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ANALOGIES FROM TRAFFIC PHENOMENA TO INSPIRE LINEAR SCHEDULING MODELS WITH SINGULARITY FUNCTIONS

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ABSTRACT

Established techniques like the Critical Path Method and Linear Scheduling Method are activity centered and exhibit schedules statically, which impedes their ability to plan and control projects holistically. Scheduling therefore should be enhanced by incorporating new capabilities of measuring and displaying the dynamic nature of projects. In another technical field that employs a time-space coordinate system, however, traffic engineering, researchers successfully apply various parameters to measure the performance of an inherently dynamic behavior, which is identified as having significant potential to be adapted for scheduling purposes. This paper identifies concepts in traffic measurement that currently lack analogies in scheduling, including signals and trajectories. They are modeled with singularity functions, range-based expressions for variable phenomena, for new application in linear scheduling. Examples demonstrate the feasibility of deriving analogies from a related engineering field, which provides a compass to navigate future research to explore concepts that emerge from interaction of dynamic elements.

1 INTRODUCTION

Construction projects unfold within a complex and dynamic environment and exhibit such behavior themselves. Accordingly, techniques to plan and control should reflect such richness by proactively providing decision-makers with relevant and intuitive quantitative information and the status and trend of their progress. Traditional scheduling methods are not sufficiently effective and efficient at intuitively communicating the dynamic nature of projects. The Critical Path Method (CPM), which currently is widely used in both industry and academia (Galloway 2006a, b) views activities as 'building blocks' toward a network schedule. Its goal is to identify the critical path and determine the total project duration from aggregating activity durations. The name-giving critical path is the *fixed sequence* of "difficult and significant activities – [to overcome] the problems of achieving the objective" (Kelley and Walker 1959, p. 160). CPM itself was established as "*management by exception*" (Kelley and Walker 1959, p. 160, emphasis in original) to replace previous simpler approaches and coincided with the advent of computer use in project management. Other approaches, albeit much less known and used, that at least explicitly recognize the progressive nature of individual activities are linear and repetitive scheduling (Harris and Ioannou 1998).

However, scheduling techniques still rely upon an essentially static view of activities as the individual elements of a schedule, which are arranged akin to puzzle pieces, rather than explicitly treating them as dynamic entities that emerge and interact within the schedule. A conceptual gap exists in that activities in schedules are not modeled in a manner that satisfactorily reflects their dynamic nature. Therefore, this paper explores a related field, traffic engineering, which is known for its significant body of knowledge on measurement and characterization of dynamic phenomena, as the source for analogous concepts. It is hoped that such inspiration will infuse realism into the theory and practice of construction scheduling.

2 LITERATURE REVIEW

2.1 Goals of Project Scheduling

Construction scheduling has several vital goals, all of which contribute to project performance. Besides determining the total project duration, or rather minimizing it, it also seeks to "identifying the specific actions to be performed… [,] relationships among the project activities [, and] … approximating the number of work periods needed to complete individual activities" (PMI 2008, pp. 50-51), i.e. their definitions, sequence, and durations within the schedule. But besides using the best available historical data to forecast realistic durations, other fundamental questions are less well explored, e.g. how to minimize the inherent risk throughout the entire schedule so that milestones and deadlines are achieved. Solution approaches may investigate selecting sequencing options that provide sufficient flexibility to be resilient to changes. Any realistic schedule optimization toward the goals of minimizing multiple performance parameters, including risk, time (activity and project durations, occurrence of delays or shifts in actual versus planned progress), cost, resource use (consumption and idle times), while maximizing productivity (active periods) and resilience (float) for a set of internal and external constraints, requires a sophisticated model that reflects the dynamic nature of such complex system. But it appears that current models are suited mostly to handling static schedules, which gives rise to the question of how schedules can gain new capabilities.

2.2 Limitations of Previous Scheduling Methods

The core of CPM has come to mean in most people's understanding that 'the' critical path provides the backbone of a schedule, which may occasionally shift somewhat due to changes or delays, but in general should be strictly adhered to – or returned to – at (nearly) all cost. Elaborate constraints in form of milestones, lead or lag durations on links, or extraneous links can be used to influence which particular sequence is deemed critical (Korman and Daniels 2003), but do not question the paradigm of the critical path itself. This perception facilitates a static view of the schedule as a vital tool for project managers. Among the shortcomings of CPM that have been voiced are a difficulty to facilitate resource continuity (Harris and Ioannou 1998) or deliberately manage beneficial modifications to the workflow, a focus on duration rather than productivity (Lucko 2009), and a widespread use of internal buffers within the schedule to compensate for duration variations, which is largely performed empirically (Russell *et al.* 2014).

Repetitive and linear scheduling methods overcome some of these defects by explicitly considering how work and time interact, either point-by-point or on a continuous basis. They typically model a linear growth of activities or segments thereof that progress within a two dimensional coordinate system of time and work. However, they essentially remained a graphical tools until the Productivity Scheduling Method (PSM) introduced a mathematical approach based on singularity functions, a type of range-based expressions, and formalized its optimization algorithm (Lucko 2009). Inputs and outputs related to project performance, including start and finishes in terms of time and work quantity, production rates, buffers, float, and critical path, can be visualized following the analysis. PSM can model not just constant activities, but also those with variable production rates. However, while it improves significantly upon CPM and previous approaches to repetitive and linear scheduling, it has fallen short of efficiently expressing the diverse ways in which projects grow and evolve, i.e. their 'flow' of individual activities and the overall project.

While earned value management (EVM) can assist in monitoring project performance, it suffers from conceptual drawbacks, including that its measure is expressed in dollar terms for schedule performance and becomes zero at the project finish, *regardless* of prior performance (Vandevoorde and Vanhoucke 2006). Advances in building information modeling (BIM) hold promise to providing rich input for creating more realistic schedules. Recent studies on data sensing are beginning to explore how to exploit visual information for updating schedule progress (Moon *et al.* 2014, Golpavar-Fard *et al.* 2011). However, they focus on the input side of extracting knowledge via image processing algorithms and machine learning techniques, whereas this study seeks to expand the capabilities of the underlying mathematical model.

2.3 Goals of Traffic Engineering

Traffic engineering is a field of research and practice whose goal is to transport persons and goods safely, rapidly, and efficiently, and is particularly focused on studying the designs and operations of the moving agents – vehicles – on the vast network of roadways and other traffic media. As such, traffic engineering research offers a rich body of knowledge in modeling dynamic phenomena, by expressing them either as movements of individuals particles or as continuous flows through the nodes and links of the network.

2.4 Need for Research on Conceptual Analogies

Since its early formalizations almost 80 years ago (Greenshields 1935), traffic engineers have observed and modeled such important measures and phenomena of highly dynamic traffic systems such as flow, density, and speed, or shockwaves that can describe jams. The latter arise through interactions of multiple sequential system elements. Importing concepts from traffic theory for project scheduling is theoretically possible because both employ 2D coordinate systems of time and work or distance, respectively, within which the trajectories of discrete elements – vehicles or activities – are tracked. As traffic engineering is experienced in modeling, analyzing, and optimizing dynamic phenomena, it is indeed surprising to find that project scheduling has not yet taken any inspiration from this field. The criticality and float of CPM suffer from limitations, because they are viewed as essentially static phenomena that are punctuated by occasional updates, rather than dynamic aspects of how a project unfolds over time. Linear and repetitive scheduling, while inherently better suited to modeling progress, lack a comprehensive theory of flow when compared with traffic engineering. Therefore, it is necessary to investigate how project scheduling, here focusing on the latter techniques, can finally be infused with urgently needed dynamic elements.

2.5 Research Objectives

To address the stated need, this paper begins an exploration of traffic engineering to identify concepts that have potential for novel and beneficial use in project scheduling. This paper can only serve to provide the beginnings of such comprehensive undertaking. Singularity functions – explained in the following section – provide the necessary enabling factor to adopt and adapt them. After introducing their mathematics and recent application to linear schedules, Research Objective 1 is to identify concepts that fulfill two criteria, being of fundamental importance in traffic engineering and being readily usable without any modification to their nature. Research Objective 2 is to convert them into singularity functions that can model activities in project schedules. Validation calculations are performed to establish credibility of the new formulas. Research Objective 3 is to identify concepts with significant potential for future interdisciplinary research.

3 SINGULARITY FUNCTIONS

3.1 Definition

Equation (1) provides the functional operator that is common to all singularity functions and has been introduced previously, e.g. to model periodic phenomena in cash flows, which was accomplished by creating signals (Su and Lucko 2013). In general, the operator can be understood as a generalization of a basic polynomial term. The behavior of the dependent variable $y(x)$ is determined by the values of three coefficients; the strength s is the intensity of $y(x)$, the activation a is the location on the x-axis whereafter $y(x)$ is evaluated for non-zero behavior, and the power n is the type of behavior, i.e. constant, linear, quadratic, or of fractional or higher orders. Here, x stands for work quantity, y is time, and z symbolizes cost for consistency with prior research (Su and Lucko 2013), but is not used in the exploratory equations of this paper, although other combinations among managerially relevant variables of the 'dimensions' of project management, such as work, time, cost, and resources are certainly possible and useful (Su and Lucko 2014).

$$y(x) = s \cdot \langle x - a \rangle^n = \begin{cases} 0 & \text{for } x < a \\ s \cdot (x - a)^n & \text{for } x \geq a \end{cases}. \tag{1}$$

3.2 Principles

Multiple operators per Equation (1) can be added into a singularity function. Such superposition enables modeling complex behaviors from more basic ones. 'Singularity' refers to any individual locations along the independent variable, here the x-axis, where a prior behavior changes; e.g. $s_0 \cdot \left[\langle x - a_1 \rangle^0 - \langle x - a_2 \rangle^0 \right]$, where $a_1 < a_2$ and $0^0 \equiv 1$, yields a constant s_0 in the interval of singularities a_1 to a_2. Note that Equation (1) is commonly defined as right-continuous (but could also be modified), so that $y(x) = s$ at a_1, but at a_2 it is $y(x) = 0$ again already. Another example, $s_1 \cdot \langle x - a_1 \rangle^1 + s_2 \cdot \langle x - a_2 \rangle^1$ first has slope s_1 from a_1 to a_2, where its behavior changes to the additively superimposed new slope $s_1 + s_2$. Extensions of singularity functions have been investigated, notably rounding operators, which are applied to an independent variable x with $n = 1$ (linear growth) "to yield [a] stepped growth" (Su and Lucko 2013, p. 3162). By subtracting from a rounded operator another one that is shifted on the x-axis, an intermittent signal is gained. It period and amplitude can be controlled to model repetitive phenomena. Even more functionality is gained if operators or entire singularity functions are combined multiplicatively (Lucko *et al.* 2014). An operator could also be nested within another singularity function, as used for compound interest (Su and Lucko 2013).

3.3 Transposition

While the meaning of axes in the coordinate system as mentioned is aligned with previous studies so that x is work and y is time, depending on use it may be necessary to treat either as the independent variable and express $y(x)$ or $x(y)$ as the output (Lucko 2011). Mathematically transposing x and y while leaving their relationship intact is equivalent to rotating the axes in the coordinate system by 90°. Equation (2) derives general transpositions for various exponents to convert between measuring time on the horizontal or vertical axis. For $n = 0$ it can be derived if it has a limited horizontal range, which becomes a vertical step of that height. It is valid for $x < a_F$. Otherwise, a second singularity function can model the upper bound.

$$y(x) = s \cdot \left[\langle x - a_S \rangle^n - \langle x - a_F \rangle^n \right],\ n = \begin{cases} 0 \Rightarrow x(y) = a_S \cdot \langle y - 0 \rangle^0 + (a_F - a_S) \cdot \langle y - s \rangle^0 \\ 1, 2 \Rightarrow \quad x(y) = a_S \cdot \langle y - 0 \rangle^0 + \langle (1/s) \cdot y - 0 \rangle^{1/n} \end{cases}. \tag{2}$$

3.4 Productivity Scheduling Method

Linear schedules are graphically represented as coordinate systems in which activities are progress curves of work quantity over their duration on the time axis. Each activity is expressed as a singularity function. While activities may be planned with constant productivity, i.e. fixed slopes for their duration, in practice they often experience changes. This is modeled by introducing change terms as explained in the previous section into the singularity function. The granularity of such segmentation within an activity can be chosen with as fine a level of detail as the available data allow. Lucko (2009) has provided details of how to analyze and optimize linear schedules with singularity functions as summarized here as the foundation for the subsequent calculations. The algorithm follows the precedence constraints and obeys any milestones for starts or finishes of activities or segments thereof. Importantly it guarantees that within the framework of those constraints the minimum total project duration is generated. It comprises two mathematical steps:

- *Stacking* activities starts at the origin of the project. Proceeding by precedence, it creates one singularity function of Equation (3) per activity, where x and y are work quantity and time; their ratio is the productivity. The intercept '*start*' indicates the project start date, which is often zero.

Among *m* activity segments, 1 through *m* – 1 are created by successively modifying the previous cumulative productivity by a new Δ*work* / Δ*time*. Each singularity function is evaluated to identify its maximum *y*-value, i.e. tentative finish date, which becomes the intercept of its successor. Buffers that specify required distances between activities can be inserted by also using Equation (3). Stacking creates a conservative schedule without concurrency among dependent activities.

- *Consolidation* performs an optimization toward the minimum total project duration. Its inputs are all singularity functions of activities and buffers from the stacking step. Following the precedence of activities from earliest to latest, their intercepts are now systematically reduced until all activities have been consolidated to a position where no further reduction is possible unless constraints would be violated. For it, first singularity functions of direct predecessor-successor pairs are subtracted, their difference is the interstitial area between them in the coordinate system. Second, the minimum distance between them must be identified. For it, the difference equation is once differentiated per Equation (4). If segments are linear, such minimum distance can only occur at a start, singularity, or finish of either predecessor or successor. Once an *x*-value of the minimum distance is found, the difference equation is evaluated at that location for the difference Δ*y*. Third, said difference is subtracted from the successor intercept. This results in consolidating it onto its predecessor. All singularity functions are updated in this manner to their final values of start and finish dates. Figures (1) and (2) illustrate the algorithm for a small example that features variable and constant productivity in activities *A* and *B*, as well as progress directions in segments C_1 and C_2.

$$y(x)_{act} = start \cdot \langle x - a_0 \rangle^0 + \frac{work_0}{time_0} \cdot \langle y - a_0 \rangle^1 + \frac{\Delta work_1}{\Delta time_1} \cdot \langle y - a_1 \rangle^1 + \ldots + \frac{\Delta work_{m-1}}{\Delta time_{m-1}} \cdot \langle y - a_{m-1} \rangle^1 . \quad (3)$$

$$\frac{d}{dx} y(x) = s \cdot n \cdot \langle x - a \rangle^{n-1} . \quad (4)$$

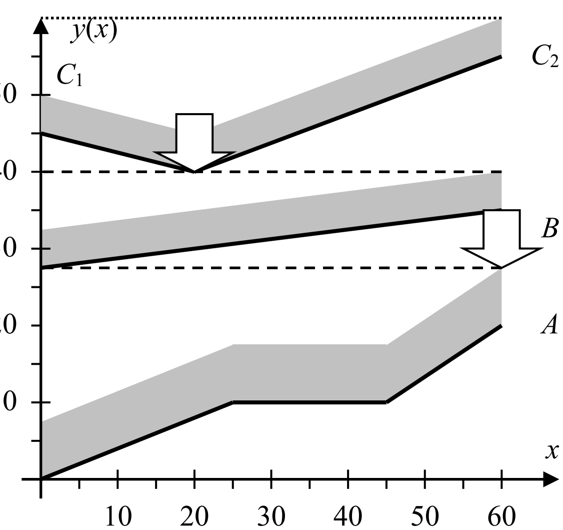

Figure 1: Productivity Scheduling Method: Stacking Step for Initial Solution

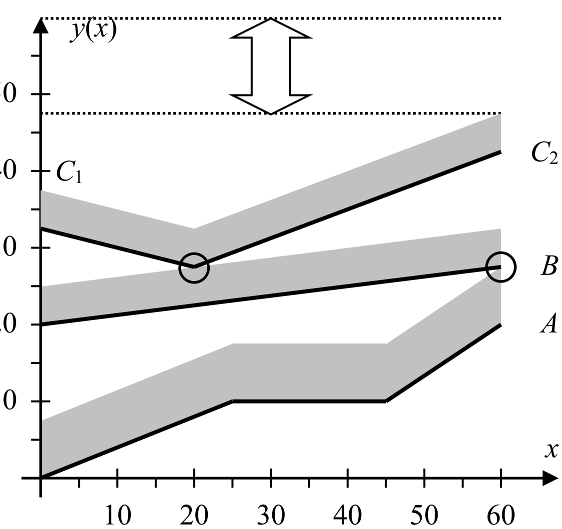

Figure 2: Productivity Scheduling Method: Consolidation Step for Optimized Solution

Note that activity *A* and *C* in Figure 1 already exhibit variability in their progress, here changing positions across the *x*-range of {25 to 45} without performing work that would consume time on the *y*-axis, and moving into two different directions before and after position *x* = 20. This, however, is planned and not induced by outside events or signals, which remains to be explored. Gray areas represent minimum buffers that must be maintained between activity pairs to prevent interference effects from congestion. Processing the activities per Equation (3) per the algorithm reduces the total project duration as shown.

4 INITIAL ANALOGIES FROM TRAFFIC ENGINEERING FOR PROJECT SCHEDULING

Before investigating more complex conceptual analogies of traffic engineering for adoption and adaption into project scheduling under future research, it is first necessary to explore how fundamental elements of these areas can be modeled and matched. These initial analogies are based on the vital measurement of progress – or lack thereof – through the time-space coordinate system. For traffic engineering, this rate of advancement is speed, for project scheduling it is productivity. Both share the time axis as the direct conceptual connection, over which the former measures distance and the latter measures work quantity. The following sections describe how the respective modeling terms are derived using singularity functions.

4.1 Signal Function for Time-Space Coordinate System

Both construction activities and traffic participants, i.e. vehicles, strictly speaking do not progress continuously, but incur changes in their rate of advancement and even encounter occasional standstills. For construction, they fall into two categories, planned and unplanned. The former comprises calendar-related items of weekends, holidays, and vacations, and shift-related items of breaks, non-working and nighttime hours. The latter includes sudden unavailability of productive resources (labor, materials, or equipment), e.g. mechanical breakdown of an excavator, unannounced inspections, accidents, weather events, or other acts of God. For traffic, they are traffic lights, which have a predictable period, stop signs, which act only upon a vehicle that arrives at any time, and yield signs, which regulate the interactions of two vehicles.

These phenomena have in common that they temporarily modify or deactivate the regular progress within a time-space coordinate system. Signal functions $w(y)$ using singularity functions have been shown to successfully model individual periodic phenomena. Equation (5) can start at any time, where a_S and λ are the integer and non-integer portions of the start date, and a_F is the finish date (Su and Lucko 2013). The previous application in cash flow analysis to provide a single 'peak' as a signal to issue a payment, however, is not yet suitable to express the alternating active and idle periods as inspired by traffic signals. This paper therefore modifies that signal function per Equation (6) to express 'block' shaped alternating active and idle periods, which of course should be able to have different durations. On and off durations n_1 and n_2 in Equation (6) repeat to infinity. For example, a signal may be on for n_1 = 10 seconds and off for n_2 = 5 seconds. Table 1 lists the verification of the calculations per Equation (7) as shown in Figure 3.

$$w(y)_{signal\_peak} = \left(\left\langle \lfloor y-\lambda \rfloor - (a_S - 1) \right\rangle^1 - \left\langle \lfloor y-\lambda \rfloor - (a_F) \right\rangle^1 \right) \\ - \left(\left\langle \lceil y-\lambda \rceil - a_S \right\rangle^1 - \left\langle \lceil y-\lambda \rceil - (a_F + 1) \right\rangle^1 \right) . \quad (5)$$

$$w(y)_{signal\_block} = \left\langle \left\lfloor \frac{y}{n_1+n_2} \right\rfloor - a_S \right\rangle^1 - \left\langle \left\lfloor \frac{y+n_2}{n_1+n_2} \right\rfloor - a_S \right\rangle^1 + 1 = w_1(y) - w_2(y) + 1 . \quad (6)$$

$$w(y)_{signal\_block} = \left\langle \left\lfloor \frac{y}{10+5} \right\rfloor - 0 \right\rangle^1 - \left\langle \left\lfloor \frac{y+5}{10+5} \right\rfloor - 0 \right\rangle^1 + 1 . \quad (7)$$

Here the singularity functions rely upon the aforementioned rounding operators, the floor operator $\lfloor\ \rfloor$ and the ceiling operator $\lceil\ \rceil$, which covert their operand into the nearest integer per Equations (8) and (9).

$$s \cdot \langle \lfloor y \rfloor - a \rangle^n = \begin{cases} 0 & for \lfloor y \rfloor < a \\ s \cdot (\lfloor y \rfloor - a)^n & for \lfloor y \rfloor \geq a \end{cases}. \tag{8}$$

$$s \cdot \langle \lceil y \rceil - a \rangle^n = \begin{cases} 0 & for \lceil y \rceil < a \\ s \cdot (\lceil y \rceil - a)^n & for \lceil y \rceil \geq a \end{cases}. \tag{9}$$

4.2 Trajectory Function for Time-Space Coordinate System

The basic expression for a vehicle moving on a road is analogous to an activity progressing along a trajectory in a schedule per Equation (10). Its strength *s* can represent speed of a vehicle or productivity of an activity. For example, per Equation (11), it may have started 1,000 m before the present location, which is treated as the origin, and moving at the speed of 20 m/s, where a_S and a_F are its respective start and finish.

$$x(y)_{trajectory} = x_S \cdot \langle y - 0 \rangle^0 + s \cdot \left(\langle y - a_S \rangle^1 - \langle y - a_F \rangle^1 \right). \tag{10}$$

$$x(y)_{trajectory} = -1{,}000 \cdot \langle y - 0 \rangle^0 + 20 \cdot \left(\langle y - 0 \rangle^1 - \langle y - 100 \rangle^1 \right). \tag{11}$$

Table 1: Calculation Verification

x	$w_1(x)$	$w_2(x)$	$w_{signal\_block}(x)$
0	0	0	1
1	0	0	1
2	0	0	1
3	0	0	1
4	0	0	1
5	0	0	1
6	0	0	1
7	0	0	1
8	0	0	1
9	0	0	1
10	0	1	0
11	0	1	0
12	0	1	0
13	0	1	0
14	0	1	0
15	1	1	1
16	1	1	1
17	1	1	1
18	1	1	1
19	1	1	1
20	1	1	1

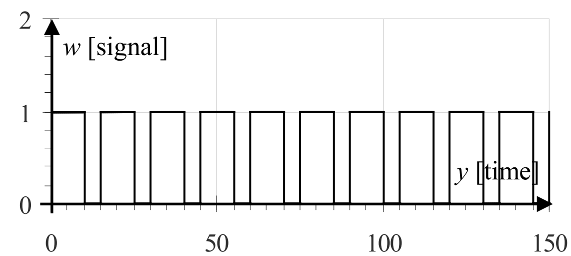

Figure 3: Signal Function with Block Shape

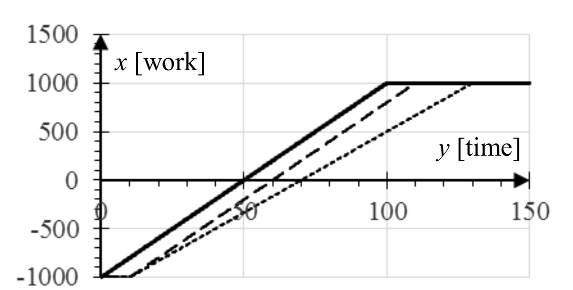

Figure 4: Trajectory Function with Shifts and Delays

Lucko (2008) has provided the general equations to perform PSM, where the slope of the singularity functions is the inverse of the productivity, if time is treated as the dependent variable for the purpose of optimization. Yet if time is treated as the independent variable, it matches the trajectory function for the vehicle of Figure 4, where distance is replaced by work quantity. Lucko (2013) has extended such linear schedule with considering shifts and delays to model how starts and finishes may change when an activity is executed. Per Equation (12), the shift d_1 is an outside influence – caused by a delayed predecessor –

that modifies both the start and finish, while a delay d_2 is an internal influence – cause by an insufficient productivity or interruption – that affects only the finish. For consistency with prior studies, an asterisk on start or finish denotes the existence of a shift or delay within dates; $a_S^* = a_S + d_1$ and $a_F^* = a_F + d_1 + d_2$. Note that the average modified productivity is also modified by a delay, as the work quantity U (which here is assumed fixed, but could be extended as well) is divided by a potentially longer duration $D + d_2$. Figure 5 shows how the activity of Equation (11) would be impacted by $d_1 = 10$ and $d_2 = 20$ time units.

$$x(y)_{trajectory\_shift\_delay} = x_S \cdot \langle y - 0 \rangle^0 + \frac{U}{D + d_2} \cdot \left(\langle y - (a_S + d_1) \rangle^1 - \langle y - (a_F + d_1 + d_2) \rangle^1 \right). \quad (12)$$

4.3 Trajectory Functions Controlled by Signals

Combining the signal and trajectory functions provides a very flexible model, which can express a vehicle stopping at a stop light, e.g. at an intersection (Liu *et al.* 2009), or an activity incurring an interruption. A vehicle trajectory can be divided into two parts, before and after a stopping location, here referred to as intersection for brevity. The intersection is located at location zero on the x-axis and alternatingly issues stop and go signals (i.e. red and green), which can be plotted over time along the y-axis per Figure 3. For clarity, the time axis is horizontal, which may apply the transposition of Section 3.3 to a linear schedule. It is assumed that the location of any possible interruption is known, whether an intersection or work position, but it is unknown if the interruption will have a negative effect or not without introducing a signal function. The times a_S^* and a_F^* in Equation (13) denote the actual start and finish when the vehicle is moving, which include the aforementioned shifts d_1 or delays d_2 from previous accumulated delays or a slower speed, respectively, where $a_{S\_after}^* = a_{S\_after} + d_{1\_after} = a_{F\_before}^* + d_{1\_after}$ and $a_{F\_after}^* = a_{F\_after} + d_{1\_after} + d_{2\_after}$. The term $x_{before/after} / (D + d_2)$ models the speed before and after the intersection. An important feature to connect the trajectory and signal functions is the shift time after the intersection d_{1_after} per Equation (14), which returns zero if the signal function is one (green). If the signal is zero (red), d_{1_after} returns the remaining duration from the time when a vehicle arrived at the intersection to when the signal will turn to green again per the term $(\lceil a_{F\_before}^* / (n_1 + n_2) \rceil - a_{F\_before}^* / (n_1 + n_2)) \cdot (n_1 + n_2)$ in Equation (14), where n_1 and n_2 are the aforementioned on and off durations of the cyclical signal. Substituting the actual arrive time at the intersection $a_{F\_before}^*$ into the signal function per Equation (6) returns the value of said signal at that time, i.e. it can check the current status. The condition term $a_{S\_after}^* = a_{S\_after} + d_{1\_after} = a_{F\_before}^* + d_{1\_after}$ in Equation (13) sets the actual start time $a_{S\_after}^*$ of the branch after the intersection equal to the finish time $a_{F\_before}^*$ of the branch before it, but only if d_{1_after} is not zero (i.e. a vehicle passes a green signal). However, if d_{1_after} is zero (i.e. the vehicle must stop at a red signal), it sets $a_{S\_after}^*$ equal to $a_{F\_before}^* + d_{1\_after}$. The term d_{2_after} of Equation (15) can incorporate a different speed of the vehicle after the intersection.

$$\begin{aligned} x(y) = x_{before} &+ \frac{|x_{before}|}{D_{before} + d_{2\_before}} \cdot \left(\langle y - a_{S\_before}^* \rangle^1 - \langle y - a_{F\_before}^* \rangle^1 \right) \\ &+ \frac{|x_{after}|}{D_{after} + d_{2\_after}} \cdot \left(\langle y - a_{S\_after}^* \rangle^1 - \langle y - a_{F\_after}^* \rangle^1 \right) \end{aligned}. \quad (13)$$

$$d_{1\_after} = \left(\left\lceil \frac{a^*_{F\_before}}{n_1 + n_2} \right\rceil - \frac{a^*_{F\_before}}{n_1 + n_2} \right) \cdot (n_1 + n_2) \cdot \left[1 - w_{signal} \cdot a^*_{F\_before} \right]. \tag{14}$$

$$d_{2\_after} = \left(\frac{x_{after}}{v_{after}} - D_{after} \right) \cdot \left[1 - w_{signal} \cdot a^*_{F\_before} \right]. \tag{15}$$

In traffic engineering, if a vehicle changes to a new speed v_{after} after having stopped at a red signal, it could be modeled by updating the previous speed $x_{after} / (D + d_2)$. In Equation (15), the planned duration D_{after} could be unequal to the actual duration x_{after} / v_{after} if a vehicle changes speed after a signal. In the example of Figure 5, $d_{2\_after} < 0$ for the third vehicle, which accelerates after the signal changes to green.

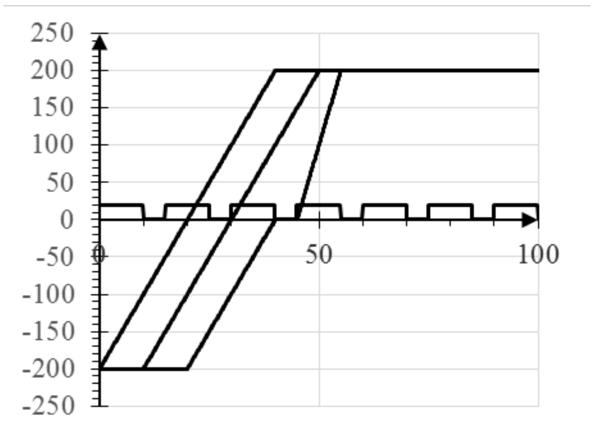

Figure 5: Multiple Trajectories with Slope Change

As with the previously investigated analogies, trajectories of multiple moving vehicles bear striking similarities to multiple sequential activities in a project schedule. Such bundles of progress curves are not merely subject to external factors such as calendars, weather, or resource limitations, but will also incur internal interactions that merit further exploration. For example, a 'shockwave', a classic phenomenon in traffic engineering that is generated by multiple vehicles that change speed in a time-staggered sequence (Wu and Liu 2011) and thus create or dissolve congestions could be modeled by an extension of the previous approach, but is beyond the scope and limited length of the initial exploration of this paper and left for future research.

5 VALIDATION WITH SINGULARITY FUNCTIONS

An example of a small linear schedule is used to validate the functioning of the newly developed model. Its parameters are selected as follows: An activity has an as-planned duration of $D = 20$ days to produce $U = 5$ work units with an initial shift of $d_1 = 0$ days that postpones its start and delay $d_2 = 0$ days that extends its duration. Actual dates that include these adjustments again are marked with an asterisk *. The activity per Equation (16) is shown in Figure 6 as a dotted upsloping line, where the slope $U / (D + d_2)$ denotes its productivity. A calendar of $n_1 = 5$ days and $n_2 = 2$ days for workdays and weekend is applied by Equation (17), which creates a dashed profile of *merlons* and *crenels* (peaks and valleys), which is amplified in the figure for clarity. More sophisticated singularity functions have been derived to calculate calendar dates for different patterns of n_1 and n_2, including different types of government holidays (Lucko 2014), but are not needed for this example. The arguments $\{n_S, n_F\}$ in the calendar signal function control its start and finish. To convert the activity (which is interrupted at weekends, akin to a vehicle obeying stop lights), into the desired calendar days, Equation (18) combines both the workday progress and the calendar signal functions and graphically plotted as the thick line in Figure 6. Such integration synchronously shifts the functions as follows: In the first week, the calendar function is applied by multiplying $x_{act\_workday}\{0,20\}$ and $w_{signal}\{0,1\}$; in the second week, $x_{act\_workday}\{2,22\}$ and $w_{signal}\{1,2\}$; andsoforth. Note that activities should only start on workdays, e.g. a $a^*_S = 6$ would be corrected to $a^*_S = n_1 + n_2 = 7$. As Figure 7 shows, this model can handle other inputs, e.g. for different work quantities, or productivities, or breaks, or shifts, delays, or calendars. It newly allows correctly calculating the vital time-related performance parameters such as productivity as the relationship of calendarized time and work quantity. Note that in CPM it would require six activities, five finish-to-start lags (weekend), productivity data would be lost, and any parameter change would require a full recalculation. CPM networks grow exponentially if a schedule grows more complex. Yet singularity functions only require modifying the parameter of an existing behavior or at the most adding one new term per change.

$$x_{act\_workday}\left\{a_S^*, a_F^*\right\} = \frac{U}{D+d_2}\cdot\left(\left\langle y-a_S^*\right\rangle^1 - \left\langle y-a_F^*\right\rangle^1\right). \tag{16}$$

$$w_{signal}\left\{n_S, n_F\right\} = \left(\left\langle\left\lceil\frac{y}{n_1+n_2}\right\rceil - n_S\right\rangle^1 - \left\langle\left\lceil\frac{y}{n_1+n_2}\right\rceil - n_F\right\rangle^1\right) - \left(\left\langle\left\lfloor\frac{y+n_2}{n_1+n_2}\right\rfloor - n_S\right\rangle^1 - \left\langle\left\lfloor\frac{y+n_2}{n_1+n_2}\right\rfloor - n_F\right\rangle^1\right) \tag{17}$$

$$x_{act\_calendar} = \sum_{i=0}^{\left\lceil\frac{D+d_2}{n_1}\right\rceil}\left(w_{signal}\left\{\left\lfloor\frac{a_S^*}{n_1+n_2}\right\rfloor + i, \left\lfloor\frac{a_S^*}{n_1+n_2}\right\rfloor + (i+1)\right\}\times x_{act\_workday}\left\{a_S^* + i\cdot n_2, a_F^* + i\cdot n_2\right\}\right) \tag{18}$$

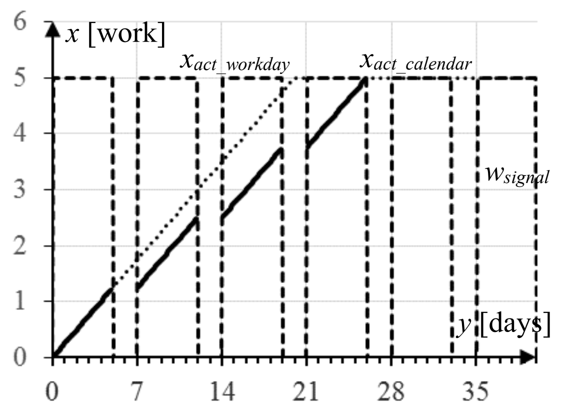

Figure 6: Initial Schedule Example with $D = 20$, $U = 5$, $d_1 = 0$, $d_2 = 0$, $n_1 = 5$, $n_2 = 2$

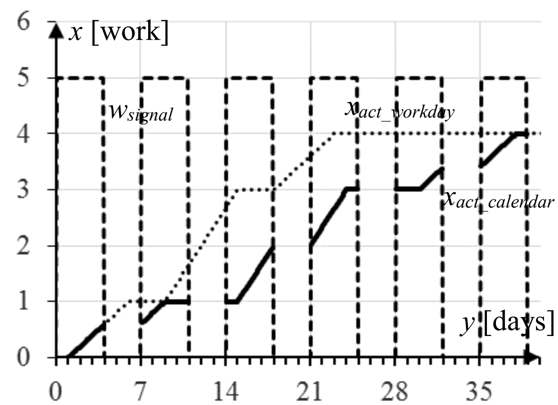

Figure 7: Modified Schedule Example with $D = 20$, $U = 4$, $d_1 = 1$, $d_2 = 2$, $n_1 = 4$, $n_2 = 3$ productivity 1/5, 1/3, 1/5 unit/d with 3 days breaks

6 CONCLUSIONS, CONTRIBUTIONS, AND RECOMMENDATIONS

Current construction scheduling techniques share the significant disadvantages of being activity-driven with a stationary perspective, thus lacking the capability of planning a construction project holistically and controlling it dynamically. Linear scheduling possesses the virtue explicitly linking time with work quantity, which can be modeled mathematically by using singularity functions and displayed graphically. However, it itself is limited by lacking an underlying theory that facilitates understanding and advancing the dynamic nature of projects. Therefore, this paper has begun to advocate *that project scheduling can be improved by exploring how traffic engineering plans and controls its dynamic flows* by employing singularity functions as the tool to transfer the relevant concepts as seamlessly as possible. Trajectories merit immediate investigation, which are matched with progress curves of construction activities. Signals to interrupt progress provide a direct constraint, which are matched with interruptions that can occur in construction, as well as calendars that govern construction projects. Both have been modeled mathematically.

Methodologically, this study has compared and aligned fundamental concepts. Its contributions to the body of knowledge include that attention has been called to the need for overcoming the current limited view of project scheduling, which by defaults treats its schedules as static. Instead, this paper has advocated a dynamic systems view that is inspired by another – initially seemingly unrelated, but in fact highly inspiring – field, traffic engineering. The necessary equivalencies can be realized by employing an integrated mathematical expression, singularity functions. New avenues to enhance linear scheduling are enabled by establishing said direct linkage between the two areas. Several further analogies, especially those arising from interactions of elements, await investigation to further advance this new approach of extending the theory of project scheduling as inspired by the performance parameters that characterize traffic.

Additional topics in project scheduling, e.g. limited availability of labor or equipment resources, and congested or restricted space on the project site could be addressed with an approach that employs signal functions. On the side of source, the body of knowledge of traffic engineering still provides an abundance of theories, concepts, and metrics that may become accessible to project management via interdisciplinary research. Table 2 lists a tentative set of potential analogies from traffic engineering for project scheduling: As discussed, the vehicle in traffic and the activity in a schedule are both entities that move through time-space coordinates systems; the slope of the progress curve is the speed of vehicle or the productivity of an activity; headway and buffer are similar concepts, which denote a mandatory safety gap between entities and can be measured in time or space units; congestion in traffic and criticality in schedules appear analogous in that they identify entities that remain without any flexibility to adapt to changes; and signals at stop lights and intersections are analogous to resource limitations or calendarization; flow and density are metrics that characterize the behavior of multiple entities, whereas project scheduling lacks such understanding; and neither last nor least, a shockwave is "the motion of an abrupt change in concentration" (Liu *et al.* 2009, p. 413), which is a dynamic effect that arises from the change in flow and density, again completely lacking in the theory of project scheduling. Even more yet unidentified concepts very likely exist. Future research will continue to exploit these analogies with the goal of deriving generalizable insights that can be converted into more efficient and effective scheduling techniques for construction managers.

Table 2: Potential Analogies from Traffic Engineering for Project Scheduling

Traffic Engineering	*Description*	*Project Scheduling*
Vehicle	Entity moving in coordinates system	Activity
Speed	Slope of progress curve	Productivity
Headway (time/space)	Safety gap between entities	Buffer (time/work)
Congestion (jam)	Entities without flexibility	Criticality
Signals	Entity moving with interruptions	Calendarization, limited labor / equipment / space
Flow, density	Behavior of multiple entities	Lacks such holistic metric
Shockwave	"[M]otion of an abrupt change in concentration" (Liu *et al.* 2009, p. 413)	Lacks such dynamic effect
Other	To be explored	Not yet existing

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