A SIMULATION-OPTIMIZATION APPROACH FOR DESIGNING RESILIENT HYPERCONNECTED PHYSICAL INTERNET SUPPLY CHAINS

Rafael D. Tordecilla
Jairo R. Montoya-Torres
William J. Guerrero

School of Engineering
Universidad de La Sabana
km 7 Autopista Norte de Bogota, D.C.
Chia, Cundinamarca, 250001, COLOMBIA

ABSTRACT
The Physical Internet (PI) is a recent paradigm in the supply chain management that proposes a framework in which standardization and optimization are key factors to raise supply chain efficiency, resilience, and sustainability. Strategic decisions are included in the PI, including the supply chain network design (SCND). In fact, structuring a (near) optimal design is essential to achieve the PI objectives. Additionally, disruptive events such as the COVID-19 pandemic, earthquakes, or terrorist attacks threaten the supply chains. These events are difficult to predict, but their effects can be simulated when addressing this problem. Hence, we propose a simulation-optimization approach that hybridizes a multi-objective multi-period mixed-integer program with discrete-event simulation to optimize both cost and resilience in the SCND. Furthermore, a network hyperconnection strategy is tested. Results show that both resilience and risk are improved after hyperconnecting the supply chain, especially when active edges are disturbed, but incur higher costs.

1 INTRODUCTION
A supply chain “consists of all stages involved, directly or indirectly, in fulfilling a customer request. The supply chain includes manufacturers, suppliers, transporters, warehouses, retailers, and customers” (Shapiro 2007). An improvement in the competitiveness of supply chains does benefit companies, as well as the economic and social development of countries, since it creates value and improves efficiency and overall performance of organizations (Ahi and Searcy 2013). However, with the continuous growth of supply chains into more and more complex systems, new challenges arise to achieve and maintain their efficiency. Indeed, according to Montreuil (2011), the current logistics paradigm is unsustainable from economic, environmental, and social points of view. The notions of resilience and sustainability have hence appeared to take over the solely economic performance of the management and design of supply chains (Montoya-Torres 2022; Tordecilla et al. 2021). In order to face these problems, the Physical Internet (PI) paradigm arises to increase the supply chain efficiency, resilience, and sustainability. The PI is based on the traditional digital Internet global system, since both have similar foundations, such as encapsulation, universal interconnectivity, openness, or standard interfaces and protocols (Montreuil et al. 2013; Montreuil 2011). For instance, the PI encapsulates the transported products in physical packets, called \( \pi \)-containers, which are characterized for being world-standard, smart, ecofriendly, and modular (Montreuil et al. 2010). Academics and practitioners have developed research and applications of the PI, as witnessed in several review papers (Maslarić et al. 2016; Saiz et al. 2016; Treiblmaier et al. 2020; Cortés-Murcia et al. 2022). Most scientific works have focused on conceptual developments and practical implementation, while research based on quantitative analyses is scarce. Hence, the objective of this paper is to deal with random
disruptions along a supply chain in order to optimize both cost and resilience. A simulation-optimization approach is proposed, in which multi-objective mathematical programming is coupled with discrete-event simulation. This paper is organized as follows. Section 2 presents an overview of academic works related to the PI. Section 3 describes the problem addressed in this paper, while the proposed simulation-optimization approach is presented in Section 4. Section 5 is devoted to present the computational experiments and analyze the results. Finally, Section 6 outlines our main conclusions and some future research lines.

2 RELATED WORK

In the search of resilience in supply chains, one of the disruptive paradigms is the PI. According to Montreuil et al. (2013), the PI is defined as “an open global logistics system founded on physical, digital and operational interconnectivity through encapsulation, interfaces and protocols. It is a perpetually evolving system driven by technological, infrastructural and business innovation”. Its main objective is to increase overall efficiency, resilience, and sustainability of supply chains. They also provide 8 foundations of the PI; this concept is strongly based on the digital Internet system and, hence, it poses a paradigm shift respect to the current logistics system. The term Physical Internet was however first used in a 2006 headline of the British popular press magazine The Economist, which contained a survey of logistics and a variety of mainstream supply chain articles (Cortés-Murcia et al. 2022).

Montreuil et al. (2010) describe the different physical elements regarding the PI, in which a fundamental one is the π-container with standard dimension for different sizes, since it ensures the encapsulation of goods and allows modularity and interlocking capabilities for transportation and storage. The π-movers are also described, which are designed to transport, convey, handle, lift, and manipulate the π-containers. Moreover, authors explain thoroughly different types of π-nodes, which are the locations where π-containers are received, sorted, picked, assembled, among other processes that are relevant to increase the system efficiency. All these elements contribute to design an efficient and sustainable logistics web in the PI context. In this same topic, Cortés-Murcia et al. (2022) present a systematic meta-review of literature with the objective of identifying how disruptive technologies and the PI impact the supply chain management. These authors analyze and synthesize 74 published literature review papers addressing these three topics and their relationship, and design a conceptual framework in which such relations are summarized: decision-making tools, real-time information processing, and business models. More recently, Nouiri et al. (2023) propose a short review of literature on the applications of the PI on the global performance of supply chains: multi-plant, multi-product supply chains and π-hub location problems are studied.

From a quantitative standpoint, research works dealing with the PI are scarce. At the operational level, Arnau et al. (2022) address a coordinated multi-vehicle routing problem in interconnected networks. A discrete-event deterministic heuristic is proposed to minimize the total routing time. Additional indicators (e.g., number of employed drivers, shipping time, total driving time) are computed as well. This procedure is extended to a probabilistic setting using a biased-randomized approach. In regard to the PI network design, the π-nodes location problem is similar to the Capacitated hub location problem (CHLP), which allows product flow through at most two hubs (Contreras and O’Kelly 2019; Ernst and Krishnamoorthy 1999). The PI has also been studied in the scientific literature jointly with other topics, such as city logistics (Crainic and Montreuil 2016), transportation (Sarraj et al. 2014), artificial intelligence (Nikitas et al. 2020), cloud logistics (Zhang et al. 2016), or the Internet of things (Tran-Dang et al. 2020). However, to the best of our knowledge, the current paper is the first work in the academic literature addressing the resilient supply chain network design (SCND) problem under the context of the PI.

3 PROBLEM DESCRIPTION

A SCND problem is addressed. It considers location-allocation decisions in a directed graph, in which three subsets of nodes are considered: suppliers, π-hubs (as so-called in the PI context), and customers. The number of suppliers, customers, and potential π-hubs are known, and we must decide which π-hubs
are open and which ones remain closed. The \( \pi \)-containers supply from each supplier, the \( \pi \)-containers demand of each customer, and the maximum capacity of each \( \pi \)-hub are known as well. Each \( \pi \)-hub has a fixed cost, and a variable cost depending on the number of \( \pi \)-containers they manage. The \( \pi \)-containers flow between nodes is performed via a set of potential edges, which have a known distance-related cost. We must also decide which edges are used, and the quantity of \( \pi \)-containers to flow via each active edge. A distance limit is imposed, i.e., any edge that exceeds this limit is not feasible, so that, for instance, in a real situation the time that truck drivers are on the road is not excessive, decreasing their fatigue and sleep deprivation. Achieving this objective is part of the PL foundations (Montreuil 2011; Treiblmaier et al. 2020). Additionally, feasible edges connect only suppliers to \( \pi \)-hubs, \( \pi \)-hubs to customers, and \( \pi \)-hubs to other \( \pi \)-hubs. Direct shipments from suppliers to customers are not allowed.

Two variants of this problem are considered. Firstly, a basic (B) problem in which a single type of \( \pi \)-container is sent. Any customer demand can be met by any supplier. Secondly, a hyperconnected (H) problem in which a single type of \( \pi \)-container is sent, and at least one path must connect each supplier-customer (S-C) pair. Figure 1 displays an example of complete solutions of both variants. Blue, orange, green, and small gray nodes represent suppliers, \( \pi \)-hubs, customers, and non-open \( \pi \)-hubs, respectively. In our work, hyperconnection means that any customer can be reached from any supplier via at least one path, such as Figure 1b shows. On the contrary, this action is not possible in the case of Figure 1a.

![](image1.png)

(a) A basic solution.  
(b) A hyperconnected solution.

Figure 1: Example of complete solutions for the B and H problem variants.

Both considered problem variants are multi-objective, since two objectives are intended to be optimized: cost and resilience. Considering resilience implies that the addressed problem is multi-period. At the beginning of the planning horizon, the supply chain is fully operative. Then a disruption occurs, perturbing any element of the graph, e.g., a \( \pi \)-hub capacity, which decreases the supply chain ability to meet the whole demand. Subsequently, the supply chain recovers gradually until it is again fully operative after a few time periods. The disruption occurrence is considered stochastic. Concretely, the number of elements to disturb (e.g., the number of \( \pi \)-hubs), the subset of elements to disturb, the recovery time, and the degradation level caused by the disruption are modeled as random variables. To deal with this randomness, a simulation-optimization approach is proposed, as explained in the following section.

As a first approximation to this problem, a few assumptions are considered as well. Firstly, we only consider a single tier of suppliers. Secondly, only one type of element is disturbed at a time, e.g., either \( \pi \)-hubs or connecting edges. Finally, lead time is not considered, i.e., each modeled period is long enough to complete a full shipment from suppliers to customers.

4 SOLUTION APPROACH

Since we seek to optimize both cost and resilience objectives after considering random disruptions over the supply chain, we have designed a simulation-optimization approach that deals efficiently with this problem.
This approach is illustrated in Figure 2. Initially, a general multi-objective mixed-integer programming (MIP) model is formulated. Table 1 shows the related sets, parameters, and variables. As depicted in Figure 2, three MIP models are solved in different phases of this procedure. Hence, the general model shown below is slightly modified to adopt the particular characteristics of each of these three MIP models.

Figure 2: Simulation-optimization approach to solve our considered problem.

\[
\text{Minimize} \sum_{i \in I} \left[ \sum_{h \in H} f_h z_h + \sum_{k \in K} \left( \sum_{(h',t) \in B} g_{h'kt} x_{h'kt} + \sum_{(h,j) \in C} g_h y_{hjkt} \right) \right] \\
+ \sum_{(i,h) \in A} c_{ih} r_{ih} + \sum_{(h',t) \in B} c_{h'kt} u_{h'kt} + \sum_{(h,j) \in C} c_{hj} v_{hjt} \\
\sum_{i \in I_{\text{in}}} \left( \sum_{k \in K} \left( \sum_{j \in J} y_{hjkt} + \sum_{j \in J} y_{hjkt-1} \right) \right) \Delta t
\]

\[
\text{Maximize} \frac{\sum_{i \in I_{\text{in}}} \left( \sum_{k \in K} \left( \sum_{j \in J} d_{jkt} + \sum_{j \in J} d_{jkt-1} \right) \right)}{2 t_{\text{max}}}
\]

\[s.t.
\sum_{h:(i,h) \in A} w_{ihkt} \leq s_{ikt}, \quad \forall i \in I, \forall k \in K, \forall t \in T
\]
\sum_{h:(h,j) \in C} y_{hjkt} \leq d_{jkt}, \quad \forall j \in J, \forall k \in K, \forall t \in T
\]
The objective function (1) minimizes the total cost, which is composed of fixed and variable costs for managing the \( \pi \)-containers in the \( \pi \)-hubs, and distance-related costs of traversing the edges. The objective
function (2) maximizes the supply chain resilience. This resilience measure is based on the work by Li et al. (2017), who computes resilience as the area under a recovery curve. This curve is defined by a known recovery function. The literature provides further resilience measures (Tordecilla et al. 2021). \( t_{\text{max}} \) is the maximum allowable recovery time, which is a parameter defined by decision-makers as a desirable threshold. Hence, Expression (2) is an approximation of this area, considering time intervals of width \( \Delta t \).

Constraint (3) ensures that the quantity of \( \pi \)-containers shipped by each supplier does not surpass their supply. Constraint (4) guarantees that the quantity of \( \pi \)-containers shipped to each customer does not surpass their demand. Constraint (5) ensures that the capacity of each open \( \pi \)-hub is not exceeded. Constraint (6) guarantees that each \( \pi \)-container entering each \( \pi \)-hub leaves. Constraints (7), (8), and (9) ensure that any \( \pi \)-container flows only via active edges. Finally, Constraints (10) and (11) indicate the variables that are integer and binary, respectively.

The first step in Figure 2 indicates that a basic model is solved. The following modifications are made to our general model to address this step: (i) the resilience objective (Expression (2)) is disregarded; (ii) \( T \) is a unit set, since we consider multiple periods only when resilience is being assessed; (iii) \( K \) is a unit set and \( K = Q \), since set \( P \) is used only to hyperconnect the supply chain; and (iv) Constraints (3) and (4) are equalities instead of inequalities. After solving this B model, we obtain the \( \pi \)-containers flow, the \( \pi \)-hubs to open, and the edges to activate. The latter two variables become input parameters for both the H model and the simulation phase. This simulation is performed as follows:

1. Determine the supply chain element and the related parameter to disturb, e.g., \( \pi \)-hub capacities, supplier supplies, edge capacities, etc.
2. Define a set containing the used elements according to the obtained results of the B model, e.g., \( \text{Open } \pi \text{-hubs} = O = \{h1, h3, h6, h7, h8, h14, h15, h17\} \) (Figure 1).
3. Define a set of the number of elements to disturb, e.g., \( \text{Number of } \pi \text{-hubs to disturb} = N = \{3, 4, 5\} \).
4. Select randomly an element \( n \in N \).
5. Select randomly \( n \) elements from the set \( O \).
6. Generate randomly the degradation level caused by the disruption and the recovery time for each disturbed element, e.g., how much capacity of the \( \pi \)-hub has been affected and when will be its full capacity restored.
7. Define a recovery function that allows the disturbed element to recover the full performance that it had before the disruption. For instance, if \( q_{h0}^{\text{ini}} \) is the \( \pi \)-hub capacity before the disruption, \( q_{h0} \) is the \( \pi \)-hub capacity at period \( t = 0 \) right after the disruption \( (q_{h0} < q_{h0}^{\text{ini}}) \), and the recovery time is \( t = \gamma \), the recovery function \( R(t) \) must go from \( R(0) = q_{h0} \) to \( R(\gamma) = q_{h0}^{\text{ini}} \).
8. Use the recovery function to compute the new disturbed parameter values in each period \( t \in T \) (Table 1), e.g., recompute \( q_{ht} \) if a subset of \( \pi \)-hubs has been disturbed, or \( s_{ht} \) if disruptions have affected the suppliers.

The disturbed parameter becomes an input for a new MIP model whose objective is to maximize the resilience. Additionally, the following modifications are made to our general model to address this step: (i) the cost objective (Expression (1)) is disregarded; (ii) \( K \) is a unit set and \( K = Q \); and (iii) the binary variables \( z_h, r_{ih}, u_{hh}, \) and \( v_{hj} \) become input parameters, according to the outputs of the B model. The goal of the latter modification is to have an unmodifiable supply chain configuration that can be tested in terms of resilience, i.e., only the \( \pi \)-containers flow is variable in this case. Hence, since this new MIP model does not have any binary variable, finding its solution is a very fast process, as well as the whole simulation phase. When the limit of simulation runs is reached, the obtained KPIs are saved. Obviously, the main KPIs are cost and resilience, but all tested configurations may be assessed as well in terms of, for instance, risk, \( \pi \)-hubs utilization, distance traveled by the \( \pi \)-containers, etc.

The outputs of the B model are employed as inputs for an H model as well. That is, the same open \( \pi \)-hubs and active edges yielded by the B model are kept as elements that contribute to hyperconnect the supply chain, however, additional edges must be activated to achieve this goal. Therefore, the following
modifications are made to our general model: (i) the resilience objective (Expression (2)) is disregarded; (ii) $T$ is a unit set; (iii) Constraints (3) and (4) are equalities instead of inequalities; (iv) Constraint (5) is replaced with Constraints (12) and (13), so that $\pi$-hub capacities are only a limitation for the $\pi$-containers flow and not for any path connecting any S-C pair; (v) the binary variable $z_h$ becomes an input parameter; and (vi) if the values of binary variables $t_{ih}$, $u_{hh'}$, and $v_{kj}$ obtained by the B model are equal to 1, they are preserved the same, otherwise, either 0 or 1 may be assigned to them when solving the H model. Then, the supply chain configurations obtained by the H model are simulated following the same procedure explained above.

\begin{align}
\sum_{k \in P} \left( \sum_{h':(h,h') \in B} x_{hh'k} + \sum_{j:(h,j) \in C} y_{hjk} \right) &\leq Mz_h, \quad \forall h \in H \tag{12}
\end{align}

\begin{align}
\sum_{k \in Q} \left( \sum_{h':(h,h') \in B} x_{hh'k} + \sum_{j:(h,j) \in C} y_{hjk} \right) &\leq q_hz_h, \quad \forall h \in H \tag{13}
\end{align}

Up to this moment of our proposed procedure only two supply chain configurations have been simulated, namely, the ones yielded by the B and the H model. Nevertheless, further configurations should be tested to assess more accurately the performance of our approach. Since the number of feasible configurations can be really high, we propose a procedure in which the model is forced to open more $\pi$-hubs than the number that minimizes the cost. Let $F$ be the set containing the number of feasible $\pi$-hubs to open mandatorily, and let $\varphi$ be each element of $F$, i.e., $\varphi \in F$. Then, we solve the B and the H model, and simulate both outputs iteratively for each element $\varphi \in F$. For example, in an instance whose optimal number of open $\pi$-hubs is 4, and the total number of available $\pi$-hubs is 10, $F = \{4, 5, 6, 7, 8, 9, 10\}$. Constraints (14) and (15) are added to the B model to force opening these additional $\pi$-hubs. As previously explained, $T$ and $K$ are unit sets in this case and, hence, they can be removed from these constraints.

\begin{align}
\sum_{h \in H} z_h = \varphi \tag{14}
\end{align}

\begin{align}
z_h \leq M \left( \sum_{i:(i,h) \in A} w_{ih} + \sum_{j:(h,j) \in C} y_{hj} \right), \quad \forall h \in H \tag{15}
\end{align}

Constraint (14) forces the model to open the given number of $\pi$-hubs. Constraint (15) guarantees two conditions: (i) a $\pi$-hub is open only if at least one $\pi$-container flows through it, and (ii) a $\pi$-hub is open only if it ensures a $\pi$-container flow between a supplier and a customer. If Constraint (15) is not added, the model opens $\pi$-hubs or groups of $\pi$-hubs that are unconnected from the rest of the network. Nevertheless, Constraint (15) has been proved to be highly restrictive for a few instances and, in this case, no feasible solution is found. Therefore, the model is solved one more time after replacing this constraint with Constraints (16) and (17). The former ensures the aforementioned condition (i), and the latter constraint guarantees that any edge connecting a pair of $\pi$-hubs is activated only if it ensures a $\pi$-container flow between a supplier and a customer, so that all open $\pi$-hubs are connected to the rest of the network. After adding both constraints all tested instances yield feasible solutions, however, models including Constraints (16) and (17) take longer computational times to find an optimal solution. Therefore, we use Constraint (15) for most instances and, only if no feasible solution is found, Constraints (16) and (17) are introduced.

\begin{align}
z_h \leq M \left( \sum_{h':(h,h') \in B} x_{hh'k} + \sum_{j:(h,j) \in C} y_{hjk} \right), \quad \forall h \in H \tag{16}
\end{align}
5 COMPUTATIONAL EXPERIMENTS

26 instances are employed to test our solution approach. Since our addressed problem has multiple particular characteristics, the literature does not provide benchmark instances that we can use without modifications. Hence, we design 6 newly-created instances, and adapt 20 benchmark instances from the CHLP (Ernst and Krishnamoorthy 1999; Farahani et al. 2013). In our designed instances, supplies, demands, and nodes coordinates were generated randomly. Resembling a global operation, we impose that suppliers have the lowest x-coordinates, customers have the highest x-coordinates, and π-hubs are located in the middle. All π-hubs have the same capacity, which is proportional to the aggregated demand. Transportation costs are the Euclidean distances between each pair of nodes, and fixed and variable costs of π-hubs are proportional to transportation costs. Ernst and Krishnamoorthy (1999) propose a set of instances for the CHLP, which we adapt to our considered problem. This set employs the Australia Post (AP) data set and has instances with 10, 20, 25, 40, 50, 100, and 200 nodes. Location coordinates, capacities, fixed costs, product flows, and distances are provided. The AP data set proposes that capacities and costs can be either loose (L) or tight (T). Additionally, we introduce the following modifications:

- We classify nodes into customers, π-hubs, and suppliers. Approximately one sixth of the nodes are suppliers, half are π-hubs, and one third are customers. The provided location coordinates remain the same, with suppliers having the lowest x-coordinates, customers having the highest x-coordinates, and locating π-hubs in the middle.
- Supplies and demands are computed as proportions of the product flows given in the AP data set.
- π-hub capacities and fixed costs remain the same as the AP’s.
- All nodes are connected each other in the AP data set. However, we consider as feasible edges only those that connect suppliers to π-hubs, π-hubs to customers, and π-hubs to other π-hubs (Table 1). Furthermore, according to the PI foundations (Montreuil 2011; Treiblmaier et al. 2020), the time that truck drivers are on the road must not be excessively long, so that their fatigue and sleep deprivation are reduced as much as possible. Hence, a distance limit is imposed when creating the feasible edges for our instances.
- We calculate transportation costs considering the Euclidean distances between each pair of nodes.
- A variable cost for managing the π-containers in the π-hubs is introduced. This cost is proportional to transportation costs.
- We create new random instances with 30 and 35 nodes, from the 100-node AP instances.

We consider 10 periods in the set T when maximizing resilience. The maximum allowable recovery time is defined as \( t^{\text{max}} = 7 \) time units and, hence, \( \Delta t = t^{\text{max}} / |T| = 0.7 \). Only for experimental purposes, the recovery time \( \Gamma_t \) is defined as a random variable that follows a Log-normal probability distribution, i.e., \( \Gamma_t \sim \log N(\mu, \sigma) \), where \( \ell \) is any disturbed element, \( \mu \) is the expected value, \( \sigma \) is the standard deviation, and \( \gamma \) is a realization of \( \Gamma_t \). Anyway, our approach is flexible enough to test alternative probability distributions. We set \( \mu = 5 \) and \( \sigma = 2 \) for all our experiments. Three sets of experiments depending on the supply chain element to disturb are performed, i.e., we disturb independently the open π-hubs, the suppliers, and the active edges. Each set of experiments is addressed following a different procedure, namely:

- Disturbing open π-hubs: the π-hub capacity is reduced by a disruption. Again, only for experimental purposes, the degradation level \( (\Theta_h) \) is a random variable that follows a Uniform probability distribution, i.e., \( \Theta_h \sim U(1, q^{\text{ini}}_h) \), where \( q^{\text{ini}}_h \) is the full π-hub capacity before the disruption. Therefore, the capacity right after the disruption is \( q_{h\theta} = q^{\text{ini}}_h - \theta_h \), where \( \theta_h \) is a realization of \( \Theta_h \).
If $D$ is the set of disturbed $\pi$-hubs, and we assume a linear recovery function $R(t)$, then the capacity of each disrupted $\pi$-hub in each period is computed by Equation (18).

$$q_{ht} = q_{h0} + \frac{t(q_{h0}^{ini} - q_{h0})}{\gamma_{h}}, \quad \forall h \in D, \forall t \in T \mid t < \left\lceil \frac{\gamma_{h}}{\Delta t} \right\rceil$$ (18)

- **Disturbing suppliers:** the supply is reduced by a disruption. In this case $\Theta_{i} \sim U(1,s_{i}^{ini})$, where $s_{i}^{ini}$ is the full supply before the disruption. Therefore, the supply right after the disruption is $s_{0} = s_{i}^{ini} - \theta_{i}$. If $D$ is the set of disturbed suppliers, and we assume a linear recovery function $R(t)$, then the supply of each disrupted supplier in each period is computed by Equation (19).

$$s_{it} = s_{0} + \frac{t(s_{i}^{ini} - s_{0})}{\gamma_{i}}, \quad \forall i \in D, \forall t \in T \mid t < \left\lceil \frac{\gamma_{i}}{\Delta t} \right\rceil$$ (19)

- **Disturbing active edges:** since the edges in our network are uncapacitated, we consider that after an edge has been disturbed, it becomes completely out of service. Hence, there are neither degradation level nor recovery function in this case. Instead, we consider that each disturbed edge is completely inactive $\forall t \in T \mid t < \left\lceil \gamma_{e}/\Delta t \right\rceil$. After this period, each disturbed edge is completely active.

Finally, besides cost and resilience, we consider the standard deviation of the simulated resilience as a measure of the risk of not meeting the demand. Small instances up to 35 nodes are employed to test our simulation-optimization approach. All experiments were performed in a PC with an Intel Core i7 processor and 16 GB RAM, using Windows 10 as operating system. The number of simulation runs is set to 500.

Table 2 shows the average results obtained when disturbing open $\pi$-hubs, suppliers, and active edges. All instances have been grouped by size, e.g., the group of instances $3x10x6$ is formed by 3 instances (see the number in parentheses). Each number in columns Resilience and Risk is an average of the results yielded after performing all simulation runs and testing all $\phi \in F$, where $F$ is the set containing the number of feasible $\pi$-hubs for each instance. Conversely, numbers in columns Optimal cost and Average cost are independent of the number of simulation runs. On the one hand, the column Optimal cost shows the cost obtained without considering Constraint (14), i.e., this is the cost yielded when the number of open $\pi$-hubs is $\phi^{*}$. On the other hand, the Average cost is computed as the average of the costs obtained $\forall \phi \in F$. Each cost column is the same for all disturbed elements, since disruptions are simulated after the optimal cost has been found. The supply chain design cost increases for every tested instance after considering the hyperconnection, since more edges must be activated. This action causes a resilience rise when disturbing open $\pi$-hubs or active edges. This growth is slight in the former case, and larger in the latter. Furthermore, hyperconnecting the network produces a generalized risk reduction for both cases, however, not all instances show this behavior in the risk. In any case, whenever the risk increases, such rise is small. Finally, the solving time also grows slightly after hyperconnecting the supply chain.

In the case where suppliers are disturbed, it has been proved that opening more $\pi$-hubs and hyperconnecting the network does not influence resilience or risk, but it obviously increases the design cost. Table 2 shows that perturbing the suppliers incurs a smaller resilience and a higher risk than perturbing the $\pi$-hubs, regardless of whether the network is hyperconnected or not. All these results let us conclude that disruptions affecting suppliers have a higher impact than those affecting $\pi$-hubs, especially considering that both resilience and risk do not improve when opening more $\pi$-hubs in the scenario where suppliers are disrupted. In the case where active edges are disturbed, results demonstrate that both resilience and risk worsen when compared to the cases in which $\pi$-hubs and suppliers are perturbed. Moreover, the column of the resilience gap between H and B models show that, whenever the edges are the disturbed element, hyperconnecting the network is a good strategy to both increase resilience and decrease risk. Figure 3 displays more clearly these differences for the cases in which either $\pi$-hubs or edges are disturbed, and considering different amounts of open $\pi$-hubs. The mean resilience is depicted by a red dashed line for the
Table 2: Average results after disturbing different supply chain elements.

<table>
<thead>
<tr>
<th>Group of instances</th>
<th>B model</th>
<th>H model</th>
<th>Gap H-B models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal cost</td>
<td>Average cost</td>
<td>Resilience</td>
</tr>
<tr>
<td>Disturbing open $\pi$-hubs</td>
<td>3x10x6 (3)</td>
<td>4072.3</td>
<td>5377.6</td>
</tr>
<tr>
<td></td>
<td>5x20x10 (3)</td>
<td>12493.9</td>
<td>15754.6</td>
</tr>
<tr>
<td></td>
<td>AP10 (4)</td>
<td>421750.9</td>
<td>557825.5</td>
</tr>
<tr>
<td></td>
<td>AP25 (4)</td>
<td>555746.2</td>
<td>727510.8</td>
</tr>
<tr>
<td></td>
<td>AP30 (4)</td>
<td>854159.5</td>
<td>1543459.3</td>
</tr>
<tr>
<td></td>
<td>AP35 (4)</td>
<td>934514.3</td>
<td>2184037.4</td>
</tr>
<tr>
<td>Average</td>
<td>431408.0</td>
<td>761898.4</td>
<td>0.9441</td>
</tr>
<tr>
<td>Disturbing suppliers</td>
<td>3x10x6 (3)</td>
<td>4072.3</td>
<td>5377.6</td>
</tr>
<tr>
<td></td>
<td>5x20x10 (3)</td>
<td>12493.9</td>
<td>15754.6</td>
</tr>
<tr>
<td></td>
<td>AP10 (4)</td>
<td>421750.9</td>
<td>557825.5</td>
</tr>
<tr>
<td></td>
<td>AP25 (4)</td>
<td>555746.2</td>
<td>727510.8</td>
</tr>
<tr>
<td></td>
<td>AP30 (4)</td>
<td>854159.5</td>
<td>1543459.3</td>
</tr>
<tr>
<td></td>
<td>AP35 (4)</td>
<td>934514.3</td>
<td>2184037.4</td>
</tr>
<tr>
<td>Average</td>
<td>431408.0</td>
<td>761898.4</td>
<td>0.8769</td>
</tr>
<tr>
<td>Disturbing active edges</td>
<td>3x10x6 (3)</td>
<td>4072.3</td>
<td>5377.6</td>
</tr>
<tr>
<td></td>
<td>5x20x10 (3)</td>
<td>12493.9</td>
<td>15754.6</td>
</tr>
<tr>
<td></td>
<td>AP10 (4)</td>
<td>237118.9</td>
<td>299323.9</td>
</tr>
<tr>
<td></td>
<td>AP20 (4)</td>
<td>421750.9</td>
<td>557825.5</td>
</tr>
<tr>
<td></td>
<td>AP25 (4)</td>
<td>555746.2</td>
<td>727510.8</td>
</tr>
<tr>
<td></td>
<td>AP30 (4)</td>
<td>854159.5</td>
<td>1543459.3</td>
</tr>
<tr>
<td></td>
<td>AP35 (4)</td>
<td>934514.3</td>
<td>2184037.4</td>
</tr>
<tr>
<td>Average</td>
<td>431408.0</td>
<td>761898.4</td>
<td>0.5162</td>
</tr>
</tbody>
</table>

**B model**, and by a black dash-dotted line for the **H model**. Hence, opening more $\pi$-hubs slightly increases resilience regardless of whether the perturbed element is a $\pi$-hub or an edge, and regardless of whether the network is hyperconnected or not.

---

**6 CONCLUSIONS**

This paper studied the problem of a supply chain network design under the Physical Internet paradigm, dealing with random disruptions along the supply chain in order to optimize both cost and resilience. A simulation-optimization approach was proposed, which hybridizes multi-objective multi-period mixed-
integer programming with discrete-event simulation. Two problem variants were considered: a basic problem in which a single type of $\pi$-container must be delivered, and any customer demand can be met by any supplier; and a hyperconnected problem in which, besides the former condition, there must be at least one path connecting each supplier-customer pair. Our approach was tested using both random-generated data sets and adapted benchmark instances from the Capacitated hub location problem. Three elements were randomly and independently disturbed: open $\pi$-hubs, suppliers, and active edges. Results show that hyperconnecting the network is a good strategy to increase the system resilience and decrease the risk of not meeting the customers demand, especially when active edges are disturbed. Nevertheless, this strategy did not produce any effect in these KPIs when disruptions affected suppliers and, hence, different strategies should be designed in this case to improve the system performance. Additionally, different quantities of open $\pi$-hubs have been tested, which enhanced resilience for both B and H problems when disturbing $\pi$-hubs and edges. Again, this strategy did not affect resilience when suppliers were disturbed. In any case, both hyperconnecting network and opening further $\pi$-hubs yielded higher design costs.

As pointed out previously in this paper, this is the first work addressing resilience in supply chains under the PI context and considering random disruptions. Hence, numerous lines for further research can be outlined. For instance, it would be very relevant to perform an analysis that identifies critical elements as, for example, which specific $\pi$-hubs or suppliers affect more intensively each KPI. Also, the study of different types of $\pi$-containers that can be transported and their impact on the supply chain performance, including sustainability metrics. Considering different definitions of hyperconnection is another open opportunity for future research, as well as additional resilience measures. Additional layers of the supply chain can be included in the analysis as well. Future work may also include perturbations of other elements, such as customer demands, and even combined perturbations of different types of elements. The consideration of random variables may lead to perform a detailed output analysis to, e.g., set a statistically significant relation between variables. Furthermore, if uncertainty is deeper and parameters of the employed probability distributions are not even known, fuzzy approaches can be employed. Finally, from the solution point of view, an interesting opportunity is designing and implementing heuristic algorithms (including metaheuristics, matheuristics or simheuristics), so that bigger instances can be solved efficiently and, hence, real data can be employed to validate our approach.

ACKNOWLEDGMENTS
This project has received funding from the Horizon 2020 Framework Programme of the European Union – Grant Agreement No. 861584 (ePIcenter Project), and Universidad de La Sabana, Colombia – Grant Agreement INGPHD-39-2020.

REFERENCES
AUTHOR BIOGRAPHIES

RAFAEL D. TORDECILLA is a postdoctoral researcher in the School of Engineering at Universidad de La Sabana, Colombia. He has a Ph.D. in Logistics and Supply Chain Management from Universidad de La Sabana, and a Ph.D. in Network and Information Technologies from Universitat Oberta de Catalunya, Spain. He holds a B.Sc. in Industrial Engineering and an M.Sc. in Logistics Process Design and Management. His research interests include logistics, transportation, and supply chain design and management, employing mathematical modeling and exact and approximate solution methods. His email address is rafael.tordecilla@unisabana.edu.co.

JAIRO R. MONTOYA-TORRES is Full Professor within the School of Engineering at Universidad de La Sabana, Colombia, where he also acts as Director of the doctoral program in Engineering and Director of the doctoral program in Logistics & Supply Chain Management. He holds a Ph.D. in Industrial Engineering from Ecole des Mines de Saint-Etienne and Université Jean Monnet de Saint-Etienne, France, and an M.Sc. in Industrial Engineering and Management from Grenoble Institute of Technology (Grenoble INP), France. His current research interests include supply chain design, urban freight transport, sustainability and resilience in logistics, and operations scheduling using optimization, simulation and hybrid procedures. His personal webpage and e-mail address are https://frmontoya.wordpress.com/ and jairo.montoya@unisabana.edu.co.

WILLIAM J. GUERRERO is an Associate Professor and the Head of the Research Group of Logistics Systems in the School of Engineering at Universidad de La Sabana, Colombia. He received the B.Sc. and M.Sc. degrees in Industrial Engineering and the Ph.D. degree in Engineering from the Universidad de Los Andes, Colombia, and the Ph.D. degree in Optimization and Systems Reliability from the Université de Technologie de Troyes, France. His research includes studies in healthcare logistics, data analytics, optimization, and simulation in Colombia and France. His research interests include transportation systems and operations research. His email address is william.guerrero1@unisabana.edu.co.