inventory. The analytical DTP formula should allow multi parameter optimization over any of the production-inventory-customer subsystems variables.

5.2 Effect of Demand Variability on DTP

Demand variability is here measured in terms of the Coefficient of Variation (CV - defined for random variable X as $CV_X = \sigma_X / \overline{X}$) and has significant effect on DTP (see Figure 8). The system in Figure 6 is subjected to different Markovian demand models having the same average demand $\overline{D} = 10.6627$, but different variabilities.

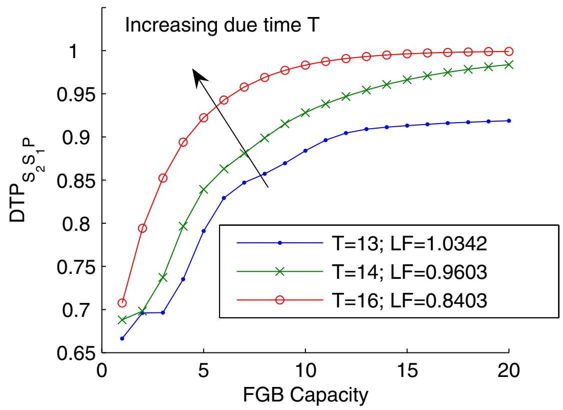

Figure 7: Overall DTP Variations with FGB Capacity and Customer Due Times

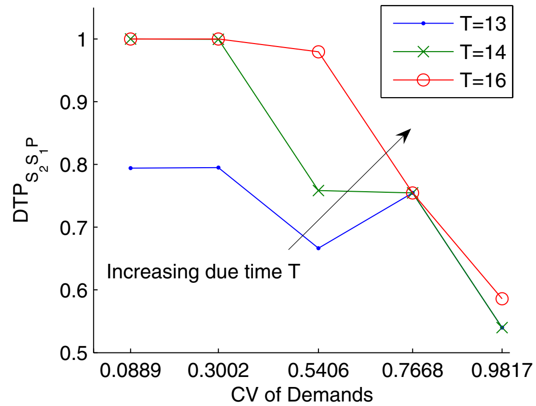

Figure 8: Overall DTP Variations with Coefficient of Variation of Demand

Table 1: Accuracy of Projected Markovian Demand Model in unit of cycle time for M machine FGB System

p_i	N_i	T	D	P	LF	P_{sim}^{proj}	P_{anl}^{proj}	% err
0.90 0.65	2 3	11	6 8	$\begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix}$	0.9439	$\begin{bmatrix} 0.5990 & 0.5739 \\ 0.4010 & 0.4261 \end{bmatrix}$	$\begin{bmatrix} 0.5901 & 0.5901 \\ 0.4099 & 0.4099 \end{bmatrix}$	$\begin{bmatrix} 1.4856 & 2.8220 \\ 2.2189 & 3.8007 \end{bmatrix}$
0.87 0.95 0.87	1 2 3	7	4 5 6	$\begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.2 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$	0.8995	$\begin{bmatrix} 0.7117 & 0.7181 \\ 0.2883 & 0.2819 \end{bmatrix}$	$\begin{bmatrix} 0.7130 & 0.7130 \\ 0.2870 & 0.2870 \end{bmatrix}$	$\begin{bmatrix} 0.1836 & 0.7055 \\ 0.4531 & 1.7967 \end{bmatrix}$
0.80 0.85 0.88 0.90 0.95	2 3 3 4 7	15	7 12	$\begin{bmatrix} 0.25 & 0.25 \\ 0.75 & 0.75 \end{bmatrix}$	0.9559	$\begin{bmatrix} 0.7192 & 0.6968 \\ 0.2808 & 0.3032 \end{bmatrix}$	$\begin{bmatrix} 0.7167 & 0.7167 \\ 0.2833 & 0.2833 \end{bmatrix}$	$\begin{bmatrix} 0.3511 & 2.8514 \\ 0.8991 & 6.5528 \end{bmatrix}$

Table 2: Accuracy of Projected Markovian Demand Model in unit of cycle time for one machine FGB System

p_1	N_1	T	D	P	LF	P_{sim}^{proj}	P_{anl}^{proj}	% err
0.65	3	10	6 8	$\begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix}$	1.0256	$\begin{bmatrix} 0.6290 & 0.6245 \\ 0.3710 & 0.3755 \end{bmatrix}$	$\begin{bmatrix} 0.4341 & 0.2819 \\ 0.7360 & 0.4688 \end{bmatrix}$	$\begin{bmatrix} 0.4341 & 0.2819 \\ 0.7360 & 0.4688 \end{bmatrix}$
0.87	3	6	4 5 6	$\begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.2 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$	0.9579	$\begin{bmatrix} 0.8260 & 0.8014 \\ 0.1740 & 0.1986 \end{bmatrix}$	$\begin{bmatrix} 0.8239 & 0.8239 \\ 0.1761 & 0.1761 \end{bmatrix}$	$\begin{bmatrix} 0.2453 & 2.8135 \\ 1.1641 & 11.3527 \end{bmatrix}$
0.5	15	25	15 25	$\begin{bmatrix} 0.5 & 0.75 \\ 0.5 & 0.25 \end{bmatrix}$	1.5200	$\begin{bmatrix} 0.5916 & 0.4988 \\ 0.4984 & 0.5012 \end{bmatrix}$	$\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.3231 & 0.2347 \\ 0.3252 & 0.2337 \end{bmatrix}$

Observations Generally the DTP of a system degrades with increasing variability of the demand model. But this is not strict and there can be counter examples like the one in Figure 8 for which $LF > 1.0$ and $D_{max} >> T$. The sharp fall in DTP at one critical value of CV indicates the system having some *bandwidth* for demand variability accommodation while sustaining higher DTP. Figure 8 can be looked upon to judge the demand variability limits (or *quantifiable risk*) that can be suitably afforded for a given production load factor and desired customer DTP range.

5.3 Demand Projection and DTP Isolation

The projection of demand and its later use in independent evaluation of each entities and their respective interaction is studied here with a simple demand model (DM_1) for supply chain in Figure 6: constant demand $D = 12$ (special case of the Markovian demand model) with due time $T = 15$, for easy interpretation of the results.

Observation Table 3 illustrates progressive increment of the load factor and associated degradation of DTP with addition of unreliability in the supplier base for supply chain in Figure 6 under demand model DM_1.

Table 3 : Load Factor and DTP decomposition for Supply Chain in Figure 6

*Isolated System	Load Factor	DTP
P	0.9067	1.0000
S_1P	1.0058	0.9552
S_2S_1P	1.0087	0.9344

* Isolated System: System decoupled from the upstream side by assuming unlimited raw material availability at its entry

However, it will be misleading to arraign only the suppliers for this reduced end-customer performance in the considered demand driven supply chain. While the upstream suppliers can create hardships by causing starvations, the downstream partners can obstruct flow through blockages. Table 4 provides some starvation and blockage statistics for the entry machine of each entity in the supply chain. $N_{s_{sim}}$ (respectively $N_{b_{sim}}$) is the number of times a machine actually starves simultaneously being up and not blocked (respectively gets blocked simultaneously being up and not starved) in one simulation run of 10000 epochs (1 epoch = 15 time units).

Table 4: Statistics of Blockages and Starvations for Supply Chain in Figure 6 under DM_1

Statistics	Tier-2 S_2 (m_1)	Tier-1 S_1 (m_2)	Assembly P (m_4)
$N_{s_{sim}}$	0 (assumption)	294	13020
$N_{b_{sim}}$	8864	885	3280
$\backslash P_{s_{sim}}$	0 (assumption)	0.0020	0.0868
$P_{b_{sim}}$	0.0591	0.0059	0.0219
$P_{s_{anl}}$	0 (assumption)	0.0027	0.1041
$P_{b_{anl}}$	0.0569	0.0043	0.0032

The assembly entry machine m_4 gets significantly starved due to the very unreliable upstream supply base. However, it also gets blocked significant number of times causing obstruction to upstream suppliers, proving that DTP degradation's responsibility assignment in a supply chain is coupled both in the upstream and the downstream directions, and can be complex. The other rows in Table 4 estimate the probabilities of these starvation and blockage events (P_s and P_b respectively) through simulations as well as aggregate analysis. The high starvation probability of the assembly plant (m_4) and simultaneously high blockage probability of the Tier-2 Supplier (m_1) should be an indicator of possible problem at Tier-1 level. The Tier-2 Supplier system though being unreliable still does a decent job as indicated by low starvation probability of machine m_2 at Tier-1 level. The DTP bottleneck indicator (Li and Meerkov 2000) also suggests that machine m_2 is the DTP bottleneck machine and its immediate downstream buffer is the DTP bottleneck buffer. (A slight increase in the production reliability of m_2 from 0.8 to 0.81 improves overall $DTP_{S_2S_1P} = 1.0000$) This indicator is however restricted to only constant demand case. We will demonstrate a new methodology for bottleneck identification by evaluating DTP of independent entities against their projected demands from downstream partner, which will hold for any demand models including constant, IID and Markovian models.

All of the above important observations including DTP calculations in Table 3 were made with respect to the end-of-line customer demands as seen *only* by the final Assembly plant. While it is important to be able to evaluate the performance of the overall supply chain against a common end metric, it is also equally important to be able to evaluate the performances of independent entities in the supply chain. The Tier-1 Supplier S_1 will surely be interested in knowing its delivery performance to meet its direct customer (the Assembly plant) orders both in the isolated (S_1 alone) and combined (S_2 and S_1 together) operating regimes. These calculations of dtp_{S_1} and $dtp_{S_2S_1}$ will require model of ordered parts placed by the assembly plant (or demand model seen by the Supplier S_1), which will be calculated using the analysis in section 4.

Figure 9 shows simulated projection of orders (in units of epoch times) by all three subsystems of the supply chain in Figure 6 under DM_1 for both the *coupled* and *isolated* modes. In coupled mode, an actual material transaction takes place with the upstream supplier only if the downstream machine is up, not blocked and not starving, unlike the isolated mode, where no upstream starvation is assumed. Generally there would exist differences between the orders placed under these two modes, the magnitude being dependent upon the starvation probability of the entry machine. For the considered case study, this difference is noticeable for the assembly plant, small for the Tier-1 Supplier, and zero for the Tier-2 Supplier (see Figure 9), as also can be inferred from the corresponding starvation probabilities in Table 4. The proposed method accounts only the downstream variations and calculates the projected order model for only isolated systems.

Figure 10 demonstrates the final results for demand projection and DTP isolation for all isolated subsystems of supply chain in Figure 6. The capitalized 'DTP' and 'LF' stands for calculations w.r.t. the original end-customer demand in epoch periods, while small 'dtp' and 'lf ' stands for calculations w.r.t. the projected Markovian demand model in unit of cycle times. The high DTP of the Tier-2 Supplier to its projected Markovian demand $d^2(t)$, given by $dtp_{S_2} = 0.9981$, inspite of its low reliability confirms the previous conclusion of it not contributory to the overall DTP degradation. The lowest DTP of the Tier-1 Supplier given by $dtp_{S1} = 0.9733$ with corresponding high load factor $lf_{S_1} = 1.0056$ identify it as a bottleneck system and suggest to improve its isolated production rate to get under unity load factor. A possible functional relationship may exist between these different isolated DTPs, but is left unexplored here.

Also noticeable in Figure 9 are the variations of the orders (as low as 8 parts and high up to 15 parts in epoch of 15 time units) that gets placed by different tier levels even for a constant demand of 12 parts under DM_1. Thus unreliability in the production system can induce significant variations in the orders passed to different tiers of the supply chain and hence quantification of these variations becomes important for employing any controls. The variability in the projected demands increases upstream supply chain (increasing CV in Figure 10) quantifying the Bull Whip effect (Chen, Drezner *et al.* 2000; Lee, Padamanabhan *et al.* 2004). However, care needs to be taken for comparing the CVs of projected demands against that of the original end-customer demands. For an overloaded system (LF > 1.0), high demand variability gets filtered at the FGB and hence remains hidden to the upstream tiers. For example, if the end-customer demand states of $\begin{bmatrix}5 & 15 & 30\end{bmatrix}$ arrive stochastically at the FGB with due time of only T=15 time units, then surely the demand of 30 parts is not getting to any upstream partners, since when only maximum 15 parts can be produced in the allotted due time. Also, it may not be a fair comparison as the original and projected demand models are in different time scales.

It is possible to extend the projection results to higher time scales and one such method is discussed here. This higher time scale analytical distribution can then be verified against simulated statistics of the projected demands from Figure 9. The projected Markovian demand model with binary [0, 1] states has one interesting property which allows to model it as a binomial distribution in time scale of epochs. The self transition probabilities of being in states '0' and '1', sums to one (i.e. $P_{00}^{proj} + P_{11}^{proj} = 1.0$). This leads the projected Markovian model steady state distribution being equal to the corresponding self transition probabilities. As established by Bagdia (2004), such Markovian demand model affects the DTP of a system exactly the same way as an IID demand model with the same states and probability mass function equal to the steady state distribution of the Markovian model. Thus the projected demand model can be looked upon as an IID (Bernoulli) demand model in steady state, as far as DTP evaluation is concerned. The probability distribution of placing exactly n orders in an epoch of T Bernoulli trials, each with probability P_{11}^{proj}, then becomes a discrete Binomial distribution as indicated in Eqn. 4.

$$\Pr(P_{11}^{proj}, n, T) = \binom{T}{n} (P_{11}^{proj})^n (1 - P_{11}^{proj})^{T-n} \quad \text{where, } 0 \le n \le T. \tag{4}$$

Figure 11 demonstrates a close match between this analytically derived binomial distribution and the simulation estimated histogram for isolated projected demands at Tier-1 and Tier-2 stages. The large deviations between the analytical and simulation achieved Binomial distribution of projected orders at the entry of the final Assembly plant arise due to inaccuracy in calculating the FGB blocking probability for shorter lines. This error gets smaller further away from the FGB buffer and hence the proposed analysis should work better for longer lines.

The projected demand statistics (e.g., mean, variance and CV) can be calculated analytically for the binomial distribution. The CV of the projected demand in the time scale of epoch also increases here along the upstream tiers of the supply chain and should be carefully compared with the CV of the end-customer demand for any meaningful inference drawing.

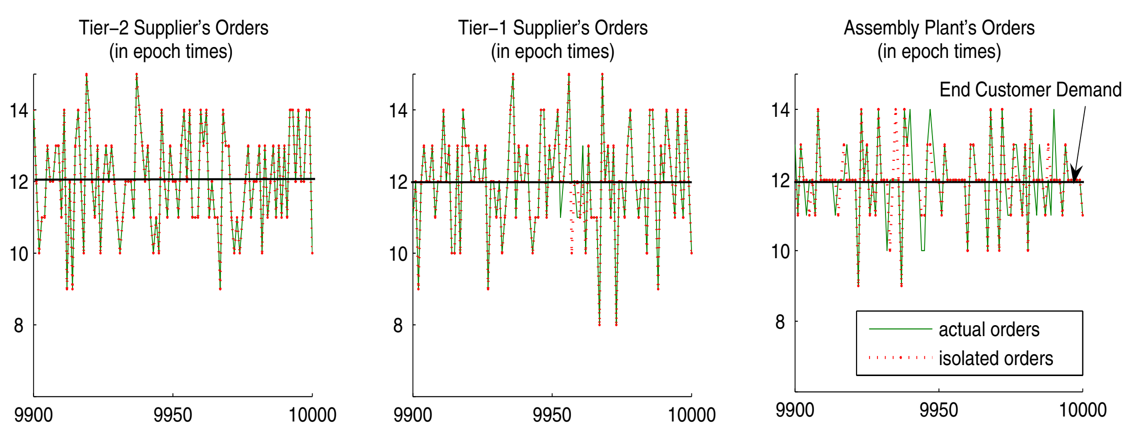

Figure 9: Actual and Isolated Orders Projection from different Entities of Supply Chain in Figure 6

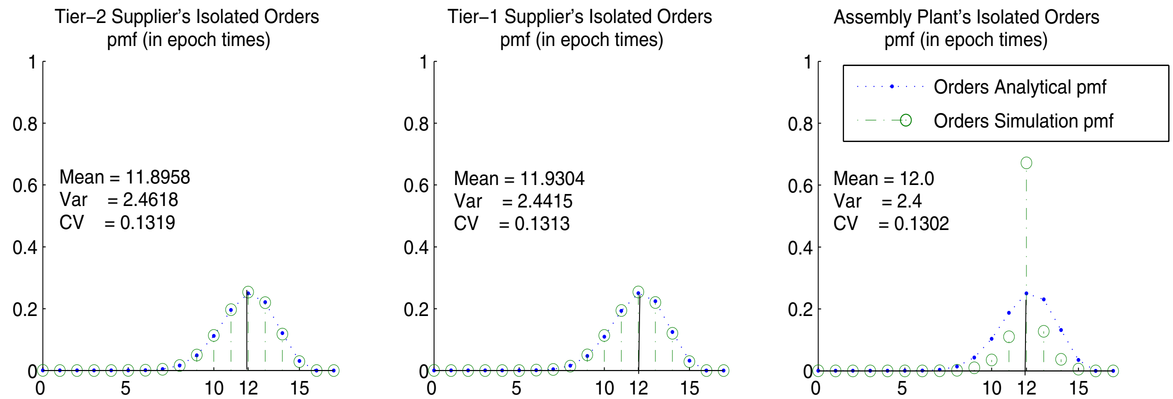

Figure 11: Projected Isolated Order's Probability Mass Function (pmf) in units of epoch times for Supply Chain in Figure 6

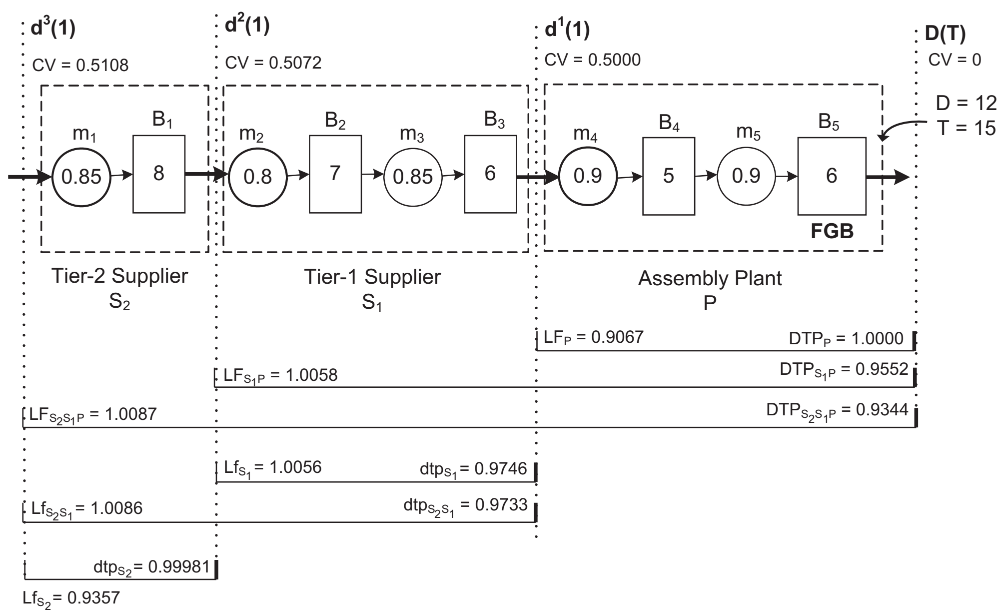

Figure 10: Final Results of Demand Projections with all Isolated Due Time Performances for Supply Chain in Figure 6

6 SUMMARY

In this paper an analytical framework for risk assessment and performance quantification of the supply chain in terms of due-time performance was presented and verified on a simple numerical example. DTP has combined consideration of throughput, lead time, order fulfillment rate, bottleneck, operations responsiveness and resiliency. The proposed framework enables projection of the random correlated end-customer demand to upstream of the supply chain and quantifies interrelationship of every entities individual performance on overall end performance of the supply chain. This information mapping via upstream demand projection also weakens the upstream supplier's dependency on the downstream partners for strategic decision making and supports decentralized control structure. The proposed model is fairly general and detailed and ties significant supply chain parameters, such as stations reliabilities, buffer capacities, demand correlations and due times. This allows not only to carry out assessment of a supply chain with given architecture and characteristics, but also can be used as a potential design tool.

The potential for such a tool was explored in the context of a relative simple numerical example, showcasing a two-tier supply chain. In this study we have looked only at the impact of a few primary parameters. Considering the potential size of the parameter space further studies need to be conducted, examining potential correlations, eventually leading to defining effective operational strategies. While the example elaborated here was fairly simple, we expect the approach to be equally effective for much more complex supply chain architectures.

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