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STOCHASTICALLY CONSTRAINED SIMULATION OPTIMIZATION ON MIXED-INTEGER SPACES

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ABSTRACT

We consider the problem of identifying solutions to an optimization problem whose domain is a subset of the mixed-integer space and whose objective and constraint functions can only be observed via a stochastic simulation. In particular, we present cgR-SPLINE, a provably efficient algorithm on integer spaces. Additionally, we present heuristics for algorithm parameter selection that have demonstrated good finite-time performance of cgR-SPLINE. Lastly, we present an extension of cgR-SPLINE for mixed spaces and provide conjectures on the performance of the proposed algorithm.

1 INTRODUCTION

We consider the largely unexplored class of simulation optimization (SO) problems over a subset $\mathbb{X}$ of the mixed-integer space $\mathbb{Z}^{d_1} \times \mathbb{R}^{d_2}$. As is characteristic of SO problems, the objective function $g(\boldsymbol{x})$ does not

$$
\begin{aligned}
P: \qquad & \min_{\boldsymbol{x} \in \mathbb{X} \subseteq \mathbb{Z}^{d_1} \times \mathbb{R}^{d_2}} g(\boldsymbol{x}) \\
& \text{subject to} \quad h_i(\boldsymbol{x}) \leqslant 0, \;\; i = 1, 2, \ldots, c
\end{aligned}
$$

have a known algebraic form, but is estimable via a stochastic simulation. A similar restriction on the set of functions $h_i(\boldsymbol{x})$, $i = 1, 2, \ldots, c$ defining the feasible region poses a much harder challenge of ascertaining feasibility of any given design point $\boldsymbol{x} \in \mathbb{X}$ while searching for an extremum.

Solution methods to this broad class of problems are recently finding popular appeal in SO literature due to their applicability to several real-world problems like, for example, scheduling problems in transportation and resource allocation problems in health care and manufacturing. For example, the objective may be to minimize the expected staffing cost of an emergency care unit while satisfying a prescribed service level, or to determine the best intervention policies that minimize the number of people expected to be affected by an emerging epidemic.

2 MOTIVATION

To motivate the need for a specialized solution framework to solve problem P and to demonstrate how the inclusion of stochastic constraints renders the problem "hard to solve", consider the following SO problem on a two-dimensional integer lattice (obtained by setting $d_2 = 0$ and $d_1 = 2$ in problem P). Its objective

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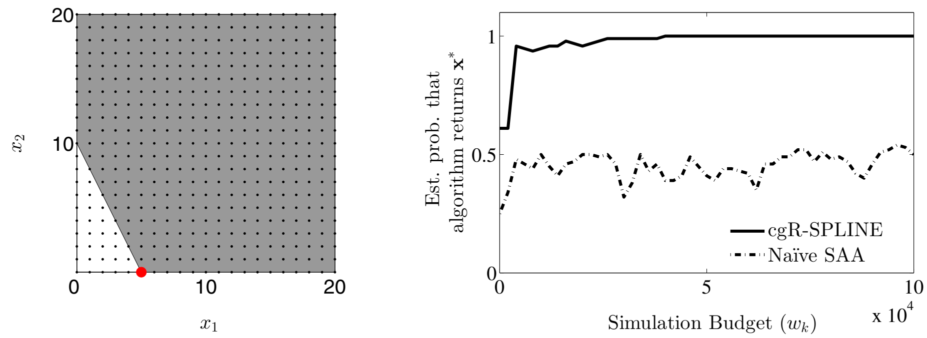

Figure 1: Figures showing (a) the search space of a two-dimensional toy SO problem, and (b) the performance of naïve SAA and cgR-SPLINE on the toy problem.

function $g(\boldsymbol{x}) = x_1^2 + 3x_2^2$ and constraint function $h(\boldsymbol{x}) = 10 - 2x_1 - x_2$. The feasible region is the set of points on the integer lattice in the shaded region of Figure 1(a) and its unique solution $\boldsymbol{x}^* = (5, 0)$, shown by the red dot, lies on the boundary $h(\boldsymbol{x}) = 0$. The estimators $\hat{g}_m(\boldsymbol{x}) = g(\boldsymbol{x})$ and $\hat{h}_m(\boldsymbol{x}) = m^{-1} \sum_{j=1}^{m} H_j(\boldsymbol{x})$, where $H_j(\boldsymbol{x})$, $j = 1, 2, \ldots, m$ are iid random variables with mean $h(\boldsymbol{x})$ and variance $\sigma^2 > 0$. Interestingly, since the solution $\boldsymbol{x}^*$ has a binding stochastic constraint, a naïve implementation of a sample-path approximation (SAA) routine fails to converge to the correct solution. This is because $\hat{h}_m(\boldsymbol{x}^*)$ approaches a normal distribution with mean 0 and variance $\sigma^2/\sqrt{m}$ and thus the probability $\Pr\left\{\hat{h}_m(\boldsymbol{x}^*) \leqslant 0\right\}$ of $\boldsymbol{x}^*$ being deemed feasible *equals* 0.5 in the limit as the sample size $m \to \infty$ (see Figure 1(b)). We believe that problem instances with solutions that have binding stochastic constraints are not pathological but fairly common in resource allocation problems with service-level constraints. Addressing consistency issues arising due to the difficulty in determining feasibility is thus crucial in the algorithmic procedure we seek.

3 CONTRIBUTIONS

We present cgR-SPLINE (Nagaraj and Pasupathy 2014), a multistart algorithm on *integer spaces* that iteratively solves a sequence of relaxed sample-path problems using an adaptation of the gradient-based SO routine R-SPLINE (Wang, Pasupathy, and Schmeiser 2013). A solution sequence that estimates the global minimum is obtained by probabilistically comparing the local solution estimator from the most recent restart with the incumbent global solution estimator. Consistency in cgR-SPLINE is guaranteed through careful relaxation of the stochastic constraints and the general convergence rate is shown to be sub-exponential. An exploration-exploitation characterization of the error rates suggests that cgR-SPLINE achieves the fastest convergence rate when the number of multistarts is proportional to the simulation budget per multistart. Additionally, we present heuristics for adaptively choosing the algorithm parameters that do not affect cgR-SPLINE's asymptotic properties. Extensive numerical experiments on problems adapted from the SO problem library (www.simopt.org) demonstrate cgR-SPLINE's good finite-time performance.

Lastly, we present ideas on extending cgR-SPLINE's solution framework to mixed spaces and provide conjectures on algorithm consistency. Ongoing work reveals that assumptions on the structure of the constraint function that were deemed reasonable in the integer-ordered context and which were crucial to showing consistency of cgR-SPLINE no longer hold in the in the context of continuous spaces.

REFERENCES

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