

**POPULARITY OR PROCLIVITY?  
REVISITING AGENT HETEROGENEITY IN NETWORK FORMATION**

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**ABSTRACT**

Agent-based modeling (ABM) approach is used to reassess the Barabasi-Albert (BA) model, the classical algorithm used to describe the emergent mechanism of scale-free networks. This approach allows for the incorporation of agent heterogeneity which is rarely considered in BA model and its extended models. The authors argue that, in social networks, people's intention to connect is not only affected by popularity, but also strongly affected by the extent of similarity. The authors propose that in forming social networks, agents are constantly balancing between instrumental and intrinsic preferences. The proposed model allows for varying the weighting of instrumental and intrinsic preferences on the agents attachment choices. The authors also find that changing preferences of individuals can lead to significant deviations from power-law degree distribution. Given the importance of intrinsic consideration in social networking, the findings emerged from this study is conducive to future studies of social networks.

**1 INTRODUCTION**

Social network analysis (SNA) has an especially long tradition in various disciplines of the social sciences (Knoke 1990, Wasserman and Faust 1994, Carrington, Scott, and Wasserman 2005, Scott 2000). It first emerged in the 1930s, as a means to investigate the impact that individual's characteristics have on the formation and shaping of different types of social networks; thus agent heterogeneity was a key factor in this early work (Lazarsfeld and Merton 1954, Freeman 1996, McPherson, Smith-Lovin, and Cook 2001). In recent decades, the scholarly interest in SNA is reenergized by the proliferation of the information and communications technologies (ICTs) like the Internet and the mobile phone network. A burgeoning literature has been devoted to exploring these large-scale complex networks (Watts 2004, Newman, Barabasi, and Watts 2006, Caldarelli 2007, Barabasi 2009).

The dramatically increased visibility of SNA is owed mainly to statistical physicists (Albert and Barabasi 2002, Brandes and Erlebach 2005, Abraham, Hassanien, and Snasel 2010). Instead of emphasizing agent heterogeneity, statistical physicists focus more on aggregate properties of large-scale networks, and highlight network's systematic regularities in spite of micro agent heterogeneity. Among many, Barabasi-Albert model (BA model) has attracted particular attention because of its novel perspective in revealing the mathematical properties of large-scale networks and its frequent appearance in a diverse range of network phenomena (Barabasi and Albert 1999, Clauset, Shalizi, and Newman 2009).

Barabasi and Albert (1999) argue that the vertex connectivities in large networks tend to follow a scale-free power-law distribution. The key underlying mechanism is that a vertex's probability to be connected is determined only by its relative position (i.e., connectiveness or "popularity") in the existing network. The formation of large networks is governed by this robust self-organizing mechanism that goes beyond

the particulars of agents or individual systems. However, in many social networks considerable deviations from scale-free behaviors have been reported (Shirazi, Namaki, Roohi, and Jafari 2013). Numerous variants of BA model accordingly have been developed to reproduce the growth process of social networks, and most of them still share the very key instrumental assumption that a vertex's probability to be connected is determined primarily by its "popularity" in a given network.

In this study, we argue that another way to advance scholarly understanding of network formation is to "bring agent heterogeneity back in." As revealed in the traditional SNA literature, people's decision to establish social ties are not conditioned solely by *instrumental* calculations of the others' position in a network (e.g., "popularity") but are also motivated by their *intrinsic* affection to join those that are "like" them. This "like" for people similar to themselves is called homophily. In other words, people are constantly weighting between popularity and proclivity in forming their social connections. The impact of this mixed preferential attachment, we argue, is particularly consequential on formation of social networks (Aral, Muchnik, and Sundararajan 2013).

Although many empirical studies have confirmed the importance of agent heterogeneity at the vertex- and dyad-level (Snijders, van de Bunt, and Steglich 2010), few studies, if any, have systematically explored its impacts on the aggregate mathematical properties of large-scale networks. In this study, we propose an integrative agent-based model (ABM) of heterogeneous attachment encompassing both instrumental calculation and intrinsic similarity. Particularly, we emphasize the ways in which agent-heterogeneity affects social network formation.

In three ways, this study contributes to current studies of network formation. First, by exploring the impacts agent heterogeneity, this studies highlights an important yet less examined mechanism in network formation, that is, intrinsic preferential attachment. This mechanism, we argue, becomes particularly important in the age of new media, in which individuals' capacity in homophilous sorting has been strongly boosted by information and communications technologies (Benkler 2006, DiMaggio, Hargittai, Neuman, and Robinson 2001, Lewis, Gonzalez, and Kaufman 2012). Therefore, an investigation of the impacts of intrinsic preferential attachment can significantly enrich our understanding about large-scale social networks. Second, by emphasizing both micro-mechanisms in governing dyad formation and macro mathematical properties of large-scale networks, we concur with Barabasi (2009) that "the structure and the evolution of networks are inseparable." Third, joining many recent works (Miller and Page 2007, Hamill and Gilbert 2009, Shirazi, Namaki, Roohi, and Jafari 2013), this study demonstrates that ABM, given its explicit emphasis on complexity and emergence, provides a promising perspective and a useful method to explore the dynamic evolution of large-scale social networks.

## 2 THE LITERATURE: SEARCHING THE MECHANISMS OF NETWORK DYNAMICS

A social network is fundamentally a complex and emergent phenomenon, which raises great challenges in conceptualizing and theorizing network dynamics. Many scholars, recognizing the impacts of network structure, focus on how network position affects various socioeconomic outcomes. One of the most influential such works is Granovetter's "strength of weak ties" (SWT) theory (Granovetter 1973, Granovetter 1983), in which weak, bridging ties are argued to be beneficial because of their potentials in introducing novel information. Burt (1997) later refines the argument by differentiating between the benefits of bridging ties and the average strength of those ties. This in turn leads to Burt's conclusion that stronger ties can be more beneficial than weak ties because they allow a greater flow of resources. More recently, Podolny (2001) argues that network structure matters not only because it serves as "pipes" of resources, but also because it acts like "prisms," revealing important information about the inherent qualities of the vertices (e.g., credibility).

## 2.1 Understanding Network Dynamics

Other scholars, rather than exploring the impacts of different network structures, focus on the network formation processes, ranging from the Erdos and Renyi's random graph model (ER model) to Watts and Strogatz's "small-world" model (Watts and Strogatz 1998, Watts 2004). However, as for large-scale complex networks, empirical results demonstrate that most of them are scale free, that is, their degree distribution follows a power law distribution (Redner 1998; Albert et al. 1999; Faloutsos 1999; Barabasi and Albert 1999; Broder et al. 2000; Newman 2001; Barabasi et al. 2001; Yook et al. 2001). BA model (Barabasi and Albert 1999) then is introduced to describe this scale-free emergent mechanism. BA model suggests that the growth of network size and preferential attachment are the necessary conditions for the emergence of scale-free networks (Albert and Barabasi 2002).

However, in many social networks, significant deviations from scale free behavior have been reported, and numerous complex network models have been proposed by updating the two key assumptions of BA model: dynamic growth and preferential attachment. It is believed that if the process that assembled the networks is captured accurately, it is possible to obtain the topology which is closer to real world networks. As for preferential attachment, Barabasi et al. (2001) and Newman (2001) estimate the functional form of preferential attachment via measuring the real world network data, including co-authorship network data, the scientific collaboration networks in physics and biology. Krapivsky et al. (2000) propose a nonlinear preferential attachment network model and conclude that scale-free nature is not observed anymore when nonlinear preferential attachment mechanism is involved.

## 2.2 Reintroduction of Agent Heterogeneity

However, Pujol et al. (2005) pointed out that the assumptions of these models usually lack sociology grounding. Wong et al. (2005) argued that many network models have not taken the advantages of sociological and psychological insights of how social networks may be formed. We also found it is problematic since it assumes all the nodes possess the same preference (instrumental preferential attachment) and overlooks the potential impacts of agent heterogeneity on network formation (intrinsic preferential attachment). When joining a real social network, people are not only driven by instrumental calculation of connecting with the popular, but also motivated by intrinsic affection of joining the like. In other words, people are constantly weighing between popularity and proclivity in forming their social connections. The impact of this mixed preferential attachment, we believe, is particularly consequential on such social networks as political communication. More importantly, we find the support to this assumption from the social theory of homophily.

McPherson, Smith-Lovin, and Cook (2001) argue that homophily is the principle that a contact between similar people occurs at a higher rate than among dissimilar people, and the similarity could be regarding to many types of personal characteristic positions, including gender, religion, social class, education and other intra-personal or behavioral characteristics. In fact, there are some models taking homophily into consideration, somehow not using the specific term but essentially the similar meaning. Robins et al. (2001) presented network models for social selection process. Although characteristic positions affecting the social relationship formation is concerned, it is broken between the local behavior and the global pattern. In other words, there is no analysis for the properties of large social networks. Newman and Girvan (2003) conducted a network model discussing the mechanism of assortative mixing, that is, the nodes with similar degree level like to link with each other. However, it actually is a special case of preferential attachment, albeit the similarity of nodes is concerned.

In this paper, the authors propose an agent based approach to address two gaps in the existing social network literatures. The first gap, as discussed by many, is how to model actual social network in a more faithful way. In order to explore the mathematic properties of dynamic networks, statistical physicians have to impose strong assumptions on human behaviors. Real world social networks, however, are a result of dynamic interactions between intentional individuals. ABM has been found particularly powerful in

modeling various actual generative mechanisms of social network, which in turn can significantly advance our understanding of a wider range of real world networks. The second gap roots also in the inherent advantages of agent based modeling (ABM). ABM provides an avenue to integrate substantive theories in various disciplines in social science with abstract network models (Hamill and Gilbert 2009). An important problem in contemporary SNA is the limited actual applications of such deductive models as BA model in social science (Bergenti, Franchi, and Poggi 2013, ?). Part of this issue is that network dynamics suggested by social theories are usually far more complex than those modeled in deductive SNA. For instance, BA model assumes uniformity of behavior of individual, which is constantly violated in real social network. Not surprisingly, in many social networks considerable deviations from scale-free behaviors have been reported (Shirazi, Namaki, Roohi, and Jafari 2013, ?). ABM, on the other hand, is flexible enough to handle heterogeneity, complexity, adaptability, and versatility of real social networks. Although lots more future study are needed for extending the current model, the authors believe that the proposed model is stepping on the right direction.

### 3 MODEL HYPOTHESIS

In this study, we argue that homophily is a key driving mechanism, comparing to the mechanism of preferential attachment, for leading the social agents to make decisions which result in the formation of different structures of the social networks. When joining a real social network, people are not only driven by instrumental calculation of connecting with the popular, but also motivated by intrinsic intention of linking with the similar people. In other words, people are constantly weighting between popularity and proclivity in forming their social connections.

#### 3.1 Heterogeneous Attachment

The impact of this mixed network formation is particularly consequential on such social networks as political communication. For instance, when people appear in a new community and start to build their network, the two endogenous driven mechanisms would lead people to build up their social networks. Under extreme conditions, by following the preferential attachment only, people would only be interested in linking with the popular people. By following the homophily only, people would focus on connecting with the other people that have similar intrinsic properties with them, thus creating a more comfortable social ambiance. Certainly, the latter is a human behavior factor, which is labeled as “intrinsic” intention to construct network in this study. We realize that in the real world, most people make decisions based on both mechanisms in different levels instead of in the extreme situations. In the following model design section, we will state the way of weighting for balancing these two mechanisms and the method of modeling homophily.

#### 3.2 Hypothesis and Heterogeneous Model

The heterogeneous attachment model proposed in this study is rested on three key assumptions.

1. **Heterogeneity:** Vertices (i.e., agents, nodes) are intrinsically different from each other on certain aspects. All of the relevant characteristics of vertices are captured by a finite set of  $c \geq 1$  types:  $\{1, 2, \dots, c\}$ . Based on this finite set of relevant characteristics, it is possible to construct  $C_i$ , representing the characteristic position of node  $i$ .
2. **Dynamic Growth:** The network continuously expands by the addition of new vertices. The network starts with a small number ( $n_0$ ) of nodes, at each time step  $t$ , a new node with  $m$  edges that link the new node to  $m$  different nodes already present in the system.
3. **Heterogeneous Attachment:** The probability that two vertices are connected is jointly determined by the connectivity of the existing vertices and the intrinsic similarity between vertices. The joint

probability that a new vertex  $t + 1$  at time step  $t + 1$  will be connected to vertex  $i$  depends on,

$$f(k_{it}, C_i, C_{t+1}) = \frac{\lambda k_{it}}{\sum_j k_{jt}} + (1 - \lambda) \cdot g(C_i, C_{t+1}) \quad (1)$$

where  $\lambda$  is a exponential weighting of the instrumental preferential probability and intrinsic preferential probability. Connectivity of node  $i$  at time step  $t$  thus is  $k_{it}$ . The probability of a new node and a random existing node are connected for intrinsic purpose at time  $t + 1$  can be captured by  $g(\cdot)$ , in which  $g(C_i, C_{t+1})$  decreases as  $C_{diff}(C_i, C_j)$  increases for  $i \neq j$ .  $C_{t+1}$  is the characteristic position of a new node entering the network at time step  $t + 1$ , and  $C_i$  is that of node  $i$  already in the network,  $i \in \{1, \dots, t\}$ .

In the original setting of the simulation, let  $m = 1$ ,  $n_0 = 2$  and  $C_i \in \{\text{“Blue”}, \text{“Red”}\}$ , and

$$g(C_{t+1}, C_i) = \begin{cases} \frac{1}{\mu N_d + N_s} & \text{for } C_{t+1} = C_i \\ \frac{\mu}{\mu N_d + N_s} & \text{for } C_{t+1} \neq C_i, \end{cases} \quad (2)$$

where  $N_s$  is the number of nodes  $C_i = C_{t+1}$ ,  $N_d$  is the number of nodes  $C_i \neq C_{t+1}$ . It can be shown that the equation 2 is a probability function, e.g., its summation equals unity. From extension of this, it can be shown that equation 1 is also an probability distribution. These proofs have not been included in this paper due to article length considerations.

#### 4 NETLOGO MODEL

In order to illustrate and model heterogeneous attachment, this study uses different colors to denote the different attributes of agents as stated in assumption 1. Specifically, for the purpose of simplicity there are two types of agents in the system: blue agents and red agents. We model assumption 2 by allowing the size of agents to increase, by one, at each time step. It should be noted that this study focuses primarily on the topology of social networks obtained on the final stage. Therefore, we assume simple dynamic growth of agents in this model. The network will stop to grow when there are 10,000 agents in the network. As implied in the formula (1) of assumption 3, when  $\lambda = 1$  a purely rationality-driven social network (i.e., a classical scale free network) is expected to emerge, and when  $\lambda = 0$ , a value-driven social network is expected to be generated. Agents in the model would take the same color agents into their homophily consideration. The same process will repeat for 30 times for each  $\lambda$ ,  $\lambda \in \{0, 0.25, 0.5, 0.75, 1\}$ .

More specifically, the simulation process can be described as the following steps:

1. Start with two connected nodes with random color (red/blue).
2. A new node with a random color (red/blue) is starting to consider about joining in the networks.
3. The nodes existent in the network (for first run, the original two nodes) are in the choosing queue.
4. The incoming node is weighing between popularity and proclivity. It choose one node from the queue to connect by calculating the probability given in Formula (1). The node with higher probability in the existing network is more likely to be connected.
5. Loop from step 3.
6. Each run of simulation stops when there are 10,000 nodes in the network.
7. Simulation stops when the model runs 30 times for each  $\lambda$ ,  $\lambda \in \{0, 0.25, 0.5, 0.75, 1\}$ .

We are interested in exploring the ways in which the inherent characteristics of individuals, that is, different predispositions of heterogeneous people, affect the structure of scale free networks. Particularly, this study intends to examine if there is any turning point or linking point for generating network topologies differently. Theoretically, this study help reveal how network formation process is affected by including the assumption of heterogeneity for autonomous agents. To do so, we simulate the formation of networks

model with different values of  $\lambda$ , where  $\lambda$  indicates to what extent people emphasize “more like me” in choosing their friends. Specifically, our experiment includes five different values of  $\lambda$  variable:

Table 1:  $\lambda$  and Its Substantial Meanings

$\lambda$	Agent Interpretation
$\lambda = 1$	People only concerned about “Are you more like me?” or “Are we in the same party?”
$\lambda = 0.75$	People concerned more about “Are you on my side,” and also care about “Do you have more links” a little bit.
$\lambda = 0.5$	People concerned these two parameters in the same level.
$\lambda = 0.25$	People concerned more about “Do you have more links,” and also care about “Are you on my side” a little bit.
$\lambda = 0$	People only care about “How many links do you have?”

After simulating our network model in NetLogo, we obtain networks with different values of  $\lambda$ . The visualization of the simulated networks can be generated as Figure 1.

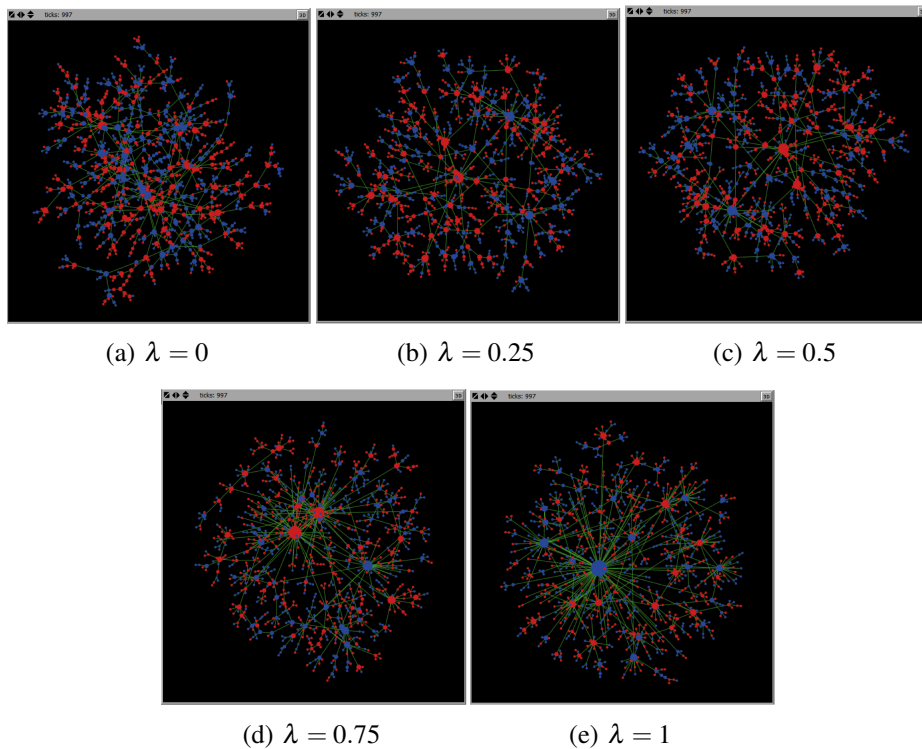


Figure 1: Visualization of simulated networks

Visually and roughly, we can observe a pattern across the four graphs as the value of  $\lambda$  increases. When the value of  $\lambda$  increases, there are more super nodes (nodes with a large number of links) in the system. When  $\lambda$  value decreases, nodes are connected more evenly and there are almost no observable super nodes. In other words, we could explain this observation as that when people make decisions according to the preferential attachment, there would be many more monopolies who own lots social resources in the emergent social networks. When people make decisions upon the heterogeneous attachment, the social resources may evenly distributed. However, this is only an observation and a rough inference based on the model visualization we have so far. As suggested Barabasi (1999), degree distribution of preferential

attachment follows a power law distribution. Degree distribution refers to the number of links of each node. Instinctively, degree distribution of heterogeneous attachment should follow exponential distribution. In next section, we conducted the statistical analysis on the degree distribution outputted from our agent-based model.

## 5 ANALYSIS

To what extent does agent heterogeneity affect network formation? A key problem in network studies is that instead of properties of the vertex themselves, network phenomena are primarily conceived and investigated through the properties of relations between and within the vertices. This in turn imposes challenges in exploring and understanding the impacts of agent heterogeneity on properties of a particular network. In this study, we explore this question by examining network properties at two levels.

### 5.1 Vertex- and local-attributes: Centrality and diameter

Researchers of SNA have long employed a variety of measurement to describe structural characteristics of a network. In this study, we focus on two key vertex- and local-level measurement: Kleinberg's centrality score and diameter. Kleinberg centrality score is an important variant of eigenvector centrality, which has been widely used to describe the distribution of connectivity for a network. Diameter, on the other hand, focuses more on local-level properties, indicating the longest of all the calculated shortest paths in a network. In other words, once the shortest path length from every node to all other nodes is calculated, the diameter is the longest of all the calculated path lengths. The diameter is representative of the linear size of a network.

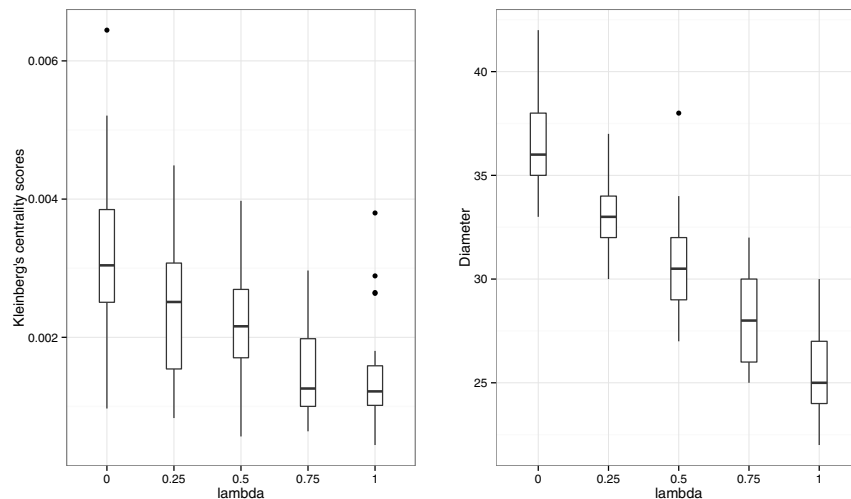


Figure 2:  $\lambda$  and vertex/local attributes

Figure 2 presents the impacts of  $\lambda$  on key structural characteristics of networks. First, as Figure 2.a indicates, as  $\lambda$  increases, the network-wide Kleinberg's centrality score become increasingly converged and smaller. In other words, as individual agents become more popularity-oriented, network-wide averaged vertex-centrality will decrease significantly. Second, the impact of  $\lambda$  on diameter is also evident. When individuals are more proclivity oriented, longer path length are observed for them to be interconnected. In other words, while the popularity-oriented network tends to decrease average centrality of vertices, it helps information diffusion by reducing the shortest path length.

### 5.2 Systematic Properties

Following statistical analysis techniques on the power law distribution provided by Clauset, Shalizi, and Newman (2009), we conduct an analysis on the degree distribution of the data generated from the simulation. The main goal of this statistical analysis is to learn how the degree distribution changes according to the different  $\lambda$  values, furthermore, to test our hypotheses that when  $\lambda = 1$ , we are reasonable to conclude that the degree distribution follows a power law distribution and when  $\lambda = 0$ , the exponential or Poisson distribution is better to describe the degree distribution.

The first group of graphs are putting 30 runs of data points in one figure with each different  $\lambda$  value. X-axis indicates the degree and Y-axis indicates the regarding numbers with the degree. We can tell from Figure 3 that there are big differences with different  $\lambda$  values. When  $\lambda$  close to 1, there are some nodes with high degree, but most of the nodes have less degree. Roughly, the degree distribution of  $\lambda = 1$  shows a feature of big tail. However, when  $\lambda$  is close to 0, the feature of big tail dispersed. Mathematically, a

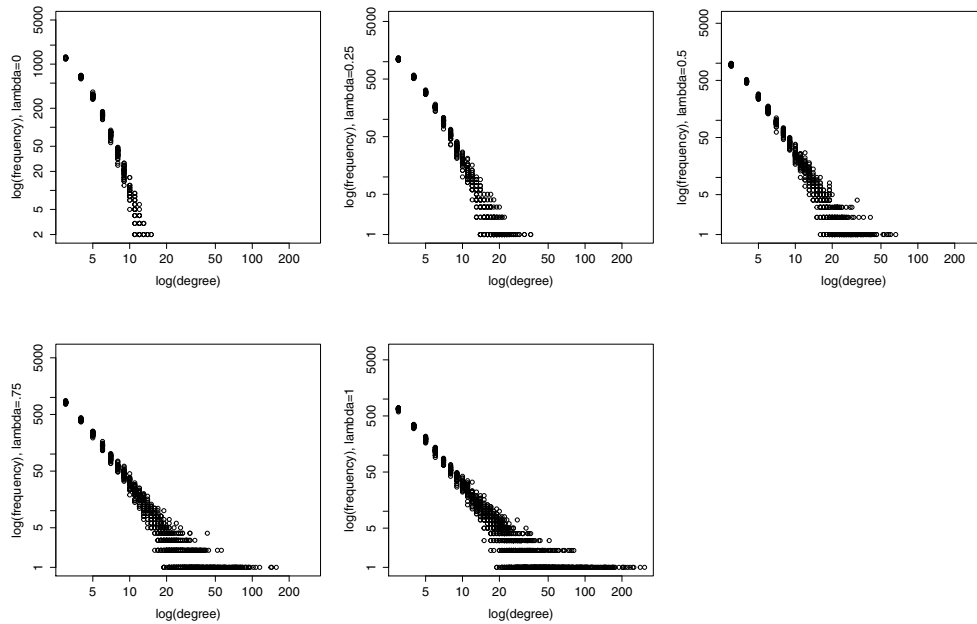


Figure 3: Degree distribution of simulated network (log/log)

quantity  $x$  obeys a power law if it is drawn from a probability distribution:

$$p(x) \propto x^{-\alpha} \tag{3}$$

As noted by Watts (2004), the probability of a randomly chosen node having degree  $x$  decays like a power of  $x$ , where the exponent  $\alpha$ , typically measure in the range of  $2 < \alpha < 3$ , determines the rate of decay (smaller  $\alpha$  implies slower decay, hence a more skewed distribution). A distinguishing feature of power-law distributions is that when plotted on a double logarithmic scale, a power law appears as a straight line with negative slope  $\alpha$ . Argued by Clauset et. al (2009), few empirical phenomena obey power laws for all values of  $x$ . Usually, the power law applies only for the values greater than some minimum  $x_{min}$ . Basically, power law distribution have two different settings: continuous distributions with the continuous real number and discrete distributions with discrete set of positive integers. Since the data of our model are positive integers, the probability distribution should follow the form of:

$$p(x) = \Pr(X = x) = Cx^{-\alpha}. \tag{4}$$



This density function diverges at  $x = 0$ , implying a lower bound  $x_{min} > 0$  to the power law behavior. The normalizing constant can be calculated as follow equations:

$$p(x) = \frac{x^{-\alpha}}{\zeta(\alpha, x_{min})}. \tag{5}$$

Based on the power law distribution, provided by Clauset, Shalizi, and Newman (2009), the approach is used for analyzing our data generated from the simulation to test how the  $\lambda$  value affects the degree distribution, so as well as the system behavior of the network formation process. We follow the below steps:

1. Estimate the parameters of power law distribution:  $x_{min}$  and scaling parameter  $\lambda$ .
2. Compare the power law with alternative hypothesis. Here, a likelihood ratio test method is appropriate. For each alternative, if the calculated likelihood ratio is significantly different from zero, then its sign indicates whether or not the alternative is favored over the power law distribution.

### 5.2.1 Estimating parameters

There are two key parameters need to be estimated in power law model:  $x_{min}$  and scaling parameter  $\alpha$ . The fundamental idea in Clauset, Shalizi, and Newman (2009) is: the lower bound  $\hat{x}_{min}$  should make the probability distributions of the measured data which are above this lower bound and the best-fit power law model as similar as possible. Kolmogorov-Smirnov or KS statistic is commonly used as a measurement of quantifying the distance between the two probability distributions, which indicates the maximum distance between the cumulative distribution functions (CDFs) of the measured data and the fitted model:

$$D = \max_{x \geq x_{min}} |S(x) - P(x)| \tag{6}$$

where  $S(x)$  is the CDF of the simulated data for the observations with the value greater than  $\hat{x}_{min}$ , and the  $P(x)$  is the CDF for the power law model that best fits the data in the region  $x \geq \hat{x}_{min}$ . Hence the  $\hat{x}_{min}$  is selected to be the  $x_{min}$  that minimizes the value of  $D$ .

Referring to the  $\alpha$  estimating, method of maximum likelihood provably gives the accurate parameters estimates with the limit of large sample size. Assuming the data are drawn from a distribution following a power law for  $x \geq x_{min}$ , the scaling parameter estimation under discrete case can be derived through maximum likelihood estimators (MLEs) as:

$$\hat{\alpha} \simeq 1 + n \left[ \sum_{i=1}^n \ln \frac{x_i}{x_{min} - \frac{1}{2}} \right]^{-1} \tag{7}$$

where  $x_i, i = 1, \dots, n$ , are the observed values of  $x$  such that  $x_i \geq x_{min}$ .

Following this procedure, we estimated the parameters for the data from the simulation. According to the experiments of our simulation, with 5 different  $\lambda$  values, we estimated 30 pairs of parameters based on 10,000 data points. The results are summarized in Figure 4.

From Figure 4, we can conclude that the means of  $\alpha$  and  $x_{min}$  are significantly changed in accordance with  $\lambda$ . When  $\lambda$  value is close or equal to 1, the mean of  $\alpha$  value falls in the range of [2,3], which is the similar to the  $\alpha$  value of real world power law distributed data. As well as the change of  $x_{min}$  value, it decreases as the value of  $\lambda$  increase. We may conclude that as  $\lambda$  increases, indicated by the decreased lower bound of  $x_{min}$ , the more data points are included and used in testing the power law distribution for next step. We see that the KS statistics are significantly affected by the change of  $\lambda$  values as well. When  $\lambda$  is close to and equal to 1, the KS statistics values turn to consistent and convergent. Conversely, when  $\lambda$  closes to and equals to 0, the KS statistics values turn to uneven and sparse.

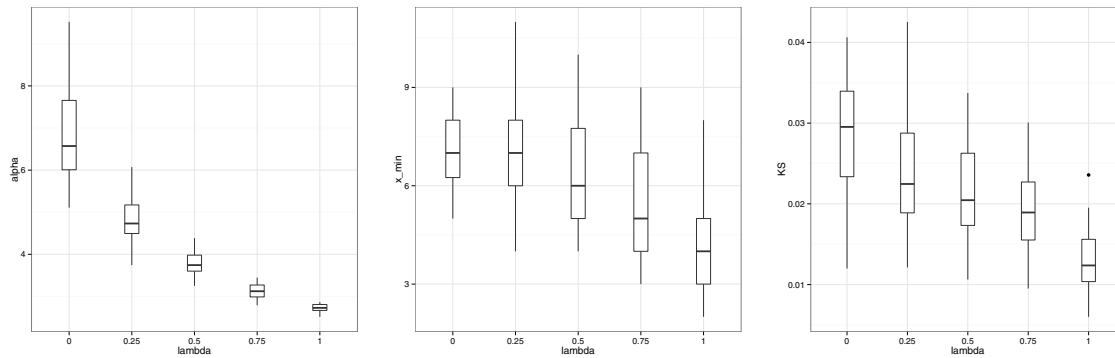


Figure 4: Estimates of scale-free network model

### 5.2.2 Comparing with the alternative distributions

While above estimated parameters indicate whether the power-law is an appropriate distribution in describing simulated networks, they provide no information about relative fitness of power-law distribution against alternative distributions. This section provides a test if our simulated data is possibly drawn from a power law distribution. Is it still possible that another distribution, such as an exponential or a Poisson distribution, might give a fit as good or better? Given by the method mentioned in the last two sections, we only need to run through the whole process again for different distribution candidates. Relying on the  $p$ -values from the test outline in Clauset, Shalizi, and Newman (2009), we make the judgment of accepting or rejecting our hypothesis.

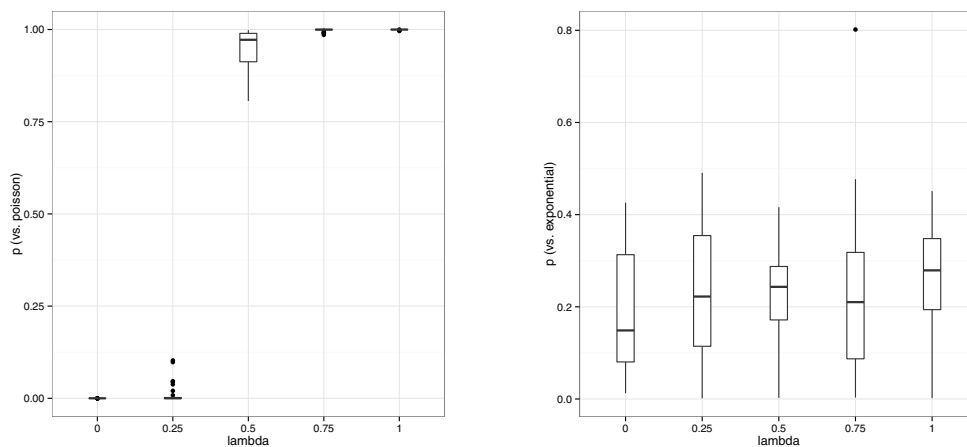


Figure 5: Comparison with alternative distributions

Figure 5 presents the results of comparing power-law distribution with Poisson and exponential distributions. Two findings stand out. First, the impact of  $\lambda$  on power-law vs. Poisson distribution is evident. Specifically, when  $\lambda = 0, 0.25$ , Poisson distribution is favored against power-law distribution. In other words, when individuals are proclivity-driven, it is unlikely to observe power-law distribution instead of a Poisson distribution. It is unclear why power-law is favored against Poisson distribution when  $\lambda = 0.5$ .

## 6 CONCLUSION

In this study, we argue that one way to advance scholarly understanding of network formation is to “bring agent heterogeneity back in.” By explicitly modeling agent heterogeneity in our ABM, we systematically

evaluate to what extent individuals' concern over proclivity/popularity affects various attributes of a network. Our simulation and analysis reveals that the impacts of individual orientation on network formation is critical and profound. Specifically,  $\lambda$  affects network attributes at vertex and system levels. The results indicate that the distribution of the vertex connectivity follows a power-law distribution when only instrumental attachment is considered, as expected, and it potentially follows a Poisson distribution when only intrinsic attachment is considered.

This heterogeneous attachment model is particularly useful for us to understand many real social networks, when individuals have to constantly weigh between the popularity ("the expert") and the similarity ("the like"). For instance, political scientists have long noted that individuals' network of political communication is strongly related to their preference in the "expert" or in the "bias" (Huckfeldt, Johnson, and Sprague 2004). However, few studies have explored various network properties associated with this kind of heterogeneous attachment. More research is thus called for to investigate how this heterogeneous attachment is manifested in different type of networks.

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