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COMPUTATIONAL METHODOLOGY FOR LINEAR FRACTIONAL TRANSPORTATION PROBLEM

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ABSTRACT

Transportation engineering is one of the most popular area of Operations Research in which fractional programming is used. In this study, we present two algorithms to find initial basic feasible solution of a linear fractional transportation problem. Also we present a methodology to find optimal solution of the stated problem. Using simulation experiments on large number of examples we compare the results with other existing methods for finding initial solution and number of iteration to find optimal solution for linear fractional transportation problem (LFTP).

1 INTRODUCTION

In the real life decision making situations, sometimes we need to construct optimization model with linear fractional objective function such as profit/cost (financial and corporate planning), inventory/sales (production planning marketing), nurse/patient ratio (health care and hospital planning) etc. The most efficient method to tackle these kind of problems is the linear fractional programming problem approach. Hence fractional programming become one of the popular research topic in Operations Research. Most of the studies in this area are on the solution methodology and its application. One of the popular application of linear fractional programming is linear fractional transportation problem (LFTP). LFTPs are special type of integer programming problem. To solve these problems we use two steps. In the first step, we find out initial feasible solution by using some standard methods (North-west corner rule, Vogels approximation method etc.), then in the second step we use some iterative method to find the optimal solution. In this solution procedure, the most important thing is the initial basic feasible solution. If the initial solution is close to the optimal solution then we have to perform less number of iterations to reach the optimal solution. Sometimes the existing methods can not find a good initial solution. So we need to develop some improved method to find a better initial solution.

The objective of this study is to develop a better solution methodology for LFTPs. We develop two algorithms to find a better initial solution of a LFTP such that less number of iteration will use to reach the optimal solution of the problem. Also we observe that the proposed methods have less computational burden than some of the existing methods. Using simulation experiments on large number of examples we compare the results with other existing methods of finding initial solution and number of iteration to find optimal solution for LFTP.

2 Proposed Methodology to Solve Linear Fractional Transportation Problem

The mathematical model of a linear fractional transportation problem can be formulated as:

$$\max : Z(X) = \frac{Z_1}{Z_2} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}} \tag{1}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad \sum_{i=1}^{m} x_{ij} = b_j, \quad \sum_{j=1}^{n} b_j \leq \sum_{i=1}^{m} a_i, \quad x_j \geq 0 \tag{2}$$

where c_{ij} is the unit transportation cost from the ith source to the jth destination and d_{ij} is the unit preference from the ith source to jth destination. a_i is the amount supply from the ith source and b_j is the amount of demand made from the jth destination. $X = (x_{11}, x_{12}, ..., x_{mn}) \in \mathbb{R}^{mn}$ is the decision vector.

To find initial basic feasible solution for above problem we present the following algorithms:

Algorithm I

Step 1: The given transportation problem is unbalanced if $\sum_{j=1}^{n} b_j < \sum_{i=1}^{m} a_i$. Balance the problem by adding a dummy column. The value of c_{ij} are 0 and the value of d_{ij} are 1 in the dummy column.
Step 2: Establish the ratio matrix $R = (r_{ij})_{m\times n}$ whose entries are given by $r_{ij} = \frac{c_{ij}}{d_{ij}}\,(d_{ij} \neq 0)$.
Step 3: Select the minimum element of the ratio matrix R and allocate as much as possible to the cell. Break the tie by choosing the cell where maximum amount can be allocated.
Step 4: Repeat step 3 until total supply or total demand become zero.
Step 5: Compute the total transportation cost (Z_1), total preference cost (Z_2) and $Z = \frac{Z_1}{Z_2}$ for the obtained feasible solution.

Algorithm II

Step 1: The given transportation problem is unbalanced if $\sum_{j=1}^{n} b_j < \sum_{i=1}^{m} a_i$. Balance the problem by adding a dummy column. The value of c_{ij} are 0 and the value of d_{ij} are 1 in the dummy column.
Step 2: Establish the ratio matrix $R = (r_{ij})_{m\times n}$ whose entries are given by $r_{ij} = \frac{c_{ij}}{d_{ij}}\,(d_{ij} \neq 0)$.
Step 3: Calculate the penalties for all the rows and columns of the ratio matrix R. The penalty of a column (or a row) is the absolute value of the difference between two smallest element of the corresponding column (or row).
Step 4: Select the row or column with the highest penalty and break the ties arbitrarily or by choosing the row or column with smallest element.
Step 5: Find the smallest element of the selected row or column and allocate as much as possible to the feasible cell.
Step 6: Repeat step 3, 4 and 5 until all the demands have been met.
Step 7: Compute the total transportation cost (Z_1), total preference cost (Z_2) and $Z = \frac{Z_1}{Z_2}$ for the obtained feasible solution.

Using any one of these two algorithm we can find initial basic feasible solution for LFTP. Then we have to check the optimality condition and have to perform some iteration for the non-optimal cases. In the solution procedure of a LFTP, at any stage a basic feasible solution can be a degenerated solution. To find optimal solution we must remove the degeneracy by allocating infinitesimally small amount e to any of the cell of the obtained solution Table. To find that particular cell we use the method proposed by Shafaat et al. (Shafaat and Goyal 1988). Then to find optimal solution of the problem we use iterative method based on modified distribution (Sivri, Emiroglu, Guler, and Tasci 2011). We develop MATLAB code based on described methodology and perform simulation experiment on large number of problems to compare the results.

REFERENCES

Shafaat, A., and S. Goyal. 1988. "Resolution of degeneracy in transportation problems". *Journal of the Operational Research Society*:411–413.

Sivri, M., I. Emiroglu, C. Guler, and F. Tasci. 2011. "A solution proposal to the transportation problem with the linear fractional objective function". In *Modeling, Simulation and Applied Optimization (ICMSAO), 2011 4th International Conference on*, 1–9. IEEE.