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A SIMULATION BASED CUT GENERATION APPROACH TO IMPROVE DEO EFFICIENCY: THE BUFFER ALLOCATION CASE

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ABSTRACT

The stochastic Buffer Allocation Problem (BAP) is well known in several fields and it has been characterized as NP-Hard. It deals with the optimal allocation of buffer spaces among stages of a system. Simulation Optimization is a possible way to approximately solve the problem. In particular, we refer to the Discrete Event Optimization (DEO). According to this approach, BAP simulation optimization can be modeled as a Mixed Integer Programming model. Despite the advantages deriving from having a single model for both simulation and optimization, its solution can be extremely demanding. In this work, we propose a Benders decomposition approach to efficiently solve large DEO of BAP, in which cuts are generated by simulation. Numerical experiment shows that the computation time can be significantly reduced by using this approach.

Pedrielli, Matta, and Alfieri (2015) proposed a general DEO framework to model and optimize queueing systems. The approach relies on the Event Relationship Graph Lite (ERG Lite) formalism to formulate integrated simulation optimization mathematical programming models. ERG Lite is an extension of the Event Relationship Graphs. The authors showed that the BAP can be solved by DEO (Matta 2008) models that contain both simulation and optimization aspects. The simulation components control the event times, by means of constraints dealing with the system dynamics. The optimization components, instead, correspond to the binary variables and related constraints used for the capacity selection and minimization of total buffer space.

Due to the presence of integer variables, the computation time required to solve the DEO model significantly increases as the number of parts increases. However, a long simulation (i.e., a huge number of parts) is needed to reduce the effect of the initialization bias and the uncertainty in the random components. Weiss and Stolletz (2015) showed that the computation time could be substantially reduced by using a Benders decomposition approach to solve the BAP. As a further step, this work targets the further reduction of computation time by simulating, instead of solving, the mathematical programming problems, to generate cuts in a Benders decomposition approach. Our approach is similar to the L-shape decomposition (Higle and Sen 1991) with the main differences that we consider simulation optimization problems instead of two stage stochastic ones, and we use simulation to generate cuts.

In general, Benders decomposition generates cuts by iterating between a *master problem* and a *subproblem* (Benders 1962). The master problem contains the integer components and the generated cuts, while the subproblem contains the continuous components and the integer variables as parameters. The procedure can be summarized in three steps: 1) solve the master problem and use the variable values as input for the subproblem; 2) solve the subproblem and use the value of the dual variables to generate either an *optimality* or a *feasibility* cut; 3) add the generated cut to the master problem. At each iteration, the solution of the master problem is a Lower Bound (LB) on the solution of the original problem, while the solution of the subproblem is an Upper Bound (UB). The procedures stops when UB=LB.

In this work, we propose an innovative method that uses simulation, instead of mathematical programming, to generate cuts in *Step 2*. The variables and constraints of the subproblem are the simulation variables and constraints in the original problem. The variables and constraints of the master problem are the optimization variables and constraints of the original problem and the optimization cuts. Figure 1 reports a flowchart of the procedure for the DEO BAP model.

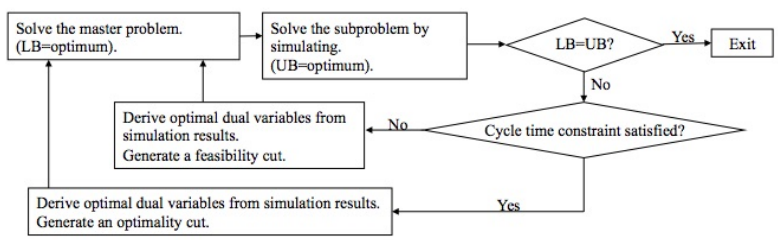

Figure 1: Benders Decomposition approach for DEO.

The main advantage of this method is the significant improvement in computation time while solving long integrated simulation optimization Mixed Integer Linear Programming (MILP) models. In particular, 1) the master problem, although it is a MILP, it is quite fast to solve due to the small number of variables and constraints and 2) the subproblem, which is a large LP, has a small computation time since it is solved by using simulation instead of optimization techniques. This avoids the exponential effort increase in the model size, since simulation shows an approximately linear increase. As a result, much larger DEO models can be solved thus improving the quality of the solution.

REFERENCES

Benders, J. F. 1962. "Partitioning procedures for solving mixed-variables programming problems". *Numerische mathematik* 4 (1): 238–252.

Higle, J. L., and S. Sen. 1991. "Stochastic decomposition: An algorithm for two-stage linear programs with recourse". *Mathematics of operations research* 16 (3): 650–669.

Matta, A. 2008. "Simulation Optimization with Mathematical Programming Representation of Discrete Event Systems". In *Proceedings of the 2008 Winter Simulation Conference*, edited by T. Jefferson, J. Fowler, S. Mason, R. Hill, L. Moench, and O. Rose, 1393–1400. Miami, Florida: Winter Simulation Conference.

Pedrielli, G., A. Matta, and A. Alfieri. 2015. "Discrete Event Optimization: Single-Run Integrated Simulation-Optimization Using Mathematical Programming". In *Proceedings of the 2015 Winter Simulation Conference*, edited by C. M. Macal, M. D. Rossetti, L. Yilmaz, I. Moon, W. K. Chan, and T. Roeder, 3557–3568. Huntington Beach, California: Institute of Electrical and Electronics Engineers, Inc.

Weiss, S., and R. Stolletz. 2015. "Buffer allocation in stochastic flow lines via sample-based optimization with initial bounds". *OR Spectrum* 37 (4): 869–902.