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EMPIRICAL STUDY OF THE BEHAVIOR OF CAPACITATED PRODUCTION-INVENTORY SYSTEMS

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ABSTRACT

Production-inventory systems model the interaction of manufacturing processes with internal and external customers. The role of inventory in these systems is to buffer mismatches between production and demand caused by process uncertainty. Often, production and demand variability is described using simplified probabilistic models that ignore underlying characteristics such as skewness or autocorrelation. These models lead to suboptimal inventory policies that result in higher costs. This work presents a novel analysis of the impact of uncertainty in the performance of production-inventory systems. It quantifies the effect of different probabilistic descriptions of production capacity and demand in systems subject to lost sales or backorders. The analysis is based on the results of discrete-event simulations. The flexibility offered by simulation allows studying diverse conditions that arise in production-inventory systems. The results clearly illustrate the importance of appropriately quantifying variability and performance for inventory management in process networks.

1 INTRODUCTION

One of the roles of inventory in manufacturing systems is to buffer temporal mismatches between production and demand. Production-inventory systems can generally be employed to represent the interactions between manufacturing plants and customers, or to model individual blocks in complex production networks. It is widely accepted that inventory control is of prime importance to industries because of its significant costs and its impact on customer satisfaction (Wilson 2013).

Inventory management in capacitated production-inventory systems under uncertainty is a difficult task. Unlike most supply chains, the capability to replenish inventory in these systems is constrained by a limited production capacity. A simplistic approach to maximize utilization might suggest to produce at full capacity and store surplus products. However, this approach may not obey the economic purpose of production systems and might lead to unstable behavior. A more rational approach is to establish an operating policy that determines production rate, inventory replenishment, and inventory depletion.

An operating policy is a set of rules that establishes the operating decisions as a function of the system's state. The right policy allows balancing the cost of inventory and the risk associated with stockouts. A production-inventory system operating under such a policy works as a push and a pull system with the inventory serving as the *Customer Order Decoupling Point* (Olhager 2010). Federgruen and Zipkin (1986) demonstrated that under a few mild conditions a modified base-stock policy is optimal for production-inventory systems with stochastic demand. The modification to the classic base-stock policy accounts for the capacity limitation of the replenishment orders. Tayur (1993) and Ciarallo, Akella, and Morton (1994) developed algorithms to find the optimal base-stock level and to calculate the average cost of these systems.

One of the main assumptions for the model developed by Federgruen and Zipkin (1986) requires demands to be independent and identically distributed (iid) random variables. This assumption might seem reasonable for long time periods but neglects autoregressive effects that impact inventory management strategies (Luong 2007). The performance of production-inventory systems can be quite sensitive to autocorrelation because of the time dependence of production, demand, and inventory levels. The novelty of this study is to analyze of the effect of skewness and autocorrelation on the performance of capacitated production-inventory systems.

The derivation of closed-form solutions that model the effect of autocorrelation in inventory policies has proved to be very challenging. Alcay, Biller, and Tayur (2012) presented a numerical approach to find the optimal base-stock in a newsvendor model (single period and unlimited capacity) with autocorrelated demand. Additionally, they consider parameter uncertainty for the probabilistic description of demand. The present study does not include parameter uncertainty but it develops a strategy that allows considering stochastic production and demand processes with arbitrary distributions.

The potential of using a simulation model to improve inventory management strategies has been well recognized (Gaither 1982). The present work proposes a methodology to study the behavior of production-inventory systems under different uncertainty characterizations based on the statistical analysis of Discrete-Event Simulations (DES). DES is a flexible approach to evaluate inventory management strategies with many realistic considerations. In particular, it allows easy implementation of base-stock policies and estimation of performance measures. DES has been extensively used to analyze inventory systems (Badri 1999, Kristianto, Helo, and Takala 2010, Kravchenko 2013) and complex production networks with diverse industrial applications (Sharda and Bury 2008, Sharda and Bury 2010, Kulkarni and Prashanth 2012, Spieckermann and Stobbe 2012, Sharda and Bury 2012). The motivation for this methodology comes from the need to develop simulation models that can leverage the diverse sources of data available in the process industry. This methodology can be used to evaluate equipment utilization and demand satisfaction levels for any production system that follows a base-stock policy.

Six production-inventory systems are tested with DES; they consider asymmetry and autocorrelation in the distributions of production capacity and demand. The results obtained are compared to assess the impact of different uncertainty characterizations on the performance of the production-inventory system. In order to gain a comprehensive insight into the system's behavior, two performance measures are considered. The comparisons indicate a noticeable difference in the production targets when these measures are considered. Additionally, a significant effect of time correlations on the performance production-inventory systems is demonstrated.

2 PROBLEM STATEMENT

The goal of this study is to analyze the response of production-inventory system to different stochastic processes describing production capacity and demand. An illustration of the system under study is presented in Fig. 1. The performance function to evaluate is the average cost in an infinite horizon. The cost has two components: inventory cost and stockouts cost. The model does not consider production costs. Two stockouts models are evaluated: lost sales and backorders. In the lost sales model, the cost in any time period is given by the holding cost (h) per unit of inventory and the penalty cost (p) per unit of unsatisfied

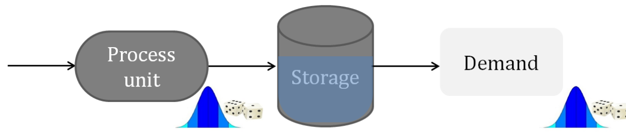

Figure 1: Illustration of the production-inventory system under uncertainty

demand. In the backorders model, demand satisfaction can be postponed to the next time period whenever stockouts occur; the penalty cost (b) per unit of backordered demand is applied in every time period.

The production-inventory system operates on a base-stock policy. The policy is characterized by the base-stock level which represents the maximum level of inventory that is desirable to store. The model assumes that demand is realized at the beginning of the time period and production is available instantly (no lead time). Following this logic, the largest demand that can be satisfied in a time period is given by the sum of the production capacity and the initial inventory.

In order to guarantee a finite average cost function, the mean production capacity is assumed to be greater than the mean demand. Furthermore, all stochastic processes under study have unique stationary distributions; they have finite expected values, variances, and covariances that are independent of time. These assumptions allow estimating the infinite horizon statistics from finite simulations.

3 DISCRETE-EVENT SIMULATION ALGORITHM

The discrete-event simulation model of the production-inventory system is based on the base-stock policy. The policy works as follows. At the beginning of a time period, the inventory is at some level (L_t). Then,

1. Random demand (D_d) is realized.
2. Production target is calculated to satisfy demand (D_t) and bring inventory level to base-stock (S).
3. Actual production is realized according to the production target and the realization of the random capacity (R_t).
4. Production and inventory is used to try to satisfied demand (D) completely.
5. Inventory level (L_{t+1}) is updated.
6. Holding or stockout costs are calculated.

Fig. 2 presents the sequence of calculations involved in the simulation. The algorithm is implemented in MATLAB R2013a in order to leverage the functions available for stochastic processes simulation.

4 CASE STUDIES

Six case studies are analyzed in order to consider different types of uncertainty. Sequences of independent and identically distributed (iid) random variables are used to represent uncertainties that are not influenced by the history of the process. The autocorrelation of realizations in consecutive time periods is modeled using the Moving Average (MA) model. Autocorrelation in the production capacity is intended to model favorable or adverse operating conditions that span during several time periods. Autocorrelation in the demand is intended to model market trends.

The first case study considers the stochastic processes characterizing production capacity and demand as sequences of iid random variables with normal distributions. The second case study analyzes the effect of skewness in the production capacity; it characterizes production capacity with a sequence of iid random variables with Pearson distributions and demand with a sequence of iid random variables with normal distributions. The third case study characterizes production capacity with MA model with lag 1 and demand with a sequence of iid variables with normal distributions. The fourth case study characterizes production capacity with a lag-3 MA model and demand with a sequence of iid random variables with normal distributions. The fifth case study characterizes production capacity and demand with lag-1 MA

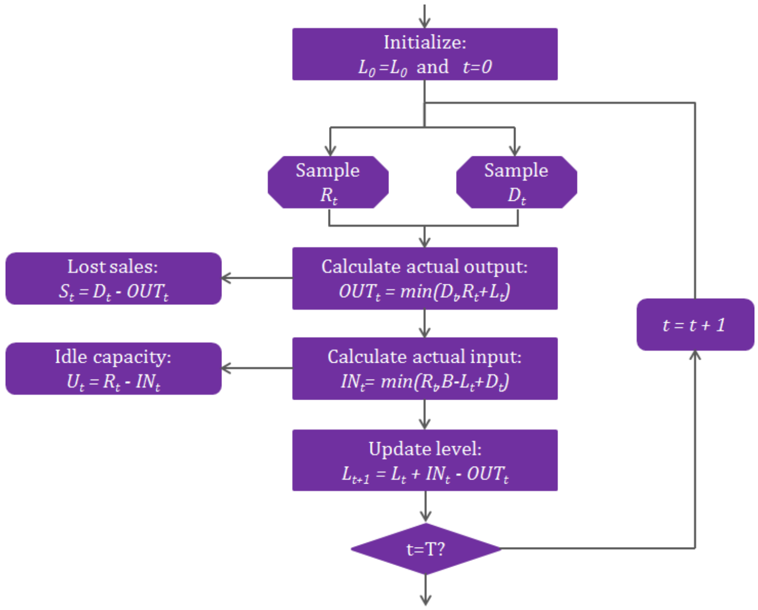

Figure 2: Algorithm for discrete-event simulation of the production-inventory system

models. The sixth case study characterizes production capacity and demand with lag-3 MA models. A summary of the parameters used for each case study is presented in Table 1.

Table 1: Parameters of the stochastic processes characterizing production capacity and demand

Case	Production capacity	Demand
Case 1	Normal: μ=110; σ=20	Normal: μ=100; σ=20
Case 2	Pearson: μ=110; σ=20; γ_1=-1; γ_2=1	Normal: μ=100; σ=20
Case 3	MA: μ=110; σ=16.33; θ_{-1}=0.5	Normal: μ=100; σ=20
Case 4	MA: μ=110; σ=17.03; θ_{-1}=0.5; θ_{-2}=0.3; θ_{-3}=0.2	Normal: μ=100; σ=20
Case 5	MA: μ=110; σ=16.33; θ_{-1}=0.5	MA: μ=100; σ=16.33; θ_{-1}=0.5
Case 6	MA: μ=110; σ=17.03; θ_{-1}=0.5; θ_{-1}=0.3; θ_{-1}=0.2	MA: μ=100; σ=17.03; θ_{-1}=0.5; θ_{-2}=0.3; θ_{-3}=0.2

All simulations are evaluated in a time horizon (T) of 200,000 time periods. The parameters of the stochastic processes are calculated to generate time series with stationary distributions that have the same mean and variance. The stationary distribution of the processes describing production capacity have a mean equal to 110 ton/period (1 ton = 1,000 kg) and a standard deviation equal to 20 ton/period. Demand processes have stationary distributions with mean equal to 100 ton/period and standard deviation equal to 20 ton/period.

Sequences of iid normal variables are characterized by their mean (μ) and standard deviation (σ); they are simulated in MATLAB using the function `normrnd(mu,sig,T,1)`. Similarly, sequences of iid Person variables are characterized by their mean (μ), standard deviation (σ), skewness (γ_1), and kurtosis (γ_2); they are simulated in MATLAB using the function `pearsrnd(mu,sig,skew,kurt,T,1)`. The autocorrelated stochastic processes are described with the Moving Average (MA) model; they are characterized by their mean (μ), standard deviation of the innovation process (σ), and the lag coefficients (θ_{-i}). The MA models are simulated in MATLAB scaling the variance of the innovation process (white noise) to obtain the desired stationary distributions.

The average cost for each case study is estimated using lost sales and backorders. The cost function includes a unit holding cost (h) of \$5/(period ton) and the stockout cost. In the lost sales model, the unit penalty cost (p) for stockouts equals \$40/ton. In the backorders model, the unit penalty cost (b) for stockouts is \$10/(period ton). The estimation of the average cost for each case is based on 100 different simulations.

5 RESULTS AND ANALYSIS

The results of the simulations are presented in Fig. 3 - 8. The figures show the average costs of the production-inventory systems for the lost sales and backorder models with varying base-stock levels. Base-stocks are evaluated in the range from 0 to 50 ton. The lines in Fig. 3 - 8 represent the mean value of the inventory cost, stockout cost, and total cost over 100 simulations. The base-stock levels that yield lowest average total costs are listed in Table 2 and can also be verified from Fig. 3 - 8.

Fig. 3 presents the results for case study 1. It can be observed that the costs of the lost sales and backorder models are very similar throughout the range of evaluated base-stocks. They both attain the minimum cost with a base-stock of 26 ton. The similarity between both models is explained by a small accumulation of backorders. The effectiveness of inventory to buffer short-term mismatches between production and demand is the result of characterizing uncertainty as sequences of iids. Additionally, the symmetry of the normal distributions provides consistent surplus production capacity over successive time periods, which avoids backorders being carried over several time periods (backorders persistence).

(a) Lost sales model (b) Backorders model

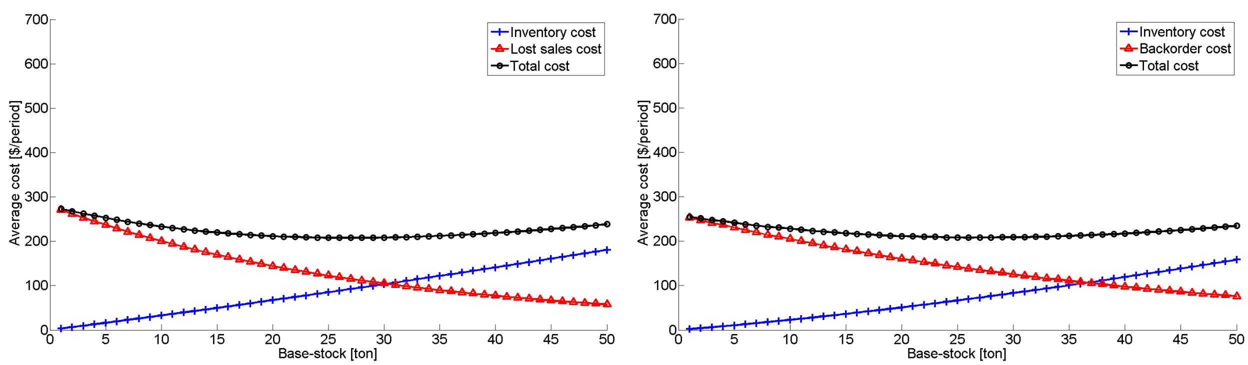

Figure 3: Average cost for case 1 with different base-stock levels

(a) Lost sales model (b) Backorders model

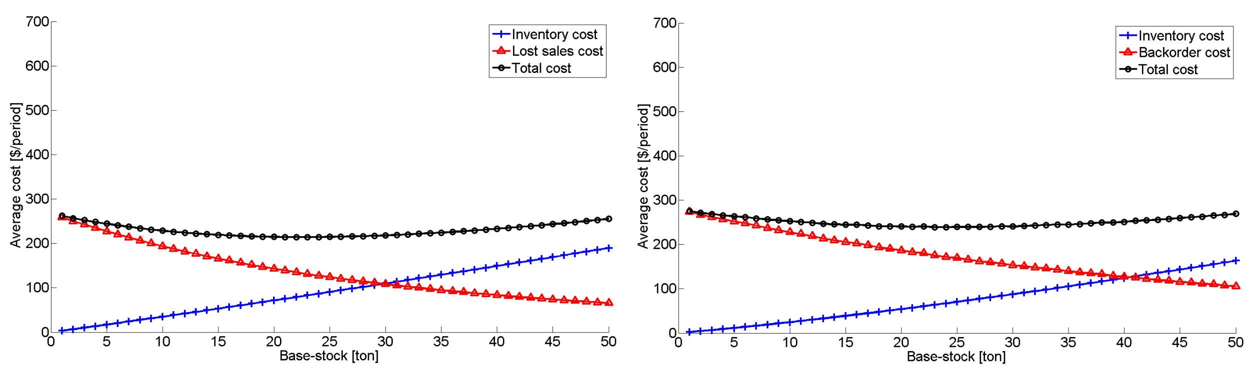

Figure 4: Average cost for case 2 with different base-stock levels

The results of the simulations corresponding to case study 2 are presented in Fig. 4. In contrast to case study 1, the lost sales model yields a lower cost than the backorders model for all base-stocks in case study

2. This behavior is explained by the high backorders persistence caused by inventory ineffectiveness. The asymmetry in the distribution of the production capacity often yields productions that are either significantly higher or lower than demand. The frequent mismatches between production and demand requires high base-stock levels that increase inventory cost. However, the values and trends observed in Fig. 4a and 4b are similar to the corresponding Fig. 3a and 3b.

(a) Lost sales model (b) Backorders model

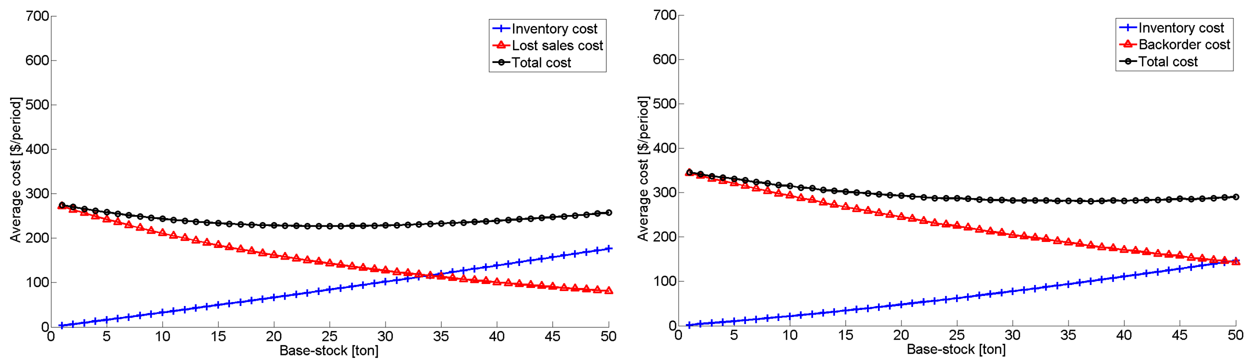

Figure 5: Average cost for case 3 with different base-stock levels

Fig. 5 presents the results for case study 3. The effect of autocorrelation in the production capacity translates in a lower sensitivity of the optimal cost to the base-stock levels. It can be observed that the slopes in Fig. 5a and 5b are less steep than the corresponding figures for cases 1 and 2. This is explained by the rapid accumulation or consumption of inventory in consecutive time periods. Autocorrelation in the production capacity requires higher base-stock levels to buffer production and demand mismatches, which leads to high inventory costs. The cost for the backorders model presented in Fig. 5b illustrates the high stockout cost that results from accumulation and persistence of backorders during several time periods.

(a) Lost sales model (b) Backorders model

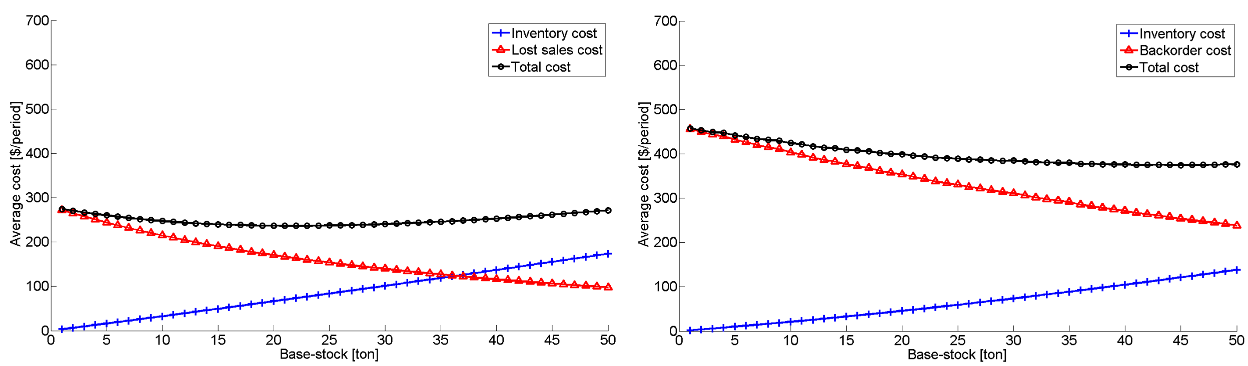

Figure 6: Average cost for case 4 with different base-stock levels

Similar trends can be observed in Fig. 6 - 8. The increasing autocorrelation in cases 4, 5, and 6 reduces the effectiveness of inventory to buffer production and demand mismatches. The lost sales models presented in Fig. 6a - 8a are affected much less by the increasing variability of the stochastic processes but there is a clear tendency to increase the average total cost. However, these models attain the minimum cost with decreasing base-stocks, which illustrates inventory ineffectiveness. The results for the backorders models presented in Fig. 6a - 8a have a different trend. In these cases, the impact of backorders persistence is so strong that high base-stocks with their associated inventory costs are preferable.

(a) Lost sales model (b) Backorders model

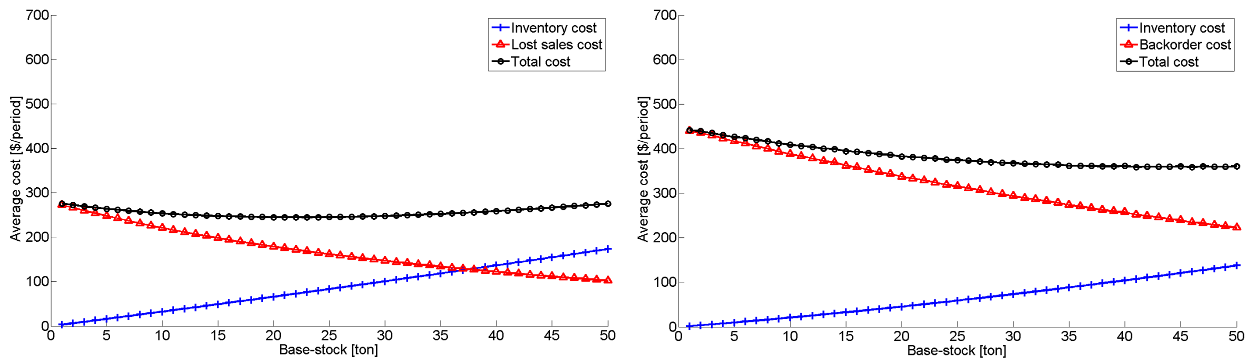

Figure 7: Average cost for case 5 with different base-stock levels

(a) Lost sales model (b) Backorders model

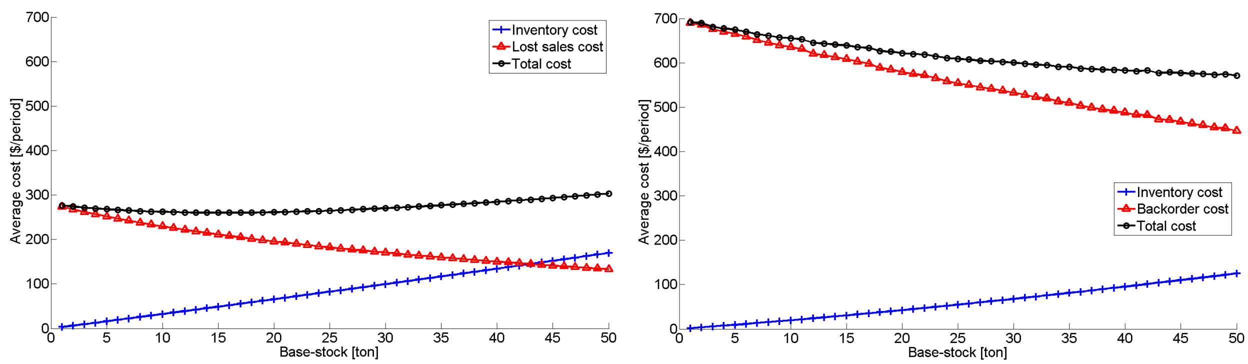

Figure 8: Average cost for case 6 with different base-stock levels

It is interesting to note that the effect of different characterizations of the production capacity and demand is moderate in the lost sales model, in contrast to their impact in the backorders model. The cost function used for the simulations considers a penalty cost four times higher for lost sales than for single-period backorders. This suggests that high stockout cost in the backorders model is the consequence of backorders that propagate for several time periods. In general, the backorders model yields lower inventory costs for any particular base-stock because the inventory level is also decreased by backorders; however, stockout costs are dominant at the optimal base-stock for all cases.

Table 2 presents the optimal base-stock levels and costs for the case studies under discussion. It can be observed that the lost sales model yields lower average total cost than the backorders model. Additionally, the optimal base-stock for the lost sales model is less or equal than the corresponding optimal base-stock for the backorders model. The cases with autocorrelated stochastic processes show a decreasing effectiveness of the inventory. In the lost sales model this ineffectiveness produces lower optimal base-stock levels to reduce the cost of inventory. The optimal base-stocks for backorders models are much higher because of the dominant effect of stockouts cost.

Table 2: Optimal base-stock levels and costs with 95% confidence intervals

Case	**Lost sales**		**Backorders**	
	Base-stock [ton]	**Cost [$]**	**Base-stock [ton]**	**Cost [$]**
Case 1	26	207.76 ($\pm$0.16)	26	208.10 ($\pm$0.45)
Case 2	23	214.07 ($\pm$0.18)	24	238.82 ($\pm$0.91)
Case 3	24	227.09 ($\pm$0.20)	37	280.68 ($\pm$0.70)
Case 4	23	236.66 ($\pm$0.20)	45	374.50 ($\pm$1.21)
Case 5	23	244.70 ($\pm$0.21)	46	358.64 ($\pm$1.23)
Case 6	14	260.28 ($\pm$0.30)	71	562.24 ($\pm$2.48)

6 CONCLUSIONS

The impact of uncertainty quantification in production-inventory systems has been analyzed with the use of discrete-event simulations. The methodology has been implemented for two stockouts models in six case studies with different characterizations of uncertainty. In particular, the models represent a single production-inventory system with variability in production capacity and demand; the stationary distribution of the stochastic processes describing the system's variability has the same mean and variance in all cases. The results clearly show that underlying characteristics of the stochastic processes such as skewness, kurtosis, and autocorrelation have an important influence on the performance of production-inventory systems. These characteristics of process variability are usually ignored in the analysis of these systems and their impact had not been quantified before.

The performance of the system evaluated with the lost sales model is much less sensitive to distributions with skewness, kurtosis, and autocorrelation. The lost sales model has a greater flexibility to balance inventory and stockout cost. In contrast, asymmetric and autocorrelated distributions have a significant impact in the backorders model. In this model, the accumulation of backorders during several time periods is the dominant element of the cost function. The effect of increasing autocorrelation only accentuates this feature.

This empirical study of the effect of uncertainty characterization in production-inventory systems clearly highlights the importance of appropriately quantifying variability and performance for inventory management. The discrete-event simulation framework and its flexibility allows studying such important considerations quite effectively. In future, this work will be expanded to analyze industrial applications and study the challenges associated with managing inventory in complex integrated process networks.

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