Proceedings of the 2014 Winter Simulation Conference
A. Tolk, S. Y. Diallo, I. O. Ryzhov, L. Yilmaz, S. Buckley, and J. A. Miller, eds.

A SIMULATION BASED INVESTIGATION OF INVENTORY MANAGEMENT UNDER WORKING CAPITAL CONSTRAINTS

Illana Bendavid

ORT Braude College of Engineering
Karmiel Snunit 51 St. P.O. Box 78
21982 Karmiel, ISRAEL

Yale T. Herer

Israel Institute of Technology
Industrial Engineering and Management
32000 Haifa, ISRAEL

Enver Yücesan

Technology and Operations Management Area
INSEAD
Boulevard de Constance
77305 Fontainebleau, FRANCE

ABSTRACT

The objective of inventory management models is to determine effective policies for managing the trade-off between customer satisfaction and the cost of service. These models have become increasingly sophisticated, incorporating many complicating factors that are relevant in practice such as demand uncertainty, finite supplier capacity, and yield losses. Curiously absent from these models are the financial constraints imposed by working capital requirements (WCR). In practice, many firms are self-financing; their ability to replenish their own inventories is directly affected not only by their current inventory levels, but also by their receivables and payables. In this paper, we analyze the materials management practices of a self-financing firm whose replenishment decisions are constrained by cash flows, which are updated periodically following purchases and sales in each period. In particular, we investigate the interaction between the financial and operational parameters as well as the impact of WCR constraints on the long-run average cost.

1 INTRODUCTION

Efficient working capital management represents a huge economic opportunity. The top 1000 U.S.-headquartered public companies have approximately $910 Billion (6.9% of U.S. GDP) tied up in working capital ("2012 US Working Capital Survey", REL Consultancy). Hence, efficient working capital management is necessary both to increase the profitability of a firm and to insure that it has sufficient liquidity throughout its operating cycle (Pass and Pike 1984).

In fact, the current focus on the management of inventory under working capital constraints was motivated by a distributor of industrial equipment that buys heavy goods such as generators and earthmoving equipment from a manufacturer and sells them to various business users. The manufacturer, which has a finite production capacity, ships orders requiring a non-negligible delivery lead time and receives full payment after a fixed number of periods following delivery. Customer demand is random as it is driven by major construction projects. Once a delivery is made to a customer, the customer must pay after a fixed number of periods. The distributor's challenge is to determine an inventory management policy that balances its inventory and shortage costs. The problem, however, is further complicated by a limit imposed on the distributor's working capital requirements (WCR). In particular, the distributor's WCR, which is defined as the quantity being financed (i.e., the value of on-hand inventory plus trade credit extended to

978-1-4799-7486-3/14/$31.00 ©2014 IEEE

customers, also known as accounts receivable) minus the quantity that is financed by the manufacturer through trade credit (i.e., accounts payable), cannot exceed a maximum value that is set by the distributor's parent company. Throughout its operating cycle, which spans procurement, inventory holding, sales and cash collection, the distributor must ensure adequate customer service in a self-financing fashion—that is, generating sufficient funds through sales to procure additional inventory.

The recent spotlight on micro-retailing is another key motivator for our study. With little or no access to external financing, small, typically family-owned firms must carefully manage their operating cycle, generating sufficient cash from sales to replenish their inventories for business continuity. (Pierson and van Ryzin 2010) describe such a micro-retail operation where the owning family needs to keep an eye on its working capital not only to effectively manage its operating cycle, but also to generate a sufficient cash surplus to make a living. In fact, it is a common practice within the wine industry in the Bordeaux region of France to sell Bordeaux futures to generate the cash needed to invest in the production of the next vintage ("A revered French winery breaks with a Bordeaux tradition," *The New York Times*, 12 April 2013).

Lack of financial resources may severely constrain operational decisions ("Credit cuts make life impossible," *The New York Times*, 21 January 2012). However, there exist external sources of financing the operating cycle. A common practice for banks has been to offer asset-based financing, i.e., affordable loans of up to 90% of accounts receivable or up to 60% of inventory value (Buzacott and Zhang 2004). However, under the current lackluster economic conditions such asset-based financing has become more difficult to obtain. Moreover, asset-based financing does not offer much relief to fast-growing firms and start-ups that typically do not have assets they could use as collateral for obtaining affordable financing (Archibald, Thomas, Betts, and Johnston 2002). An alternative to asset-based financing is trade credit, which is the single most important source of external finance for firms (Cunat 2007). In 2007, 90% of the global merchandize trade ($25 trillion) was financed through trade credit ("World Bank urged to lift trade credit finance," *Financial Times*, 11 November 2008). Trade credit, however, represents a double-edge sword as 46% of small businesses experience payment defaults (1998 National Survey of Small Business Finance).

In this setting, cash flows generated by a firm have to be explicitly incorporated into the operational decision making process—beyond the common practice of imposing an exogenous budget constraint on activities such as procurement, production, and sales (Kaiser and Young 2009).

In the literature, however, until recently there has been little investigation of the relationship between finance and operations, and, in particular, of the interactions between material flows and cash flows. As a result, these decisions are often made separately without a unifying perspective on how these trade-offs affect the performance of the firm. The objective of this paper is to construct a model to depict the interactions between material and cash flows as well as to quantify the impact of these interactions on performance. Through this study, we identify some unintended consequences of financial constraints.

The traditional approach for modeling inventory management challenges in the face of demand uncertainty is the newsvendor framework. While our modeling efforts are anchored in this framework, we will extend it by incorporating cash flows that might constrain replenishment decisions. We also explicitly consider the lead times within the operating cycle (i.e., replenishment lead time, holding period for on-hand inventory, payment period for accounts payable, and collection period for accounts receivable). In addition, instead of setting a known, exogenously determined budgetary constraint as some models do, we model the available cash in each period as a function of assets and liabilities that are updated through the procurement and sales activities.

The key messages of our study can be summarized as follows:

1. The persisting economic crisis makes external financing increasingly difficult to secure. As a result, firms must be increasingly prudent in managing their operating cycle, ensuring sufficient and timely generation and collection of cash for business continuity.

2. The imposition of A myopic limit on WCR renders an otherwise stable operating cycle unstable by leading to shortfalls that become impossible to eliminate, ultimately forcing the firm out of business.
3. Supplier financing has been an emerging mechanism for relieving the operational decisions from financial constraints, thereby enabling a more effective way to manage the operating cycle.

In the next section, we review the literature with a focus on papers that are directly relevant to our study. We then introduce our Base Model to analyze the impact of the WCR constraint on the performance of the firm. The model enables us to demonstrate the unintended consequences of the limit on WCR. In addition, we describe some recent developments on supplier finance as a stabilizing solution.

2 LITERATURE REVIEW

Due to the fundamental result in corporate finance theory (Modigliani and Miller 1958) whereby, in perfect and competitive markets, financing decisions can be made independently of investment and production decisions, there is traditionally little overlap between the extensive literature on cash management and the vast literature on production and inventory management. This separation is further reinforced in practice as the two functions normally reside in separate "silos" within the company, the former being part of treasury while the latter being part of operations.

In addition to the rich literature on corporate finance (Hawawini and Viallet 2007), there exists a large body of papers addressing the cash management problem aimed at finding the best balance between the sources and uses of cash (Porteus 2002), through an equally broad set of techniques, including deterministic linear programming (Mao 1968), stochastic dynamic programming (Kallberg, White, and Ziemba 1982), and option pricing (Birge and Zhang 1999). While these papers focus on short-term financial planning, they do not consider the potentially significant impact of operational decisions such as procurement, production, and sales on cash flows. Similarly, the overwhelming majority of the extensive literature on production and inventory management (Hopp and Spearman 1996) assumes that the firm is always able to secure the cash it needs to deploy "optimal" materials management policies. There are a few papers (e.g., (Rosenblatt 1981)), where an exogenously fixed budget is imposed as a side constraint. (Hu and Sobel 2008) analyze a model of short-term operational and financial decisions in which the long-term capital structure is explicitly captured. In a simpler setting, (Dada and Hu 2008) consider the single-period inventory procurement problem of a capital-constrained newsvendor (CCNV). In a sequential decision model, (Archibald, Thomas, Betts, and Johnston 2002) argue that start-up firms should be more cautious in their procurement strategy than their well-established counterparts if they wish to maximize their likelihood of survival. (Li, Shubik, and Sobel 1997) characterize the optimal production, borrowing, and dividend decisions of a firm that aims to maximize the expected present value of the infinite-horizon flow of the dividends under uncertain demand, capacity, borrowing, and liquidity constraints. (Babich and Sobel 2004) focus on the coordination of financial and operational decisions to maximize the expected discounted proceeds from an initial public offering, modeled as a stopping time in an infinite-horizon Markov decision process.

In two separate settings, (Birge and Zhang 1999) study the interactions between financial and operational decisions. First, in a deterministic environment, they use a linear programming formulation to consider different assets that a firm can use to finance its operations for maximizing its profits over a finite horizon. Second, in a single-period newsvendor setting, they analyze a Stackelberg game between a bank and a retailer.

The persistent economic crisis, however, makes external financing increasingly difficult to secure ("Bank loans becoming harder to get in euro zone," *International Herald Tribune*, 2 February 2012). Settings, where access to external funding is restricted, consequently deserve further attention. For example, (Chao, Chen, and Wang 2008) analyze the replenishment decisions of a self-financing firm with the objective of maximizing its expected wealth at the end of a finite horizon. Their paper differs from ours mainly in the fact that they have a penalty for violating the working capital constraint while we consider this to be a hard

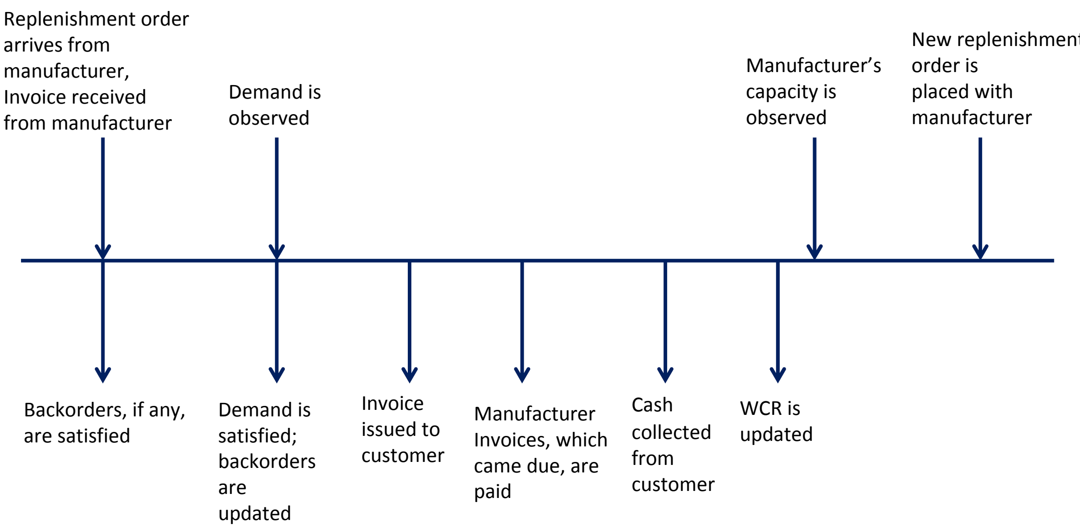

Figure 1: Order of Events in a Given Period.

constraint. In addition, our model, which will be introduced in the next section, incorporates the working capital requirements during the operating cycle as well as non-negligible replenishment lead times.

3 BASE MODEL

Consider a firm that procures goods from a manufacturer at a fixed purchase price to satisfy random demand. In particular, the firm faces an infinite-horizon newsvendor setting with backorders. In addition, the manufacturer has uncertain capacity that is independent of the order quantity. Mean demand is assumed to be smaller than mean capacity. The manufacturer delivers the product after a fixed order lead time. The firm has to pay for the received goods after a given payment period following delivery. When the firm sells a unit at a given selling price, the customer has to pay for the product after a fixed collection period. The goal is to minimize the long-run average cost of demand-supply mismatch while taking into account the uncertainty in customer demand and manufacturer capacity.

This challenge is further complicated by a limit on the firm's WCR during its operating cycle. As a result, the firm may not have the funds to finance the totality of the desired replenishment quantity. In the Base Model, whose order of events in a particular period is depicted in Figure 1, the manufacturer ships the minimum of: 1) the quantity ordered by the firm (which is the minimum of the quantity needed to attain the firm's order-up-to level and the quantity corresponding to the monetary value of the working capital available to the firm) and 2) its own realized capacity. The number of items received in each period allows us to update the values of the end-of-period inventory, the end-of-period backorders, the quantity sold, and the working capital "consumed." The cost in a period will be the cost of the end-of-period on-hand inventory or the cost of the end-of-period backorders. The consideration of the long-run average cost by itself, however, masks some interesting system behavior; we therefore keep track of two additional service-based performance metrics, namely the proportion of the time in which order quantity is limited by the WCR constraint as well as the proportion of the time in which we in fact violate the WCR constraint. These measures aid us in assessing the impact of the WCR constraint on system performance.

3.1 Notation and Problem Formulation

Throughout the paper, we will use the following notation:

Random variables

D_t random variable describing the I.I.D. demand in period t, with cumulative distribution function F_D, mean μ_D, variance σ_D^2, and realization d_t

K_t random variable describing the I.I.D. manufacturing capacity available in period t, with cumulative distribution function F_K, mean μ_K, having a finite variance, and realization k_t

R_t random variable describing the I.I.D. shortfalls/residuals at the end of period t, with cumulative distribution function F_R and realization r_t

Problem parameters

c per unit purchase price from the manufacturer

L_O order delivery lead time from the manufacturer

PP duration of the payment period, i.e., the number of periods during which the firm carries accounts payable (A/P)

V per unit selling price

CP duration of the collection period, i.e., the number of periods during which the firm carries accounts receivable (A/R)

h holding cost per unit per period

p shortage cost per unit per period

CR newsvendor critical ratio given by $p/(p+h)$

W maximum allowable value of the Working Capital Requirements (WCR), henceforth referred to as the WCR limit

ρ capacity utilization rate or the system load on the manufacturer, defined as μ_D/μ_K

Auxiliary and Decision variables

S order-up-to level

Q_t quantity ordered by the firm in period t taking into account the order-up-to level, the available capacity, and the WCR limit.

I_t net inventory at the end of period t

I_t^+ on-hand inventory at the end of period t $(= \max\{0, I_t\})$

I_t^- backorder level at the end of period t $(= \max\{0, -I_t\})$

WCR_t the working capital requirements at the end of period t

We calculate the system cost as the cost of excess inventory plus the cost of backorders, i.e.,

$$\sum_t (hI_t^+ + pI_t^-), \tag{1}$$

driven by the following system dynamics:

$$I_t^+ = \max\{0, I_t\} \qquad \text{for all } t > 0 \tag{2}$$

$$I_t^- = \max\{0, -I_t\} \qquad \text{for all } t > 0 \tag{3}$$

$$I_t = I_{t-1} + Q_{t-L_O} - D_t \qquad \text{for all } t > 0 \tag{4}$$

$$Q_t = \max\left\{0, \min\left\{S - I_t - \sum_{m=t-L_O+1}^{t-1} Q_m, K_t, \frac{W - \mathrm{WCR}_t}{c}\right\}\right\} \qquad \text{for all } t > 0 \tag{5}$$

$$\begin{aligned} \mathrm{WCR}_t = {} & \mathrm{WCR}_{t-1} + c\left(I_t^+ - I_{t-1}^+\right) + \\ & V\left(\min\{I_{t-1}^+ + Q_{t-L_O}, D_t + I_{t-1}^-\} - \min\{I_{t-CP-1}^+ + Q_{t-CP-L_O}, D_{t-CP} + I_{t-CP-1}^-\}\right) - \\ & c\left(Q_{t-L_O} - Q_{t-PP-L_O}\right) \qquad \text{for all } t > 0 \end{aligned} \tag{6}$$

$$I_t = I_t^+ = I_t^- = 0 \qquad \text{for all } t \le 0 \tag{7}$$

$$\mathrm{WCR}_t = Q_t = 0 \qquad \text{for all } t \le 0 \tag{8}$$

Equations 2 and 3 convert the net inventory to on-hand inventory and backorder level. Equation 4 is the standard inventory balance relationship. Equation 5 defines the quantity ordered in period t, as the minimum of the quantity needed to attain the order-up-to level S, the actual capacity, and the quantity corresponding to the monetary value of the remaining working capital available to the firm. The last element of this minimum highlights the difference between W and WCR_t, the former being the amount of wealth to which the firm has access and the latter being the amount of this wealth that the firm is using at time t. Equation 5 also guarantees that the order quantity will be non-negative. This is required for situations in which WCR_t exceeds W. Equation 6 updates the WCR by taking the previous value of the WCR and changing its components. In particular, we add the change in the monetary value of the inventory. We also add the change to the A/R, adding the value of sales in period t (note that the amount sold in period t is the minimum between the quantity available for sale $(I^+_{t-1} + Q_{t-Lo})$ and the demand for products in period t $(D_t + I^-_{t-1})$) and subtracting the value of sales in period $t - CP$. Finally, we subtract the change in the A/P, the difference in the value of items delivered in period t and the items delivered in period $t - PP$. Equations 7 and 8 initialize the system.

3.2 Determination of S^*

Systems with stochastic supplier capacity and random customer demand have received considerable attention since (Federgruen and Zipkin 1986). In particular, uncertain supplier capacity has typically been modeled in one of two ways (Wang and Gerchak 1996): in a binary fashion with either full or zero capacity or as a random capacity following a probability distribution. In this paper, we adopt the latter approach, modeling the manufacturer capacity with a multinomial distribution.

In a system with stochastic supplier capacity and random customer demand, (Glasserman and Tayur 1995) analyze a modified base stock policy. The policy is modified in the sense that limited production capacity may preclude restoring inventories to their base-stock levels in a single period. (Ciarallo, Akella, and Morton 1994) show that base stock policies are optimal in a multi-period setting under the discounted cost criterion. (Güllü 1998) extends this result to the long-run expected average cost per period criterion and characterizes the optimal order-up-to level. His characterization is based on establishing an analogy between base stock policies (with stochastic capacity and demand) and random walks associated with G/G/1 queues. While this analogy allows one to find the optimal order-up-to levels for particular distributions, it does not readily handle typical demand distributions. We thus use the result from (Güllü 1998) that an order-up-to policy is optimal for our system with the capacity constraint, but without the WCR constraint, and determine our order-up-to levels via simulation following the same logic. We note that, in the presence of the WCR constraint, the ordering policy is likely to be dependent on the on-order inventories, accounts receivables, and accounts payable. However, we note that, due to its ease of implementation, a base stock policy is commonly used in the literature even when it is not optimal (see e.g. (Glasserman and Tayur 1995) and (Song and Tong 2012)).

Most of the literature that examines the effect of working capital through the consideration of trade credit does not model the working capital as a hard constraint; rather, in these papers “trade credit is used to facilitate supply chain coordination” (Yang and Birge 2013). Moreover, in this literature a base stock policy has been shown to be optimal in various settings. For example, (Luo and Shang 2013) show that, when the payment period is less than or equal to the collection period, a base stock policy is optimal. Our model further includes manufacturer leadtime and a capacity constraint, both of which have been shown in the literature not to affect the optimality of base stock policies.

Key to the results in (Güllü 1998) is the analysis of the shortfall distribution where the shortfall is defined as the difference between the order-up-to quantity and the current inventory position. He demonstrates that the shortfall distribution is independent of the order-up-to quantity. In particular, the shortfall at the end of period t can be recursively defined as

$$R_t = \max(0, R_{t-1} + D_t - K_t). \tag{9}$$

Once the shortfall distribution is obtained, the distribution of the inventory position at the end of period t is simply $S - R_t$ and the distribution of the post-demand net inventory, L_O time periods later, is the inventory position after ordering minus the lead time demand ($\sum_{s=t+1}^{t+L_O-1} D_s$) and the demand in the period itself (D_{t+L_O}). That is, the distribution of the inventory after demand in period $t + L_O$ is simply equal to:

$$S - R_t - \sum_{s=t+1}^{t+L_O-1} D_s - D_{t+L_O}. \tag{10}$$

Note that while R_t is I.I.D., it is shown with a subscript for clarity. Using the fact that for this newsvendor type problem, it is optimal to have the no-stockout probability equal to the critical ratio and that the distribution of the end-of-period inventory is "linear" in S, once we know the distributions of R and D, we can easily find the optimal order-up-to level.

Since the determination of R is not readily available for most distributions commonly used in inventory control ((Güllü 1998) determines this distribution for random variables with an Erlang-k distribution), we use a simulation-based procedure to compute S^* numerically. In particular, for given distributions of the capacity and demand, and for given values of the critical ratio, $CR = p/(h+p)$, and the order lead time, L_O, we use the following quantile estimation algorithm ((Law and Kelton 2000), page 517):

Algorithm: simulation procedure for finding S^*

1. Generate a vector of shortfalls using Equation (9) and thin out the vector of realizations to reduce correlation between successive observations.
2. Add to the instances of the shortfall L_O independent demand realizations generated from the demand distribution.
3. Sort the resulting vector in descending order.
4. Choose S^* such that a CR fraction of the elements of the vector are greater than S^*.

3.3 Experimental Design

We use simulation to assess the impact of the WCR constraint on system performance. The model parameters can conceptually be divided into two sets. The first set, referred to as the *financial parameters*, includes the WCR limit, W; the purchase price, c; the payment period for A/P, PP; the selling price, V, and the collection period for A/R, CP. These parameters impact the solution only through the WCR constraint; in other words, without the WCR constraint the system performance would be unaffected by their values. In contrast, the remaining parameters, referred to as the *newsvendor parameters*, affect the solution through the underlying capacitated newsvendor problem; in other words, they modify the value of S^*, but they do not directly appear in the WCR constraint. The newsvendor parameters include the critical ratio, CR; the system load, ρ; the delivery lead time, L_O, and demand variance, σ_D^2.

The parameters used in our numerical study are presented in Table 1. Note that in our experiments demand is distributed according to a discrete uniform distribution; when $\sigma_D^2 = 850$, demand is between 100 and 200 and when $\sigma_D^2 = 7550$, demand is between 0 and 300. In particular, demand always has an expected value of 150. Finally, note that when $\rho = 1/1.1$, manufacturing capacity has a multivariate distribution with possible values of $[0, 97, 146, 194, 243]$ and associated probabilities of $[0.1, 0.1, 0.15, 0.5, 0.15]$, respectively, whereas when $\rho = 1/1.3$, manufacturing capacity has a multivariate distribution with possible values of $[0, 115, 172, 230, 287]$ and associated probabilities of $[0.1, 0.1, 0.15, 0.5, 0.15]$, respectively.

We use a full factorial design (nine factors, each at two levels, leading to 2^9 problem instances) with thirty replications at each design point. This leads to $2^9 \cdot 30 = 15,360$ simulation runs. For each simulation run, we estimate the long-run average cost over 9,000 periods after an initial warmup period of 1,000 periods. To reduce estimator variance, we use common random number streams where possible. To obtain S^* for use in the simulation runs, we execute the algorithm described in Section 3.2 to generate a total of 100,000 shortfall values; thinning this vector by taking one observation in every 100 generated values leaves

Table 1: Parameter values for the experimental design

	Financial parameters					Newsvendor parameters			
	W	c	PP	V	CP	CR	ρ	L_O	σ_D^2
Low	10,000	8	1	13	1	0.6	1/1.1	2	850
High	20,000	12	3	17	3	0.95	1/1.3	4	7550

us with 1000 shortfall values. Since the value of S^* is unaffected by a change in the financial parameters, as a variance reduction technique, we generate S^* only once for each set of these parameters.

The two main performance measures of interest are: Long-run average cost (AC)and percentage of time the WCR constraint was violated (PV). That is, the percentage of time the working capital requirements (WCR_t) were larger than the imposed limit (W). An additional performance measure that was recorded during the simulation runs was the percentage of times the quantity needed to attain the order-up-to level was higher than the quantity corresponding to the monetary value of the working capital available, i.e., even without considering the limited capacity, we would not have been able to order-up-to S because of the WCR constraint. While this performance measure,which will be denoted by PM, is not of direct interest, it helps us better understand the system behavior.

3.4 Observations

Our first observation from the experimental setup described in Section 3.3 was that for some combination of parameters the system becomes unstable. By unstable we mean that, as time progresses, the inventory drifts towards negative infinity; see Figure 2 for one such instance. As a result, the average cost per period reported by the simulation (which was over a finite time) was large, but finite, while the true average cost per unit time is infinite. Note that this phenomenon occurs despite the fact that, for *all* problem instances, the expected manufacturer capacity is greater than the expected customer demand (maximal load on the manufacturer was $\rho = 0.91$ and, for the problem instance in Figure 2, it was 0.77). Accordingly, in the analysis below when we compare two alternatives we *do not* compare their average performance; rather we report the fraction of times one value of interest was greater than its paired value.

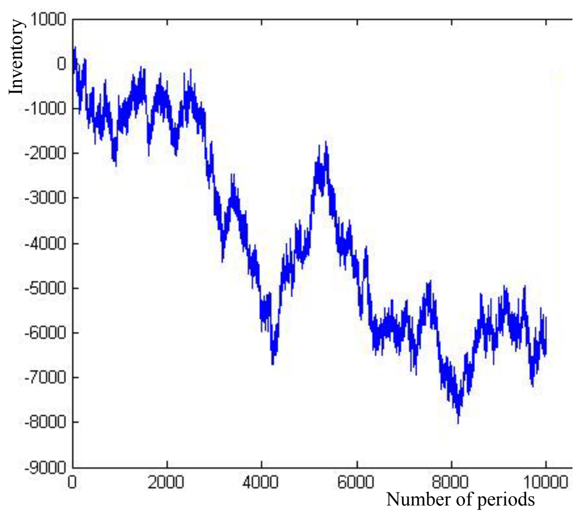

Figure 2: The end of period inventory in the Base Model for one simulation run for the problem instance with $W = 10,000$, $c = 8$, $PP = 1$, $V = 17$, $CP = 3$, $CR = 0.95$, $\rho = 1/1.3$, $L_O = 4$, and $\sigma_D^2 = 850$.

In Table 2, we present the results of our simulation experiment organized by parameter. Each triple in Table 2 indicates the percentage of time the larger parameter value resulted in a larger performance measure value, the percentage of time the larger parameter value resulted in a smaller performance measure value, and the percentage of time the larger parameter value resulted in the same performance measure value. Each triple is supported by 7,680 comparisons ($30 \cdot 2^9 = 15,360$ paired simulation runs based on the parameter of interest). Please note that, according to the sign test, all the differences in Table 2 are significant at the 95% level.

Table 2: The effects of the various parameters on the model. The table provides the percentage of time the larger parameter value yielded the indicated effect on the performance measure.

	AC			*PM*			*PV*		
parameter	larger	smaller	equal	larger	smaller	equal	larger	smaller	equal
W	12	43	45	0	55	45	0	36	64
c	21	16	63	34	3	63	13	10	77
PP	9	29	63	1	37	63	4	20	76
V	27	8	65	33	2	65	23	4	74
CP	35	8	57	42	1	57	28	2	70
CR	11	89	0	41	0	58	29	1	70
ρ	98	2	0	27	12	61	22	5	73
L_O	75	25	0	25	8	67	22	2	76
σ_D	100	0	0	41	5	54	32	0	68

We begin our analysis with the financial parameters (the first five rows of Table 2). Recall that the working capital requirements incorporate the monetary values as follows:

$$WCR = \text{Inventory} + \text{Accounts Receivable(A/R)} - \text{Accounts Payable(A/P)}. \tag{11}$$

W As expected, when W is larger (WCR constraint is looser), AC, PV, and PM all tend to be smaller. As W increases, there are fewer restrictions on the replenishment quantity; thus, there is a better match between supply and demand. Similarly, since the WCR limit is larger, there is less chance of violating the WCR constraint.

V We would expect that a larger V would lead to larger AC, PV, and PM; this is indeed the case. This is due to the fact that V comes into play only through an increase in the value of the A/R—hence, the "consumption" of a larger part of the WCR—thereby limiting our ability to order. Moreover, as V increases, the instantaneous increase in WCR that occurs when a sale is made is greater and thus PV is larger.

CP We would expect that a longer CP would lead to larger AC, PV, and PM; this is also the case as a longer CP means that an increasing WCR limits our ability to order. Moreover, when WCR grows beyond W due to an increase in A/R, this increase persists over CP periods.

PP We would expect that a longer PP would lead to smaller AC, PV, and PM; this is indeed the case. A longer PP means that we are essentially receiving funding from our suppliers, effectively reducing the WCR—enabling us to order more.

c The impact of c is harder to predict than the other financial parameters because c comes into play in three places: twice in the calculation of WCR (once with a "plus" in inventory investment and once with a "minus" in A/P) and in the calculation of the number of units that can be ordered (see Equation 11). From Table 2 we see that, overall, a larger c results in a larger AC, PV, and PM.

Analysis of the newsvendor parameters (CR, L_O, ρ, and σ_D^2) requires further consideration, given that a change in the parameter affects the system even without the WCR constraint. We now examine each of these parameters in turn.

σ_D^2 In the version of the problem where WCR is unconstrained an increase in demand variance would lead to a larger *AC*. Moreover, since the same increase in demand variance would lead to a larger S^* (note that $CR > 0.5$ for all problem instances), and thus increased inventories, we would expect the WCR constraint to be active and violated more often (i.e., an increase in *PM* and *PV*, respectively) and thus a further increase in *AC*. This is indeed what is observed in Table 2.

L_O Same as σ_D^2.

ρ Same as σ_D^2.

CR The effect of the critical ratio is more complex. On the one hand, when working capital is unconstrained, an increase in critical ratio (with fixed $h+p$ as is the case here) leads to *reduced* cost. This result has been shown analytically for the uncapacitated version of the problem. Intuitively, as p and h diverge (increased *CR*), the inventory planner can take advantage of this divergence to reduce the expected mismatch cost. In particular, inventories are increased so that the cheaper mistake (overage) is more likely than the more expensive one (underage). On the other hand, an increase in *CR* causes an *increase* in S^* and thus an *increase* in *PM* and *PV*. These increases, in turn, cause an *increase* in *AC*. To summarize, we would expect that an increase in *CR* would increase *PM* and *PV*, but due to the two opposing effects on *AC* we cannot predict if it would increase or decrease. In Table 2, we see that the larger *CR* is associated with a smaller *AC* and a larger *PM* and *PV*.

In summary, the worst-case scenario in terms of all three performance measures is clearly created by a small W, large V, long *CP*, short *PP*, large σ_D^2 (as long as $CR > 0.5$), large L_O, and large ρ. In addition, a large *CR* (given it is greater than 0.5) clearly contributes to a worst-case scenario for the performance measures of *PM* and *PV*. Additionally, from our experiment, we saw that a large c contributes to a worst-case scenario for all three performance measures while a small *CR* contributes to the worst-case scenario for *AC*.

In addition to the main effects, we note an interesting two-way interaction between the parameters V and *CP*. A priori one would suspect that these two parameters would interact as they both are financial parameters that affect the WCR in a multiplicative fashion through the A/R (see Equation 11). To see the interaction, examine Table 3 where we 'split' the row from Table 2 associated with the parameter V into two parts, the first for a short *CP* and the second for a long *CP*. As can be observed in Table 3, for a short *CP*, there is virtually no difference in cost between a small and large V (the difference of 11 versus 9 percent failed to be significant with the sign test while all the other differences in Table 3 are significant). No difference in cost occurs because the need to use additional capital (the difference between the large and small V) to finance the A/R for a short *CP* is not significant. However, when the *CP* is long, the additional capital has quite a significant impact on system performance.

Table 3: The interaction between V and *CP*. The table provides the percentage of time the larger parameter value yielded the indicated effect on the performance measure.

	AC			*PM*			*PV*		
parameter	larger	smaller	equal	larger	smaller	equal	larger	smaller	equal
V overall	27	8	65	33	2	65	23	4	74
V for short *CP*	11	9	79	18	2	79	5	1	44
V for long *CP*	42	7	51	47	2	51	18	3	29

4 MANAGING THE OPERATING CYCLE

As a well-established financial indicator, WCR provides a snapshot of a firm's health. The Base Model captures how the WCR measure is typically "operationalized" in practice to manage the operating cycle. However, we question whether WCR is a logical operational measure (*without* questioning it as a financial measure). As we view it, organizations using WCR as an operational measure are trying to limit the capital they invest in their operating cycle. Thus, WCR increases when the accounts payable are paid and decreases when the accounts receivable are collected. However, there is another point at which WCR changes, namely, when a unit is sold. At this point, the inventory value is reduced by c while A/R increases by V. The net effect is therefore an increase in WCR by $V - c$ (> 0) for each unit sold. Thus, each sale ends up 'consuming' a portion of WCR, which, in turn, constrains the firm's ability to place a replenishment order. Ultimately, the more inventory we sell, the tighter the constraint becomes on the firm's ability to place replenishment orders. We contend that, while it may limit the collection risk, such a practice does not make operational sense. From a cash flow point of view, no money changes hands when a sale is made. From a risk management point of view, the risk has actually been reduced. Now that the inventory has been sold, we have a collection risk; before the inventory was sold, however, we had an obsolesce risk combined with a future collection risk—if the unit does not become obsolete. Thus, once the unit is sold we are, operationally, strictly better off.

As an alternative, a number of financial institutions are offering 'supplier finance' (the retailer in terms of this paper) to provide some relief to the WCR constraint. This financing scheme, also referred to as reverse factoring, is further analyzed in (Tanrisever, Cetinay, Reidorp, and Fransoo 2012). Pre-shipment financing is also receiving increased attention (Reindorp, Lange, and Tanrisever 2013). Similar schemes are becoming increasingly popular in commodity trade finance ('Tea with FT Middle East: Muneef Tarmoom,' *The Financial Times* 1 May 2012).

REFERENCES

Archibald, T. W., L. C. Thomas, J. M. Betts, and R. B. Johnston. 2002. "Should Start-up Companies be Cautious? Inventory Policies which Maximize Survival Probabilities". *Management Science* 48:1161–1174.

Babich, V., and M. J. Sobel. 2004. "Pre-IPO Operational and Financial Decisions". *Management Science* 50:935–948.

Birge, J. R., and R. Q. Zhang. 1999. "Risk Neutral Option Pricing Methods for Adjusting Constrained Cash Flows". *Engineering Economist* 44:36–49.

Buzacott, J. A., and R. Q. Zhang. 2004. "Inventory Management with Asset-Based Financing". *Management Science* 50:1274–1292.

Chao, X., J. Chen, and S. Wang. 2008. "Dynamic Inventory Management with Cash Flow Constraints". *Naval Research Logistics* 55:1161–1174.

Ciarallo, F. W., R. Akella, and T. E. Morton. 1994. "A Periodic Review Production Planning Model with Uncertain Capacity and Uncertain Demand—Optimality of Extended Myopic Policies". *Management Science* 40:320–332.

Cunat, V. 2007. "Trade Credit: Suppliers as Debt Collectors and Insurance Providers". *The Review of Financial Studies* 20:491–527.

Dada, M., and Q. Hu. 2008. "Financing Newsvendor Inventory". *Operations Research Letters* 36:491–502.

Federgruen, A., and P. H. Zipkin. 1986. "An Inventory Model with Limited Production Capacity and Uncertain Demands: I. The Average Cost Criterion". *Mathematics of Operations Research* 11.

Glasserman, P., and S. Tayur. 1995. "Sensitivity Analysis for Base Stock Levels in Multi-Echelon Production-Inventory Systems". *Management Science* 41:263–281.

Güllü, R. 1998. "Base Stock Policies for Production/Inventory Problems with Uncertain Capacity Levels". *European Journal of Operational Research* 105:43–51.

Hawawini, G., and C. Viallet. 2007. *Finance for Executives*. Ohio: Thomson.

Hopp, W. J., and M. L. Spearman. 1996. *Factory Physics*. Chicago: Irwin.

Hu, Q. J., and M. J. Sobel. 2008. "Capital Structure and Inventory Management". Technical report, Dept. of Operations Research, Weatherhead School of Management, Case Western Reserve University, Cleveland, OH.

Kaiser, K., and S. D. Young. 2009. "Need Cash? Look Inside Your Company". *Harvard Business Review* 54:3–14.

Kallberg, J. G., R. W. White, and W. T. Ziemba. 1982. "Short Term Financial Planning Under Uncertainty". *Management Science* 28:670–682.

Law, A. M., and W. D. Kelton. 2000. *Simulation Modeling & Analysis*. 3rd ed. New York: McGraw-Hill, Inc.

Li, L., M. Shubik, and M. Sobel. 1997. "Control of Dividends, Capital Subscriptions, and Physical Inventories". Technical report, Yale School of Management, New Haven, CT.

Luo, W., and K. Shang. 2013. "Managing Inventory for Entrepreneurial Firms with Trade Credits". Technical report, Fuqua School of Business, Duke University, Durham, NC.

Mao, J. C. T. 1968. "Applications of Linear Programming to Short Term Financing Decisions". *Engineering Economist* 13.

Modigliani, F., and M. H. Miller. 1958. "The Cost of Capital, Corporation Finance and the Theory of Investment". *American Economic Review* 48:261–297.

Pass, C., and R. Pike. 1984. "An Overview of Working Capital Management and Corporate Financing". *Management Finance* 10:1–11.

Pierson, M., and G. van Ryzin. 2010. "Frogtek: Mobile Technology for Micro-Retailing". Technical report, Columbia Business School, Columbia University, New York.

Porteus, E. L. 2002. *Foundations of Stochastic Inventory Theory*. Palo Alto: Stanford University Press.

Reindorp, M., A. Lange, and F. Tanrisever. 2013. "Pre-Shipment Financing: Credit Capacities and Supply Chain Consequences". Technical report, Technical University Eindhoven, Eindhoven, NL.

Rosenblatt, M. J. 1981. "Multi-Item Inventory System with Budgetary Constraint: a Comparison Between the Lagrangian and the Fixed Cycle Approach". *International Journal of Production Research* 19:331–339.

Song, J., and J. Tong. 2012. "A New Accounting Framework for Supply Chain inventory Finance". Technical report, Fuqua School of Business, Duke University, Durham, NC.

Tanrisever, F., H. Cetinay, M. Reidorp, and J. Fransoo. 2012. "The Value of Reverse Factoring in Multi-Stage Supply Chains". Technical report, Technical University Eindhoven, Eindhoven, NL.

Wang, Y., and Y. Gerchak. 1996. "Periodic Review Production Models with Variable Capacity, Random Yield, and Uncertain Demand". *Management Science* 42.

Yang, S. A., and J. R. Birge. 2013. "How Inventory is (Should be) Financed: Trade Credit in Supply Chains with Demand Uncertainty and Costs of Financial Distress". Technical report, Booth School of Business, University of Chicago, Chicago, IL.

AUTHOR BIOGRAPHIES

ILLANA BENDAVID is an Assistant Professor of Industrial Engineering at Braude College of Engineering. Her email address is illana@techunix.technion.ac.il.

YALE T. HERER is an Associate Professor of Industrial Engineering and Management at the Technion – Israel Institute of Technology. He received his Ph.D. in 1990 from Cornell University. He has won the IIE Transactions Best Paper Award and recently won INFORMS Waganer Prize. His email address is yale@technion.ac.il.

ENVER YÜCESAN is a Professor of Operations Management at INSEAD in Fontainebleau, France. He received his PhD in 1989 from Cornell University. He is currently serving as the representative of the INFORMS Simulation Society on the WSC Board of Directors. His email address is enver.yucesan@insead.edu.