

SMALL-SCALE: A NEW MODEL OF SOCIAL NETWORKS

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ABSTRACT

Despite social network analysis has been subject to scientific interest, further study of the dynamics of networks have been recently developed by a new interdisciplinary science, rooted in sociological research and in the evolution of Graph Theory: the Science of Networks. Researches in this area subverted predefined concepts, presenting revelations about the interconnected social universe, highlighting the demystification of six degrees of separation with the confirmation of “small world” phenomenon. Grounded on these findings, we propose in this paper a new model of social networks called small-scale networks, engendered by the improvement of existing models. As a way of validation, we have built the Fluzz application, able to simulate the generation of social networks through this new model, and through other major literature models (random, small-world and scale-free networks), graphing the networks conceived. The simulation results have shown that small-scale networks are a real alternative to the evolution of society.

1 INTRODUCTION

The subtle entanglement of humans in a unique and immense open, interactive and cooperative fabric generated currently an absolutely unprecedented situation for society: the era of global connectivity. For this era could be understood, researchers from various fields such as mathematics, physics, biology, sociology and computer science have joined forces to describe it scientifically, through intensive study of networks. The main objective was to find alternatives to traditional dealings that considered the formation of networks, including social networks, a purely random process.

This transdisciplinary foundation challenged the scientific community to explore the adveniente revolution: the Science of Networks (Barabási 2003). Originally proposed by the researchers Duncan Watts, Steven Strogatz and Albert Barabási, this science adopts a vision of society oriented by networks in evolution and continuous self-constitution (Watts 2003). The preeminent idea prescribed that a hidden pattern is behind the way people interact, and despite population growth, the distance between individuals is decreasing, leaving them connected by only six degrees of separation in a “small world”.

The transposition of analysis focus, from the knowledge of isolated individuals to their complex interaction dynamics in network, allowed to the researchers a new light on the understanding of emergent organizational patterns that make social networks to produce organically their settings. Currently, the branch of mathematics known as Graph Theory, proposed by mathematician Leonhard Euler, is the basis of thinking about networks. In many ways, the result obtained by Euler symbolizes an important message from this work: the construction and structure of graphs or networks are the key to understanding the complex world that surrounds mankind.

Under the aegis of this new science’s concepts we designed a new model of social networks, which presents itself as a natural evolution of previous models. Called small-scale networks, this model merges

the thoughts of Watts and Strogatz to the Barabási's, eliminating flaws, and engaging the ideas like pieces of a puzzle to form a more coherent image.

As a way of validating the small-scale networks, we built the Fluzz application, designed to simulate the generation of networks according to this new model, and according to the other main models proposed in the literature: random networks, small-world networks, and scale-free networks. To facilitate the cognitive process of interpretation, we added to Fluzz mapping and graphical transformation resources to represent the networks conceived. All these theoretical and practical apparatus developed were essential for our exploratory analysis of indications, with consequent confirmation of evidences. When we compared the data generated by simulations of all models, we were able to conclude that the small-scale networks are a real alternative to the evolution of social networks.

The remainder of this paper is organized as follows: section 2 presents the concept of social networks at the prospect of the Science of Networks, in addition to explaining the “small world” problem, and the enigma of six degrees of separation. Section 3 presents the main network models already created by the researchers, emphasizing their particular properties and applicability, especially in the social context. Also in this section, the small-scale network model is introduced, with all the details of their design being clarified. Section 4 shows how the Fluzz application is designed to validate the new model of network created, detailing its features and displaying the simulations results. With a thorough analysis we argue how this new model was able to balance the conflicting forces of two major previous network models: small-world networks and scale-free networks. Finally, the last section contains the concluding remarks on the theoretical and practical benefits of the work, besides exposing the possible future research in context.

2 SOCIAL NETWORKS

Basically, a social network is a set of people and their connections with each other. In the absence of a regulatory center, there is no meticulous designs behind this type of network, that self-organizes offering a vivid example of how the independent actions of millions of nodes and links lead to an emergent behavior. Social networks evolve organically from the natural tendency of people make their friendships, and these choices are exactly what make each individual occupy a specific location in the network.

For the Science of Networks, there is a fundamental aspect in the emergence and evolution process of social networks, related to the specific pattern of ties that connect the components in a system: the topology. This pattern interferes in the network connection rules, making social ties often more important than the individuals themselves. People apparently “similar” may have actually quite different structural positions, which make them susceptible to heterogeneous influences. However, without a complete mapping of the network, there is no way to have this comprehension (Christakis and Fowler 2009). This structural perspective designs a new light on many social processes.

2.1 The Small-World Problem and the Six Degrees of Separation

In 1967, the social psychologist Stanley Milgram conducted an experiment to investigate a hypothesis which stated that the world, seen as a huge network of social relations, was in some sense “small”. The small-world problem, as it became known, ensuring that anyone could be reached through a network of friends in just a few steps (Milgram 1967).

The Milgram's goal was to estimate more precisely the “distance” between any two people in the United States, since at that time, there was little consensus about this measure, being a typical estimate in the hundreds. The experiment involved the shipment of letters to randomly selected residents, asking them to try to contact the target person through their network of friendship. Surprisingly, 43 of the 160 letters arrived at their destinations, allowing Milgram calculate the average number of intermediate persons, on the occasion 5.5, rounded to the famous six degrees of separation.

In 2003, the researchers Peter Dodds, Roby Muhamad, and Duncan Watts replicated the Milgram's experiment on a global scale using e-mail as a mode of communication. They recruited thousands of

volunteers to send a message to the targets around the world, and roughly six steps were needed on average to send the email to each target, confirming the Milgram's estimate (Dodds, Muhamad, and Watts 2003).

The six degrees of separation intrigue the scientists for two reasons. First, they suggest that global society can be navigated by social links from one person to another, and second, because in a network with more than 7 billion people, any pair of nodes is on average six links far from the others.

In an attempt to model the problem, Watts exemplified the situation where a person, Ego, has a hundred friends (Watts 2003), with each Ego's friend also having a hundred friends. Therefore, with one degree of separation, Ego is connected to one hundred people, and with two degrees, ten thousand. With three degrees, it reaches one million, with four, a hundred million, and with five, ten billion people. Under this reasoning, illustrated in Figure 1 (a), if everyone in the world had one hundred friends, in six steps would be possible to reach the entire population of the planet.

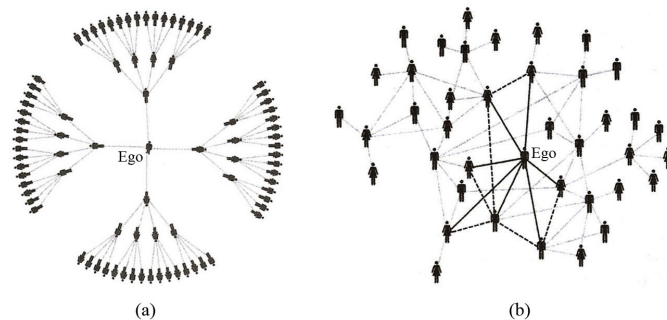


Figure 1: (a) Hypothetical Social Network. (b) Real Social Network.

Nevertheless, with minimal sociological inclinations, it is possible to realize that the real social networks resemble more the image in Figure 1(b). They exhibit clustering, because the individuals's friends are also friends with each other, creating redundancy (Watts 2003). The paradox of social networks that the Milgram's experiment has shown, is that despite high social clustering, people can still contact any individual in an average of a few steps, intriguing the scientists to figure out how this could be possible.

3 NETWORK MODELING

Because of their sophistication, simplify complex networks as the society in graphs is something that raises some challenges. Discovering a unique model to describe this, and other different systems may seem, at first glance, an insurmountable task, boldly accepted by the Science of Networks. The concealment of particularities revealed the laws that governing the evolution of society in web form, and allowed scientists to understand, that in a world that could conceivably be devoid of any discernible order, in reality the order abounds, even in a context of overwhelming disorder.

3.1 Random Universe

In the 1950's, the mathematicians Paul Erdős and Alfréd Rényi proposed a mathematical formulation to describe all complex graphs with a single schema. As each system obeys disparate rules in its configuration, both mathematicians admitted that the real networks were random (Karonski and Rucinski 1997).

Erdős found that, regardless of the amount of nodes, a small percentage of random links is always enough to connect the network as an almost completely connected component, and that the necessary percentage decreases as the network grows (Buchanan 2003). In a network of 300 points, no more than 2% of the links between them ensure the formation of a single connected component. To 1000 points, the fraction is less than 1%, and to 10 million points, only 0.00016%.

Mathematically, in a network with N vertices, the percentage of connections needed to tie the network into a single component is given by: $\frac{\ln N}{N}$, where $\ln N$ is the the natural logarithm of N (Buchanan 2003).

For a network with seven billion people, the fraction shown by Erdős would not be more than 0.000000004. This number implies that if people really were connected randomly, a typical person would have to know about one person every 305 million for the world population to form a connected graph. That means only 23 acquaintances to each person, a low number compared with the established by society.

Under this reasoning, it would not be surprising that any two people in the world can be connected through a social connections path. However, for more sophisticated than random graphs theory be, recent discoveries about real networks suggest that they are not similar to the Erdős and Rényi graphs. If people really chose their friends randomly, would be much easier meet friends from another continent than in their own neighborhoods. From everyday experience, it is known that a person's friends tend to know themselves, therefore random graphs can not be a good representation of the society.

3.2 Clustered and Connected Universe, Not Random

Inspired by the networks work of the mathematician Anatol Rapoport (Rapoport 1963), the sociologist Mark Granovetter investigated what could be the secret of small worlds (Granovetter 1973). For Granovetter, a parameter disregarded by scientists was a crucial piece to this puzzle: the strength of social ties.

Overall, he called strong ties, those that exist between members of the same family, or between good friends, while weak ties join just acquaintances. The Granovetter's insight was to realize that with the evolutionary dynamics of networks, strong ties tend to appear in triangles being responsible for the social clustering. In practical terms this amounts to saying that two strangers, strongly linked to a third in common, tend to relate to the time, being incomplete triangles found mainly in groups dominated by weak ties.

From this perspective, if a strong tie of a social network is removed, it will cause little effect in the number of degrees of separation, because as they almost always appear in triangles, it would still be possible to go from one point to another of the broken link in just two steps, moving along two remaining edges of the triangle (Buchanan 2003). Nevertheless, when weak ties are removed, the inevitable consequence may be the network fragmentation in disconnected components. For Granovetter, weak ties create shortcuts or bridges that shorten social distances, providing a structural arrangement for the existence of a small world.

The Granovetter argument proposes a society very different to the random universe from Erdős and Rényi. This is a collection of complete graphs, joined by weak ties between some known belonging to different circles of friends, as shown in Figure 2 (Barabási 2003).

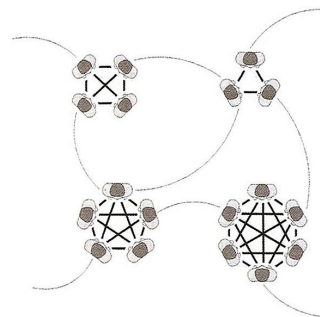


Figure 2: The Topology of Granovetter's Society.

3.3 Small-World Networks

The theory of random networks conciliated with the social structure described by Granovetter, and with the discovery of six degrees of separation performed by Milgram, prepared the ground for a scientific revolution that is reverberating today in different areas, including sociology. It is this foundation that has enabled the researchers Duncan Watts and Steven Strogatz find a different way to connect the dots in the graph, fundamental to the process of revealing the enigma of six degrees of separation.

Initially, Watts and Strogatz introduced a metric for determining the rate of network clustering: the clustering coefficient. This measure is obtained by dividing the number of links between the friends of someone, by the number of possible links of friendship. From the Granovetter's perspective, the society should have numerous clusters, what would determine a high value for this metric.

Aiming to create a simplified model of social networks, the researchers began to interpolate networks passing through all the intermediate stages between complete order and complete disorder. Named *Beta Model*, this model was based on a regular reticulate as shown in Figure 3(a), in which each node is connected to a fixed number of nearest neighbors in the ring (Watts 2003). Then, they randomly picked links in the ring and started rewiring them. In practice, they assigned a value between zero and one to the adjustable parameter *beta*, randomly reconnecting each link in the reticulate with *beta* probability. When $\beta = 0$, no reconnection occurs as in Figure 3(a), and when $\beta = 1$, all links are rewired, as in Figure 3(c).

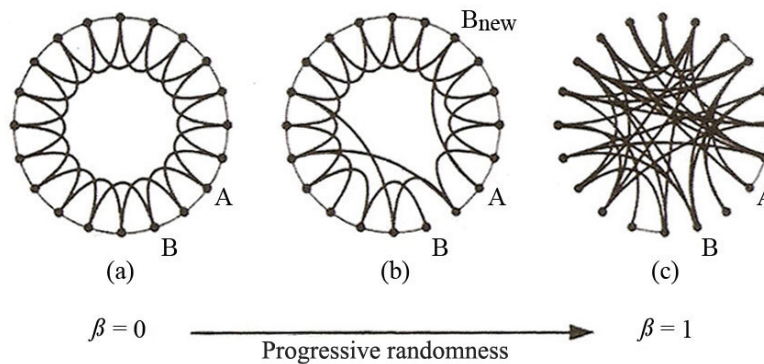


Figure 3: Beta Model.

With a small probability of rewiring, illustrated in Figure 3(b), the resulting object resembling a regular reticulate, despite the presence of some random connections. When Watts and Strogatz examined the clustering coefficient of rewired people, they realized that it remained high because most of their friends still knew each other. However, the length of the paths changed dramatically. As the links were randomly rewired, the highest probability was they were connected to someone far (Watts and Strogatz 1998). This way, few random links tended to create shortcuts that shortened the paths between nodes previously distant.

Under this conjecture, the networks would be able to display high clustering, plus a small distance between their components. Watts and Strogatz called this class of *small-world networks* (Watts and Strogatz 1998). Their properties have been identified in several different types of real networks, such as Internet, cells, the ecosystem, or society, implying a possible organizational pattern of nature. This pattern always seems arise when it is incorporated some degree of order and disorder in the constitution of a system.

3.4 Scale-Free Networks

When Watts and Strogatz's work was published, the research group of Albert-László Barabási was also trying to understand the structure of complex networks. For this purpose, they conducted a Web mapping, identifying the network created by many websites and their links. When Barabási and his colleagues embedded the histogram of node connectivity in a log-log graph, the result showed that the links distribution on these sites followed a mathematical expression called power law (Barabási and Albert 1999).

The assumption made for Watts and Strogatz about networks in general, is they follow a normal degree distribution $p(k)$, similar to Figure 4(a), known as bell curve. This means there would be not only an average degree well defined as most nodes would have a degree k not far from this average. However, the power law distribution differs crucially from normal distributions for two reasons. First, a power law does not have a peak at its mean value, starting at the maximum and decreasing to infinity, as shown in Figure 4(b). Second, the decay rate of power laws is much slower than the normal distribution.

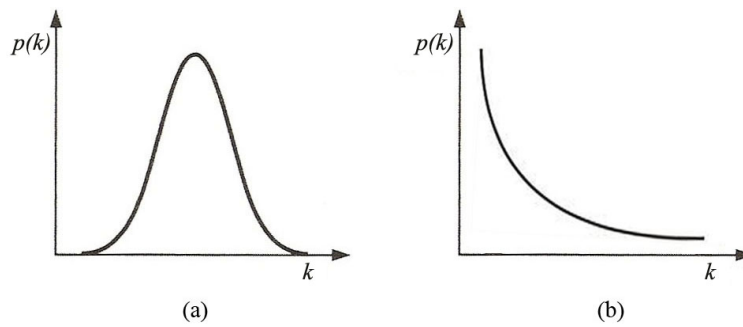


Figure 4: (a) Normal Distribution. (b) Power Law Distribution.

After the discovery about the Web, the Barabási's group found the same distribution pattern in various other types of networks, including social networks. Given the evidence exposed by this research group, power laws have forced scientists to abandon the idea of a scale or a feature node in the networks studied, being therefore described by the group as *scale-free networks* (Barabási and Albert 1999).

In order to explain the formation of hubs and power laws, Barabási along with Réka Albert proposed a mechanism by which the networks could evolve over time, based on two fundamental laws: growth and preferential attachment (Barabási and Albert 1999). Any type of network certainly starts with few nodes, grows incrementally adding new nodes, and reaches the current size. This growth process however, according to the researchers, is governed by other important precept, the preferential attachment.

The researchers concluded that in real networks, the connection is never random, because it depends of popularity (Barabási and Albert 1999). Websites with more connections are more likely to be connected again, and similarly, in social networks, people who are more well known make more new friends. Guided by this law, individuals unconsciously add links to a higher rate for choosing who are already highly connected, leading to a phenomenon known as rich-get-richer.

To answer whether these laws are sufficient to explain the hubs and power laws, the researchers have proposed a network model that incorporates both laws through the following rules (Barabási 2003):

- Growth: for each iteration, add a new node to the network;
- Preferential attachment: each new node must connect to existing nodes with two links, with the probability of selection proportional to the number of links of nodes.

This model shown in Figure 5, combining growth and preferential attachment, was the first successful attempt to explain the hubs, since it was able to generate power laws (Barabási 2003). The networks that emerged through this model, named *Scale-Free Model*, were always the same in terms of architecture, being clustered, with short paths between elements, and revealing the peculiarity of hubs. For the Barabási's group, this certainly would also be the evolutionary pattern of social networks, directly responsible for the condition of small world that surrounds society.

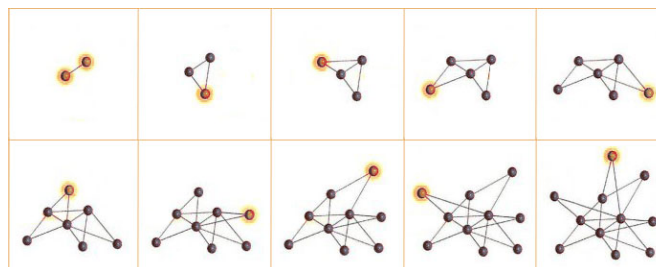


Figure 5: Scale-Free Model.

3.5 Gamma Model

Faced the enigma of six degrees of separation, two distinct theories were presented by scientists. On one hand, Duncan Watts and Steven Strogatz demonstrated how some shortcuts can make a difference in a clustered network. Nevertheless, they did not explain the existence of hubs, elements that by having many links, play a role similar to the Granovetter's bridges. On the other side, Barabási and Albert model neglected the conditioning elements of social structure, assuming that the preferential attachment is the only engine that drives the evolution of social networks.

In order to define relevant criteria for a comparative analysis of the models presented, and that could be significant to the proposition of a new model of social networks, we analyzed four observable features of this kind of network: the clustering coefficient, the average geodesic distance, the size of the largest hub and the network degree distribution. Table 1 presents roughly on a scale from 0 to 5, the evaluations of small-world and scale-free models before these characteristics, together with the assessment of the random model that serves as a reference parameter.

Table 1: Evaluation of the Network Models of Literature.

	CC	AGD	LH	DD
Random Networks	0	4	2	Normal
Small-World Networks	5	2	1	Normal
Scale-Free Networks	1	5	5	Power Law

Label: CC = Clustering Coefficient, AGD = Average Geodesic Distance, LH = Largest Hub, DD = Degree Distribution.

It is important to clarify that such evaluations do not serve as isolated metrics and should be used only as a comparative parameter between the different models. A scale-free network can not be characterized as weakly clustered. Nevertheless, if these networks are compared with small-world networks, certainly they will present much smaller clusters. This happens basically because scale-free networks do not use the adjacent social structure as a major factor in the connection process. Therefore, in this feature, each network respectively received the evaluations 1 and 5.

In contrast, another process seems to point a weakness in small-world networks. Separately, these networks can not be labeled as those that have typical long paths. However, when comparing them with scale-free networks, the typical length is usually much longer. The reason is the inability of small-world networks produce hubs, elements that if present, shorten distances quickly as they become better connected.

According to our perception, to get closer to real topology of society, the new model of social networks to be proposed should be able to positively standardize the four features measured. Reached this purpose, the model would represent a new universe for networks generation, stabilizing weaknesses, and allowing the findings of previous models were conciliated.

What we have done with the so-called *Gamma Model* was basically to place the patterns of small-world and scale-free networks under the same aegis. Using the same reasoning of Watts and Strogatz when they designed the Beta Model, we defined an adjustable control parameter *gamma*, which was the interpolation variable to be adjusted between these two types of network.

Consequently, we assigned values between zero and one to the parameter, determining with this procedure, which would be the connection pattern of the vertices added to the networks. If $\gamma = 0$, the graphs would present small-world networks topologies, and at the other extreme, when $\gamma = 1$, scale-free networks would dominate the graphs structure. Figure 6 presents the evolutionary dynamics of this model.

The uncertainty was what happened in the middle of the process. When the gamma value was small, the social structure became highly important for determining the links, but certainly some preferential attachment would be conducted. However, when the parameter value was high, preferential attachment dominated the connective intention of the vertices, however some local connections could be made. The expectation is that at some intermediate region, something amazing could be discovered.

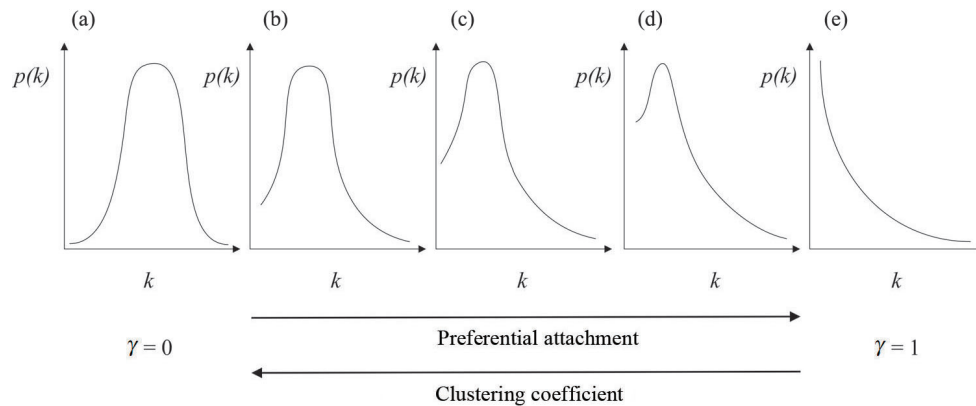


Figure 6: Gamma Model.

4 THE FLUZZ APPLICATION

The preeminent idea for the design of the Fluzz application was to provide practical resources to simulate the generation of social networks, following the concepts defined by each network model presented above, including the new model which should be validated. The simulation results can be analyzed graphically in the Fluzz interface, since visual mapping and transformation capabilities were also incorporated to it.

We use three different types of open source technology to design the application. The development platform was JavaSE (Java Platform Standard Edition), in version 7.0 (Oracle 2013). The data generated by the simulations are stored in a database provided by the database management system PostgreSQL, in version 9.0 (Group 2013). Finally, the graphic composition of the networks is performed by the framework JUNG (Java Universal Network/Graph Framework), in version 4.2 (Team 2010), a library that provides a language for modeling, analysing and visualizing data that can be represented as graphs.

4.1 Simulating the Networks Generation and Graphically Representing them

The Fluzz's top panel has the feature that enables simulating the networks generation. Initially there should be made a choice between the models available: Erdős and Rényi, Watts and Strogatz, Barabási and Albert, and the new network model designed in this work, referenced in the application as Marin/Carvalho.

Once the model to be used was informed, it is still necessary to fill two fields to accomplish the networks generation procedure. First, it must be informed the total number of vertices to be produced, and second, the amount of connections to be established for each new vertex created. The graph to be produced will reproduce the essential features of the model reported in its generation.

Figures 7(a), 7(b), 7(c), and 7(d) comprise networks with 50 vertices generated by the four models mentioned above, with two connections being added for each new vertex created. In this example, the graphs were visualized by Fruchterman-Reingold force-directed algorithm for node layout (Fruchterman and Reingold 1991). The main features of this algorithm are to minimize the crossing edges and unify their lengths, which places nodes more connected at central regions in the graph. Additionally, we have created a vertex pigmentation resource that accompanies its connection degree, which as can be seen in the pictures, makes the vertices more yellow as they become more connected.

Figure 7(a) represents the network generated with the Erdős and Rényi model. According to data calculated by the application, the clustering coefficient of this network was 0.089, the average geodesic distance, 2.870, and the vertex most connected presented 11 connections. Figure 7(b) corresponds to the network generated following the Watts and Strogatz model. For the clustering coefficient, the value found was 0.410, the typical length of the paths, 3.953, whereas the vertex most connected obtained 6 connections. Figure 7(c) indicates the network created by the Barabási and Albert model. The clustering coefficient was 0.152, the average distance of paths, 2.587, and the most connected vertex got 24 connections.

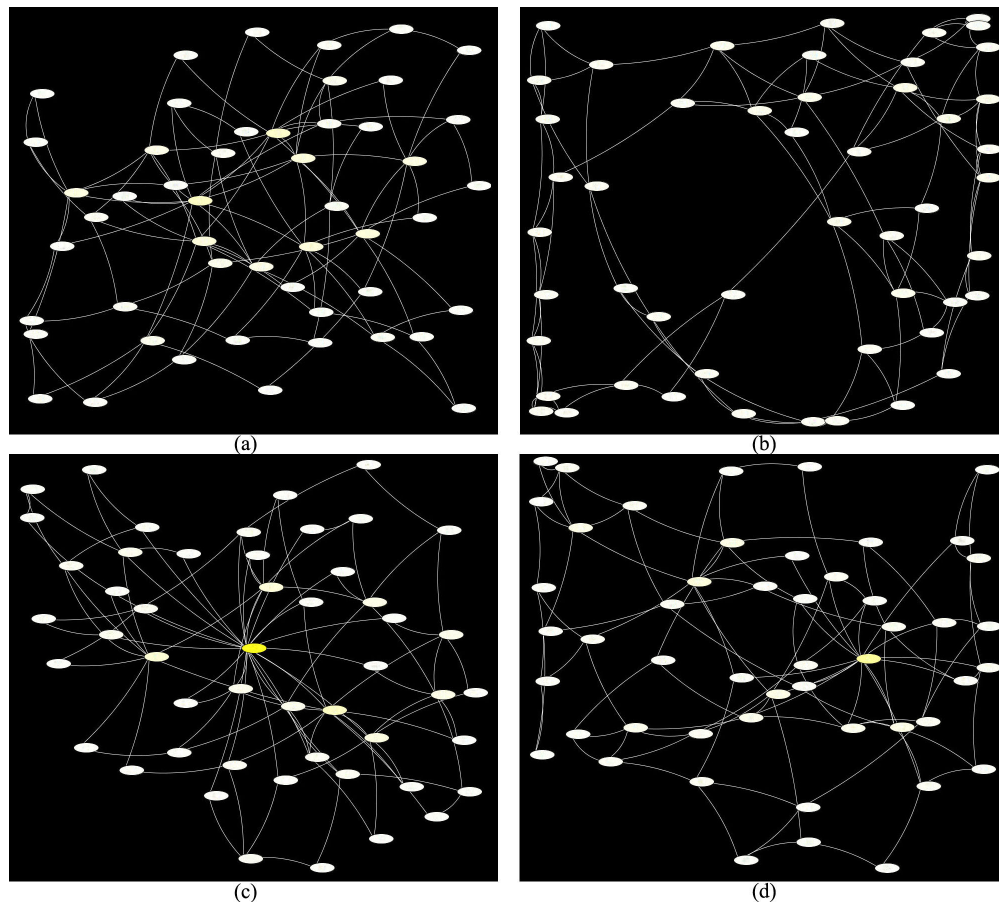


Figure 7: Networks Generated by the Following Models in Fluzz: (a) Erdős and Rényi. (b) Watts and Strogatz. (c) Barabási and Albert. (d) Marin and Carvalho.

Even these networks being relatively small, they were able to provide the expected standard features provided in Table 1. Random networks must have negligible clusters, small distances between vertices, and little chance to the emergence of hubs. Small-world networks are excelente for clustering, can relatively shorten the paths, but are poor to generate hubs. In contrast, scale-free networks are experts to create these hubs, with high performance for shortening paths, and low performance for clustering.

The Erdős and Rényi model implemented in Fluzz follows one specific rule: the randomness. the Watts and Strogatz model creates in 80% of the time local connections, and for the 20% remaining, random connections, based on the small-world networks pattern. The procedure of creating local connections tries to find the closest vertex that still has not been connected to the current vertex. To simulate the Barabási and Albert model, the application also follows a single rule: the preferential attachment. This rule states that the probability of a vertex receive a connection is proportional to its connection degree.

4.2 Small-Scale Networks: Verifying and Validating the Gamma Model

The goal of creating the Gamma Model was to positively standardize the fundamental features of social networks analyzed in Table 1. In practice, this means establishing acceptable values for each of these features in a new model, capable of reconciling all discoveries about social networks previously made by scientists. In this context, networks of different sizes and densities were generated according to the three models of literature, and according to three configurations of the Gamma Model. Table 2 presents the values obtained for these models, displaying the growth of the original networks.

Table 2: Values Obtained Through Simulation of the Network Models

N(V/L)	E/R			W/S			B/A			GM-40			GM-50			GM-60		
	CC	AGD	LH	CC	AGD	LH	CC	AGD	LH	CC	AGD	LH	CC	AGD	LH	CC	AGD	LH
100/2	0.03	3.37	13	0.37	4.57	6	0.12	3.02	19	0.23	3.81	11	0.13	3.53	13	0.15	3.51	12
100/5	0.12	2.28	23	0.47	2.56	15	0.24	2.20	41	0.34	2.41	17	0.28	2.34	24	0.19	2.29	25
100/10	0.24	1.86	39	0.57	2.02	25	0.35	1.84	66	0.44	1.92	30	0.36	1.89	36	0.32	1.87	38
200/2	0.02	3.79	14	0.36	5.30	7	0.05	3.33	27	0.19	4.23	15	0.10	4.08	16	0.11	3.95	14
200/5	0.07	2.59	27	0.45	3.01	15	0.15	2.44	56	0.31	2.76	24	0.22	2.69	26	0.15	2.63	30
200/10	0.13	2.09	50	0.52	2.37	27	0.22	2.03	97	0.35	2.20	33	0.28	2.14	44	0.24	2.13	46
500/2	0.01	4.48	19	0.35	6.41	8	0.03	3.79	41	0.19	5.00	22	0.11	4.67	21	0.07	4.61	18
500/5	0.02	2.93	32	0.45	3.63	17	0.07	2.74	92	0.29	3.21	30	0.21	3.12	32	0.14	3.03	39
500/10	0.06	2.43	61	0.50	2.75	30	0.12	2.33	155	0.31	2.56	42	0.24	2.51	56	0.17	2.48	62
1000/2	0.00	4.91	20	0.33	7.14	10	0.03	4.15	57	0.19	5.60	25	0.12	5.18	26	0.07	5.08	21
1000/5	0.01	3.25	33	0.44	4.00	17	0.01	3.63	400	0.25	4.59	57	0.18	4.38	67	0.12	3.30	49
1000/10	0.03	2.65	67	0.48	3.02	31	0.07	2.54	212	0.30	2.80	52	0.23	2.74	69	0.15	2.69	83
2000/2	0.00	5.37	22	0.34	8.02	10	0.02	4.33	87	0.19	6.03	26	0.12	5.70	30	0.07	5.46	27
2000/5	0.01	3.54	39	0.44	4.37	19	0.02	3.19	184	0.26	3.86	43	0.18	3.68	46	0.12	3.60	66
2000/10	0.01	2.85	72	0.48	3.34	32	0.05	2.71	282	0.29	3.05	62	0.21	2.97	79	0.15	2.90	101
5000/2	0.00	5.89	23	0.34	9.09	11	0.01	4.72	140	0.18	6.81	33	0.13	6.32	37	0.07	6.05	36
5000/5	0.00	3.90	42	0.44	4.92	20	0.01	3.44	277	0.26	4.30	49	0.19	4.10	56	0.18	3.94	81
5000/10	0.01	3.15	89	0.47	3.71	34	0.02	2.91	463	0.28	3.39	78	0.20	3.28	101	0.14	3.21	134
10000/2	0.00	6.35	24	0.33	9.69	12	0.00	5.00	198	0.18	7.34	41	0.12	6.81	46	0.07	6.44	43
10000/5	0.00	4.17	47	0.43	5.37	22	0.01	3.63	400	0.25	4.59	57	0.18	4.38	67	0.11	4.25	103
10000/10	0.00	3.39	94	0.47	3.40	38	0.01	3.04	649	0.28	3.63	90	0.20	3.52	126	0.14	3.46	169
20000/2	0.00	6.86	27	0.33	10.42	12	0.00	5.25	295	0.19	7.81	48	0.12	7.41	55	0.07	6.78	54
20000/5	0.00	4.44	54	0.44	5.70	23	0.00	3.84	558	0.25	4.90	69	0.18	4.65	81	0.12	4.48	130
20000/10	0.00	3.59	98	0.46	4.28	39	0.01	3.17	901	0.28	3.86	106	0.20	3.75	148	0.13	3.65	208
50000/2	0.00	7.38	27	0.33	11.40	12	0.00	5.61	472	0.18	8.52	61	0.12	7.85	79	0.07	7.34	72
50000/5	0.00	4.79	64	0.44	6.24	23	0.00	4.07	870	0.26	5.28	83	0.18	5.05	102	0.11	4.84	176
50000/10	0.00	3.85	112	0.46	4.67	40	0.00	3.44	1451	0.28	4.18	132	0.20	4.05	185	0.13	3.91	282
100000/2	0.00	7.76	29	0.33	11.38	13	0.00	5.83	692	0.18	8.98	72	0.12	8.31	94	0.07	7.72	87
100000/5	0.00	5.06	69	0.43	6.64	26	0.00	4.31	1231	0.26	5.61	92	0.18	5.32	120	0.12	5.1	224
100000/10	0.00	4.08	118	0.46	4.97	42	0.00	3.59	2031	0.28	4.42	158	0.20	4.25	216	0.13	4.13	345
200000/2	0.00	8.15	31	0.33	12.58	14	0.00	6.11	993	0.18	9.54	77	0.12	8.78	108	0.07	8.14	102
200000/5	0.00	5.30	73	0.44	6.99	27	0.00	4.48	1762	0.26	5.88	111	0.18	5.62	154	0.12	5.37	295
200000/10	0.00	4.29	129	0.46	5.21	44	0.00	3.77	2869	0.28	4.66	179	0.20	4.50	252	0.13	4.35	414
500000/2	0.00	8.69	32	0.33	13.58	15	0.00	6.47	1560	0.18	10.01	88	0.12	9.24	134	0.07	8.52	131
500000/5	0.00	5.65	76	0.44	7.41	30	0.00	4.68	2722	0.26	6.30	130	0.18	5.95	185	0.11	5.67	386
500000/10	0.00	4.57	141	0.46	5.55	47	0.00	3.90	4453	0.28	4.92	213	0.20	4.75	331	0.13	4.63	549

Label: E/R = Erdős and Rényi Model, W/S = Watts and Strogatz Model, B/A = Barabási and Albert Model, GM-40 = Gamma Model with 40% of preferential attachment, GM-50 = Gamma Model with 50% of preferential attachment, GM-60 = Gamma Model with 60% of preferential attachment, N(V/L) = Total Number of Vertices and Number of Links Created for Each New Vertex Added, CC = Clustering Coefficient, AGD = Average Geodesic Distance, LH = Largest Hub.

Each configuration of the Gamma Model defined the border between local and preferential connections. The GM-40 configuration assumes that the probability of preferential attachment is 40%, while the remainder will be used for local connections. The GM-50 configuration provides the same probability for both connection types, and the GM-60 configuration assumes that 60% of connections are performed preferentially.

During the measurements, we realized that values below 40% of preferential attachment make unfeasible the hubs creation. In contrast, values above 60% of this connection type, significantly affect the network clustering. For this reason, the Gamma Model's settings used for determining the best fit point between the two connection types, local or preferential, were between the extremes 40% and 60%.

The variation of Gamma Model's settings influences at the starting position of the degree distribution curve on the ordinate axis, as we predicted in the intermediate images of Figure 6. The explanation is that the more preferential attachment exists, higher the curve starts in this axis. Furthermore, this variation also influences the rate of the curve decay, which is softer as it becomes more attached preferentially.

Checking the data in Table 2, we observed that the Gamma Model GM-50 assumed the best values for the desired positive standardization of the analyzed network features. Obviously, the generation process of these networks may vary throughout its execution, making the connective configurations alternate according to several variables that involve individuals and environment. However, the GM-50 configuration reflected the best range for these variations, correcting the models' weaknesses.

When we fit the histogram of node connectivity for the Gamma Model GM-50 in a graph, the result was that the links distribution in the generated networks follows a hybridization between normal and power law distributions, as provided in Figure 6. The curve starts at the minimum connections number of a vertex. Then, rises following a normal distribution to the maximum point, which is approximately the initial value plus the intermediate amount of connections to be added for each new vertex created. Finally, from this climax, the curve descends relentlessly by the most of the remaining network data, following a power law.

For this property of merging two different types of networks, with a small region disposed under a normal distribution and the remainder under power laws, the Gamma Model GM-50 set a new network model called *small-scale networks*. Table 3 presents the analysis of its features, showing that with no weaknesses, this model puts clusters, short paths and hubs under the same aegis. With this validation, this model becomes a real alternative proposed for a more coherent definition of the society’s topology.

Table 3: Evaluation of Small-Scale Networks

CC	AGD	LH	DD
3	3	3	Hybrid (Normal and Power Law)

Label: CC = Clustering Coefficient, AGD = Average Geodesic Distance, LH = Largest Hub, DD = Degree Distribution.

Figure 7(d) shows the simulation results of the networks created via the small-scale model, in the same condition of the other models used. The clustering coefficient calculated was 0.210, the average distance of paths, 3.106, and the most connected vertex got 15 links, following the pattern presented in Table 3.

The images in Figures 8(a), 8(b), 8(c) show respectively the degree distribution of the small-world, scale-free and small-scale models, according to the simulation done in section 4.1. The bar colors represent the number of links, and the height of each bar represents the number of vertices. The curves which emerge in the graphic closely follow the patterns of each expected degree distribution, showing that the application has been verified and that the computational models have been correctly implemented.

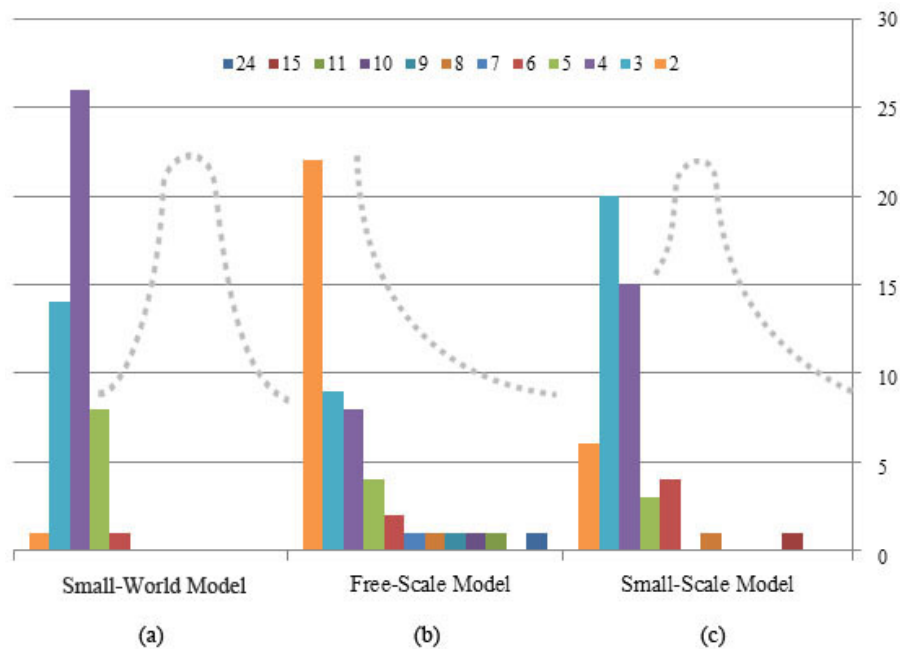


Figure 8: Graphical Analysis of Degree Distribution of the Network Models.

5 FINAL CONSIDERATIONS

In this paper we proposed the creation of a new model of social networks. The so called small-scale networks have led to a kind of unified theory of the small-world and scale-free network models. The validation of this new model was performed through simulations executed in the Fluzz application, built to provide computing resources that favored the thorough analysis of data generated by each network model.

The value of this result is not only the identification of a new type of architecture, which is appreciated by its differences before the previous conceptions about social networks, but how this tool can facilitate in

practice the understanding of the interconnected society. It is one more indication that to deeply understand the connectivity, we must recognize that different classes of networks require that different types of network properties be simultaneously explored, especially when the networks are formed by complex human beings.

Merging the main findings of scientists who joined the Science of Networks, the small-scale network model can be used for researches about the evolutionary dynamics of social networks. The intent of this research was to understand the properties of these networks, enabling them to be used for the benefit of humans. Understanding the organizational patterns of the network model proposed was the first step, that should be followed by the provision of techniques and methods capable to influence them.

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