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ON-LINE FORECASTING OF CALL CENTER ARRIVALS VIA SEQUENTIAL MONTE CARLO

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ABSTRACT

We consider the intra-day forecasting of call center arrivals, a real-time challenge faced by call center managers in practice, under the dynamic doubly stochastic Poisson process model. This model stipulates that the randomness of the arrival rate is dynamically evolving rather than static as many existing models do. A major difficulty associated with the model is to estimate the posterior probability distribution of the arrival rate given the observed arrival counts and update the forecasts in a sequential manner. In this paper, we apply the sequential Monte Carlo method to solve this computational challenge.

1 INTRODUCTION

A common managerial challenge in service systems is to achieve balance between service quality and operational efficiency. This is certainly the case for call centers. In order to strategically staff call centers, it is essential for managers to develop accurate forecasts of future call volumes. Our focus in this paper is the short-term forecasting because in addition to accuracy, its "on-line" nature requires that the forecasting must be done in a timely and sequential fashion as fresh information becomes available. We will study the forecasts using the dynamic doubly stochastic Poisson process (DSPP) model proposed in Zhang (2013). The parameter estimation was addressed in Zhang (2013) using the Markov chain Monte Carlo (MCMC) method. However, the computational inefficiency makes the MCMC method inadequate for the short-term forecasting. By contrast, the sequential Monte Carlo (SMC) method estimates the posterior distributions of the arrival rates one at a time, thereby greatly facilitating the on-line forecasting of the future arrivals.

2 SEQUENTIAL MONTE CARLO METHOD

We briefly describe the model for the arrival process here and refer interested readers to Zhang (2013) for supportive evidence. Let $N(t)$ denote the cumulative number of arrivals by time t. We model the arrival process $(N(t) : t \geq 0)$ as a doubly stochastic Poisson process with arrival rate process $(\lambda(t) : t \geq 0)$, i.e. conditional on $\Lambda(t) \triangleq \int_0^t \lambda(s)\mathrm{d}s$, $N(t)$ has the Poisson distribution with mean $\Lambda(t)$. Moreover, we assume that the arrival rate $\lambda(t)$ is of multiplicative form with $\lambda(t) = \mu(t)e^{x(t)}$, where $\mu : [0,\infty) \to \mathbb{R}_+$ is a positive deterministic function, and $x(t)$ is an Ornstein-Uhlenbeck (OU) process

$$\mathrm{d}x(t) = -\kappa x(t)\,\mathrm{d}t + \sigma\,\mathrm{d}B(t), \tag{1}$$

where κ and σ are positive constants, and $(B(t) : t \geq 0)$ is a standard Brownian motion. The deterministic function $\mu(t)$ is used to capture the predictable patterns such as time-of-day or day-of-week effects, whereas the stochastic process $x(t)$ characterizes the dynamic evolution of the randomness underlies the arrival rate.

The on-line intra-day forecasting problem that we attempt to solve is as follows. Suppose that the parameters have already been estimated based on from historical data by virtue of the MCMC method developed in Zhang (2013). Given the parameters of our model, we compute the posterior distributions of the arrival rates as new data becomes available in a sequential fashion and continuously update the forecasts of future arrival counts a few hours in advance. The SMC method recursively estimates $p(X_n|Y_{0:n})$ based on the prior estimate of $p(X_{n-1}|Y_{0:n-1})$, thereby much more efficient.

The central idea of the SMC method is to simulate a set of random samples $\{X_n^{(i)} : i = 1,\dots,L\}$ and their associated importance weights $\{w^{(i)} : i = 1,\dots,L\}$. These weighted particles then form a discrete approximation to the posterior $p(X_n|Y_{0:n})$. Let $\Theta = (\theta_i : i = 1,\dots,k) = \mu(t)$, $X_i = x(i\delta)$ and $Y_i = N((i+1)\delta) - N(i\delta)$. If the weighted particles $\{(X_{n-1}^{(i)}, w_{n-1}^{(i)}) : i = 1,\dots,L\}$ approximate $p(X_{n-1}|Y_{0:n-1})$, then by Bayes' rule, the marginal posterior can be written as

$$p(X_n|Y_{0:n}) \approx \frac{p(Y_n|X_n)}{p(Y_n|Y_{0:n-1})} \cdot \sum_{i=1}^{L} w_{n-1}^{(i)} p(X_n|X_{n-1}^{(i)}) \propto p(Y_n|X_n) \cdot \sum_{i=1}^{L} w_{n-1}^{(i)} p(X_n|X_{n-1}^{(i)}),$$

where the particle $X_n^{(i)}$ can be simulated by the Gaussian transition kernel $p(X_n|X_{n-1}^{(i)})$ of O-U process, and its weight $w_n^{(i)}$ can be computed by $w_n^{(i)} \propto w_{n-1}^{(i)} \cdot p(Y_n|X_n^{(i)})$, where $p(Y_n|X_n)$ is Poisson with rate $\theta_{I(n)} e^{X_n}$.

We summarize the SMC algorithm for our model as follows.

Table 1: Bootstrap Filter with Resampling at Random Times

Initialization. At $n = -1$:

(i) Determine the prior distribution $p(X_{-1})$, and the number of particles L.

(ii) For $i = 1,\dots,L$, simulate particles $X_{-1}^{(i)}$ from $p(X_{-1})$ and compute $\tilde{w}_{-1}^{(i)} = p(X_{-1}^{(i)})$

(iii) For $i = 1,\dots,L$, normalize the importance weights: $w_{-1}^{(i)} = \frac{\tilde{w}_{-1}^{(i)}}{\sum_{j=1}^{L} \tilde{w}_{-1}^{(j)}}$.

Iteration. For $n \geq 0$:

(i) For $i = 1,\dots,L$, simulate particles $X_n^{(i)}$ from $p(X_n|X_{n-1}^{(i)})$ and compute $\tilde{w}_n^{(i)} = w_{n-1}^{(i)} \cdot p(Y_n|X_{n-1}^{(i)})$

(ii) For $i = 1,\dots,L$, normalize the importance weights: $w_n^{(i)} = \frac{\tilde{w}_n^{(i)}}{\sum_{j=1}^{L} \tilde{w}_n^{(j)}}$.

Given the weighted particles $\{(X_n^{(i)}, w_n^{(i)}) : i = 1,\dots,L\}$, the point forecast for future arrival Y_{n+j} is

$$\mathbb{E}[Y_{n+j}|Y_{0:n}] \approx \sum_{i=1}^{L} w_n^{(i)} \theta_{I(n+j)} \delta \cdot \exp\left(X_n^{(i)} e^{-j\kappa\delta} + \frac{\sigma^2(1-e^{-2j\kappa\delta})}{4\kappa} \right), \tag{2}$$

where the last equality follows from that $p(X_{n+j}|X_n^{(i)})$ is a Gaussian kernel. Distributional forecasts can be generated with different simulation paths therefore.

3 CONCLUDING REMARKS

We have developed a sequential Monte Carlo algorithm for the DSPP model for on-line forecasting of call center arrivals. The method is easy to implement and produces both point forecasts and distributional forecasts.

REFERENCES

Zhang, X. 2013. “A Bayesian Approach for Modeling and Analysis of Call Center Arrivals”. In *Proceedings of the 2013 Winter Simulation Conference*, edited by R. Pasupathy, S.-H. Kim, A. Tolk, R. Hill, and M. E. Kuhl.