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SIMULATION-BASED OPTIMIZATION FOR SPLIT DELIVERY VEHICLE ROUTING PROBLEM: A REPORT OF ONGOING STUDY

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ABSTRACT

Due to the complexity of split delivery vehicle routing problem (SDVRP), a simulation based optimization approach is proposed. A simulation model is used to capture the dynamics and uncertainties of the system and evaluate the system performance. Three split policies, LOS-policy, LDD-policy and LWT-policy are designed to implement the order split/consolidation. To optimize the route of orders in a consolidation, a genetic algorithm is developed and integrated with the simulation model. Experimental results showed that the average order size has significant impact on consolidation and split policies. Split delivery outperforms non-split delivery significantly when the average order size occupies about 60% of a truckload. Large arrival rate of orders also benefits split delivery. Sparse distribution of customers deteriorates the performance of split delivery. In various experimental scenarios, LDD-policy is always better than LOS-policy and LWT-policy.

1 INTRODUCTION

Since the split delivery vehicle routing problem (SDVRP) was introduced by Dror and Trudeau (1989), it has been attracting the interests of many researchers. Comparing with classical vehicle routing problem (VRP), the demands of a customer can be split and satisfied by multiple deliveries in SDVRP. Empirical studies show that split delivery may lead to cost savings (Dror and Trudeau 1989; Archetti et al. 2008). Nowak et al. (2008, 2009) conducted research on SDVRP and did many sensitive experiments to demonstrate the relationship between important system parameters and the benefits of split delivery in third-party logistics. Their research results show that more cost saving can be obtained by split delivery if the order size of each customer occupies 51% to 60% of truckload. Ho et al. (2004) indicates that split delivery can save transportation capacity and transportation distance. Split delivery may also be beneficial to customers. Chen et al. (2007) point out that split delivery can help the customers receive their part of goods more quickly so as to improve their service satisfaction.

It seems there are some studies that focus on the benefits of split delivery (Dror and Trudeau 1989; Archetti et al. 2008; Salani et al. 2011; Nowak et al. 2009). However, most current methods solving/optimizing the SDVRP emphasize analytical analysis via mathematical programming models. For example, Jin et al. (2008) proposed a column generation approach for solving SDVRP. Comparing with cutting plane algorithm, their method can obtain better upper and lower bound. Desaulniers (2010) designed a branch-and-price-and-cut method for SDVRP with time windows considered to minimize the transportation cost. This optimization approach does very well when the demands are determined and the number of customers is less than one hundred. Gulczynski et al. (2010) studied a kind of SDVRP with minimum

delivery amounts. They built a mix integer programming model to optimize the transportation cost. Lin et al. (2011) developed a multi-objective integer programming model for the emergency supply of critical items in the aftermath of a disaster. Multi-items, multi-vehicles, multi-periods, soft time windows and a split and prioritized delivery strategy scenario were considered in their optimization framework. To effectively solve this model, they limited the number of available tours for delivery vehicles. Hertz et al. (2012) studied a cement delivery problem with a heterogeneous fleet of vehicles and several depots where the demands of the customers are larger than the capacity of the vehicles. A two-phase integer linear programming model was proposed in their paper to solve this split delivery vehicle routing problem.

Due to the complexity of real-life SDVRP, the analytical methods will usually face the curse of dimensionality, which makes it hard to solve the large-scale vehicle routing problems. Furthermore, there exist many uncertain factors such as stochastic order arrivals, fluctuation of order size and transportation time in the process of logistics service. It is very necessary to consider these uncertainties in the optimization model because they have important impacts on system performance (Clara and Robert 2009). To address these issues in SDVRP, a simulation-based optimization methodology is proposed in this paper. The simulation is employed to model the transportation problem, obtain the system performance such as total cost and service level, and capture the uncertainties and dynamics of the system. A genetic algorithm (GA) is introduced to optimize the transportation routes. Several split policies are designed to implement the split and consolidation of customer orders. The rest of this paper proceeds as follows. Section 2 describes our simulation-optimization framework and the genetic algorithm. Section 3 gives the details of split policies and the conceptual simulation model. Section 4 shows the simulation experiments and analyzes the experimental results. We will conclude our research in section 5.

2 SIMULATION-OPTIMIZATION FRAMEWORK FOR SDVRP

A simulation-based optimization that integrates genetic algorithm (GA) and discrete event simulation (DES) within an evolutionary computing framework is proposed to optimize the split delivery vehicle routing problem. Figure 1 shows the overall architecture describing the integration of simulation sub-system and optimization sub-system. The architecture has two main models that work independently and interact with each other: a simulation model and an optimization model. Order arrives at RDC (Regional Distribution Center) randomly. The order information such as order size (weight or volume), due date, location and so on will be input into the simulation model, where some split and consolidation policies will be applied first to determine if this order can be combined with other ones to build a bigger batch and be delivered together by one carrier. If the split/consolidation can be done immediately, the information of the orders in this split/consolidation will be transferred to the optimization model where "good" vehicle routes will be produced by some optimization algorithms (e.g., genetic algorithm). Otherwise, the order (entity) will be held in a queue and wait for consolidation. When order split and consolidation happens, the optimization model (implemented here by GA) will first build a population (routes for the orders in this consolidation). This vehicle routes are then fed back to the simulation model and the transportation process is simulated as well as the system's performance such as total cost are obtained and will be transferred back to the optimization model. If a satisfied route is achieved, the split, consolidation and routing results will be finally output to the real-life system. Otherwise, the evolution process of GA, i.e. selection, crossover and mutation will be carried out so that better routes can be found. To interact between the simulation model and optimization model, an interface is usually needed.

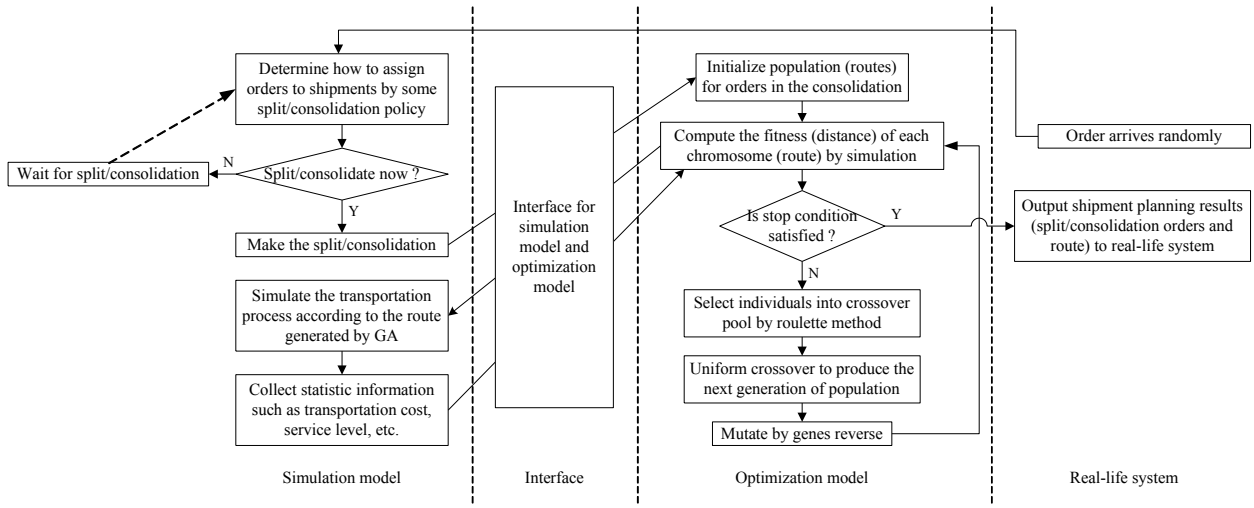

Figure 1: Overall simulation-optimization framework.

In the design of GA for routing optimization, we adopt decimal numbers to encode a solution/route. For example, gene string "3, 1, 4, 5, 2" stands for a chromosome (solution/vehicle route) that five orders are combined together into a batch for transportation by one vehicle. And order 3 will be first delivered, then order 1, 4, 5. When the last order (order 2) is delivered, the vehicle will come back to RDC. Roulette method is used to select individuals. Uniform crossover is developed to produce next generation of population and genes reverse is introduced as mutation operator for GA (Gen and Cheng 2000).

Simulation is powerful for modeling and analyzing complex systems where uncertainties and dynamic processes are main issues. While genetic algorithm does well in finding good solutions for the problem defined. As a result, such an integrated approach provides a flexible and effective way for solving complex optimization problems (Pan et al. 2011; Zhou et al. 2013). With this simulation-based optimization framework, other optimization methods such as Tabu Search, Simulated Annealing can also be integrated with simulation.

3 SPLIT POLICIES DESIGN AND CONCEPTUAL SIMULATION MODEL

The type of systems under this study is a RDC that carries inventory to serve customers (e.g., retailers) located within a geographic region. The uncertainties of demands such as arrival time, order size, due date, and the uncertainties of transportation such as transportation time are considered in our simulation system. One consolidation policy (Q-policy) and three split policies (LOS-policy, LDD-policy and LWT-policy) are studied in this paper. In Q-policy, the orders are processed via FIFO (first in first out) rule. Suppose there are m orders waiting in the consolidation queue when the ith order comes in ($m \geq 0$ and $i \geq 1$). We define s_k as the size (volume or weight) of the kth order, Q as the total order size ($Q = s_i + s_1 + s_2 + \cdots + s_m$), and C as the truck capacity. If $Q < C$, the ith order will wait in the consolidation queue according to FIFO rule. If $Q = C$, the ith order will be consolidated with all the m orders in the queue to form a batch and delivered together to the customers in one route. If $Q > C$, the m orders in the queue will be consolidated and delivered in a batch and the ith order will wait for next consolidation. Q-policy is a classical consolidation policy. So it is regarded as a benchmark of non-split policy to compare with the split policies (LOS-policy, LDD-policy and LWT-policy) proposed in this paper.

As described above, only quantity factor is considered in Q-policy. To improve the split/consolidation effectiveness, we design three split policies (LOS-policy, LDD-policy and LWT-policy) in which the quantity and time factors are considered simultaneously. When an order entity enters the simulation system, a new attribute named *CanWaitTime* is assigned. The attribute *CanWaitTime* is used to determine how long the order can be "hold" in the consolidation queue according to its lead time and estimated

transportation time. When *CanWaitTime* goes by, the order will be delivered. The other orders in the consolidation queue will then be consolidated and delivered together with this order as many as possible. Pan et al. (2011) indicated the details for estimating *CanWaitTime*. Due to the space limit, we do not show it in this paper. LOS-policy, LDD-policy and LWT-policy are all based on the estimation of *CanWaitTime*. The difference among these split policies is the mechanism of selecting split order. When split occurs in LOS-policy, the order with largest order size is selected to be split. While in LDD-policy, the order with latest due date is selected. In LWT-policy, the order with largest *CanWaitTime* is selected to be split.

4 SIMULATION EXPERIMENTS AND RESULTS ANALYSIS

To test and validate the conceptual model proposed, we use general-purpose simulation platform MATLAB© to develop executable models for experimental studies. M-function embedded in MATLAB© is employed to implement the genetic algorithm and regarded as an interface to integrate simulation with optimization. We omit the implementation details in this paper since they are mainly realized by computer programming with C language in MATLAB©.

4.1 Benchmark and its Simulation Results

We first present an exemplary case that twenty customers are clustered in the religion of RDC. Table 1 gives the input parameters of current operations (benchmark). Using the integrated model that combines simulation with optimization, we analyze the system performance of Q-policy and the three split policies proposed under an uncertain environment. Some important performance indices are defined as follows:

- Total late cost = Unit late cost × Total late time;
- Total fixed cost = Fixed cost per truckload × Number of truckload;
- Total transportation cost = Unit transportation cost × Total transportation distant × Total transportation quantity;
- Total cost = Total transportation cost + Total late cost + Total fixed cost;
- Average fixed cost = Total fixed cost ÷ Total size of orders;
- Average late cost = Total late cost ÷ Total size of orders;
- Average transportation cost = Total transportation cost ÷ Total size of orders;
- Average total cost = Average fixed cost + Average late cost + Average transportation cost.

Table 1: Simulation input parameters.

parameter	value
Truck capacity (*C*)	5 tons
Unit transportation cost	3 RMB/(ton·km)
Order size	UNIF(2.5, 3.5) ton
Fix cost	3000 RMB/truckload
Arrival interval time	EXPO(1/20) hour
Lead time	TRIA(6, 8, 10) hour
Unit late cost	10 RMB/hour late
Truck speed	TRIA(40, 70, 100) km/hour

Note: UNIF - uniform distribution; EXPO - exponential distribution; TRIA - triangle distribution; RMB - The Chinese currency unit.

Table 2 shows the simulation results of the benchmark. As shown in Table 2, LDD-policy has the minimal average total cost (1420.077 RMB/ton) while Q-policy has the maximal average total cost

(1556.494 RMB/ton). LOS-policy and LWT-policy are not as good as LDD-policy in average late cost, average transportation cost and average total cost. However, both of them outperform the non-split policy (Q-policy) in average total cost. These results demonstrate that split delivery can also reduce average total cost under uncertain environment. By comparing average fixed cost with average transportation cost and average late cost, we can see that the main reason of the decrease in average total cost of split delivery is as follows. Truckload via split delivery may increase the transportation distance of orders in one route (there are more orders in the consolidation due to order split), which leads to higher average transportation cost and average late cost. However, truckload can result in fewer vehicles so that the average fixed cost declines obviously, which makes the average total cost in split delivery lower. In the benchmark, the average order size is about half of the truck capacity. Consequently, it is hard to form order consolidation in Q-policy and most orders will be delivered separately. So the average late cost and average transportation cost of Q-policy is the smallest one and the average fixed cost of Q-policy is the largest one among these four policies.

Table 2: Simulation results of the benchmark.

Consolidation/split policies	Average Total Cost (RMB/ton)	Average Transportation Cost (RMB/ton)	Average Fixed Cost (RMB/ton)	Average Late Cost (RMB/ton)
Q-policy	1556.494	556.6113	999.8815	0.001443
LOS-policy	1471.185	828.3003	604.2635	38.62097
LDD-policy	1420.077	781.7886	600.9658	37.32263
LWT-policy	1476.917	823.978	600.2229	52.71572

4.2 Effects of Split Policies Under Different Conditions

To analyze the effects of split policies under different conditions, some sensitivity experiments about order size, arrival rate, and geographical distribution of customers are done in our study. All the experiments are based on the benchmark. When a system parameter is changed to do the experiments, the other parameters keep unchanged.

Table 3 compares the average total cost of Q-policy and the three split policies when average order size changes (the distribution pattern and the variance of order size keep unchanged). As we can see, order size has significant impact on system performance. With the decrease of average order size, the average total cost goes up because there are more orders in a consolidation need to be delivered when the order size is small, which increases the transportation cost and late cost apparently. While in Q-policy, a peak value appears when the average order size is 3 tons (60% of truckload). The reason is as follows. Most orders with 3 tons has to be delivered separately (the truck capacity is 5 tons), which leads to low loading rate and high fixed transportation cost. However, orders with small size (e.g., 2 tons) are easier to be consolidated and orders with large size (e.g., 4 tons) result in higher loading rate, which makes the average total cost decrease when order size is small or large in Q-policy. Table 3 also indicates that split delivery is not always profitable, especially when the order size is very small. When the average order size occupies about 60% truckload, split delivery reaches its best performance. In general, LDD-policy is better than LOS-policy and LWT-policy under the condition of different average order size.

Table 3: Average total cost of consolidation and split policies at different order size.

Consolidation/split policies	Average order size (tons)					
	1	2	2.5	3	3.5	4
Q-policy	1969.06	1443.022	1450.125	1556.494	1335.238	1168.572
LOS-policy	2333.498	1681.074	1555.272	1471.185	1409.383	1359.329
LDD-policy	2315.653	1649.632	1511.854	1420.077	1359.435	1319.942
LWT-policy	2345.526	1708.289	1579.912	1476.917	1402.802	1373.067

The comparison of different split/consolidation policies at different arrival rate is shown in Table 4. The simulation results indicate that the change of arrival rate has no significant impact on non-split policy while it affects the performance of split policies apparently. With the increase of arrival rate, more orders enter the simulation system and it is easier to form consolidation/split. As a result, the average total cost drops down in split delivery. Similarly, the LDD-policy is the best among these three split policies.

Table 4: Average total cost of consolidation and split policies at different arrival rate.

Consolidation/split policies	Average arrival rate (number of arrival orders per hour)					
	2	4	6	8	10	20
Q-policy	1555.011	1561.04	1559.193	1554.343	1554.119	1556.494
LOS-policy	1515.595	1512.588	1507.162	1501.319	1488.777	1471.185
LDD-policy	1492.502	1486.754	1471.561	1457.617	1449.926	1420.077
LWT-policy	1523.43	1500.864	1511.293	1491.767	1486.437	1476.917

To analyze the effects of geographical distribution of customers, we keep the number of customers (i.e. twenty customers) unchanged while the area coverage extends from 300 kilometers to 500 kilometers. The customers is scattered among this area randomly. Namely, the distribution of customers is sparser. Table 5 shows the average total cost of consolidation and split policies in both compact geographical distribution (benchmark) and sparse distribution of customers. As we can see, due to the longer transportation distance in sparse distribution, the average total cost goes up no matter which policy is adopted. The simulation results also demonstrate that in sparse distribution, non-split delivery outperforms split delivery because there are more orders need to be delivered and the longer transportation distance leads to higher transportation cost and late cost when orders are split into several parts. LDD-policy is still better than LOS-policy and LWT-policy in sparse distribution.

Table 5: Average total cost of consolidation and split policies at different geographical distribution of customers.

geographical distribution of customers	Consolidation/split policies			
	Q-policy	LOS-policy	LDD-policy	LWT-policy
Compact distribution	1556.494	1471.185	1420.077	1476.917
Sparse distribution	1698.378	1797.895	1785.34	1824.43

5 CONCLUSIONS

This paper focuses on the issues of split delivery vehicle routing problem (SDVRP). To overcome the shortcomings of traditional analytical methods when solving this kind of complex problem, a simulation-based optimization approach is proposed. Our contribution lies in two folds. On the one hand, we design a general-purpose simulation-optimization framework that integrates discrete-event simulation with genetic algorithm to solve SDVRP. Simulation is used to model the complex logistics system, capture the dynam-

ic characteristics and obtain the system performance. Genetic algorithm is developed to find a "best" vehicle route for the consolidated orders by evolution computing with the guidance of system's performance obtained via simulation. With this framework, other optimization methods can also be employed. On the other hand, three split policies, LOS-policy, LDD-policy and LWT-policy are proposed. The results of simulation experiments indicate that split delivery is not always effective. Some important conclusions are summarized as follows: (1) the average order size has significant impact on consolidation and split policies. When the average order size occupies about 60% truckload, split delivery is better than non-split delivery apparently. (2) The average arrival rate of orders is more sensitive to split policies than non-split policies. The larger the arrival rate is (i.e. the more the orders enter the logistics system), the better performance can be obtained via split delivery. (3) Sparse distribution of customers makes the effectiveness of split delivery deteriorate and even worse than non-split delivery. (4) Among the three split policies proposed in this paper, LDD-policy outperforms the other split policies in various simulation scenarios. As for the future research, we are interested in studying the choice behavior of customers in SDVRP. For example, some customers may accept split delivery while others may not. Some customers may be willing to receive their goods in several batches if they can get a discount in price, while others may not. If customers have these personalized services, what is the influence to the distribution system and how to balance the economic performance of split delivery and customer satisfaction?

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