EQUIPMENT MAINTENANCE STUDIES USING A COMBINATION OF DISCRETE EVENT AND CONTINUOUS SYSTEM SIMULATION

Harold G. Hixson
Advanced Logistics Systems Center
Research Division (ACLR)
Wright-Patterson AFB, Ohio 45433

Abstract

These studies were undertaken to investigate the costs of alternative equipment maintenance procedures. The paper describes the scope of the investigation, the need for both discrete and continuous representation, the study costs and results, and the potential for using this model for additional applications.

1. INTRODUCTION

Simulation is a technique which is quite appropriate for use in actuarial studies since actuarial science deals with time-related events. However, a problem may arise when discrete event simulation is applied for such events occurring in extremely large populations. The problem is that the computer memory size required for storing attribute(s) of simulated entities normally exceeds the size of available memory. While hardware and software exists for solving this problem (3) and this hardware (a computer direct access mass storage device) is widely available, the required software is not yet generally available.

1.1 APPLICATIONS

Although the major use of actuarial methods is in the management of insurance and annuity plans (1), the Department of Defense has used these methods for the past 20 years in the management of aircraft engine logistics and illuminating engineers have applied them in attaining economies in lamp replacement ((5) and (6)). Two similar applications will be addressed in this paper.

1.2 SYNOPSIS OF ACTUARIAL METHOD

A short synopsis of the actuarial method follows, emphasizing its general applicability to individuals in populations whose failure probabilities vary as a function of age. For simplicity of the computational examples in this paper, each population discussed herein satisfies all the conditions necessary for it to eventually become stationary. These conditions include the stipulations that there will be no immigration into, or emigration from, the population, and that each of the age-specific failure rates remain constant with the passage of calendar time. In practice, when these conditions do not apply, computational complexity must be increased to represent changes in the composition of the population. The populations described in this paper do not become stationary within the simulated period only because the length of calendar time simulated was insufficient for this to occur.

2. TERMINOLOGY

Terminology used in actuarial computations includes the theoretical number of living individuals aged x in the population (represented as l_x) and the number of individuals dying during an infinitesimal age interval dx following age x (represented as dl_x). The probability of individuals aged x dying (or failing) during the interval dx is referred to as the mortality rate and is denoted by q_x. The process begins with a number of individuals aged zero (l_0). Prior to the end of a small increment of aging time (dx), a certain number of individuals (dl_0) fail to survive depending on the magnitude of q_x. The number of failures (dl_0) is computed by $dl_0 = l_0 \cdot q_x$. The number of individuals surviving to age 1 (l_1) is computed by $l_1 = l_0 - dl_0$. The process described in this paragraph continues throughout all age intervals in the same manner. However, it presupposes that, as regular increments of calendar time (dt) pass, the number

FIGURE 1

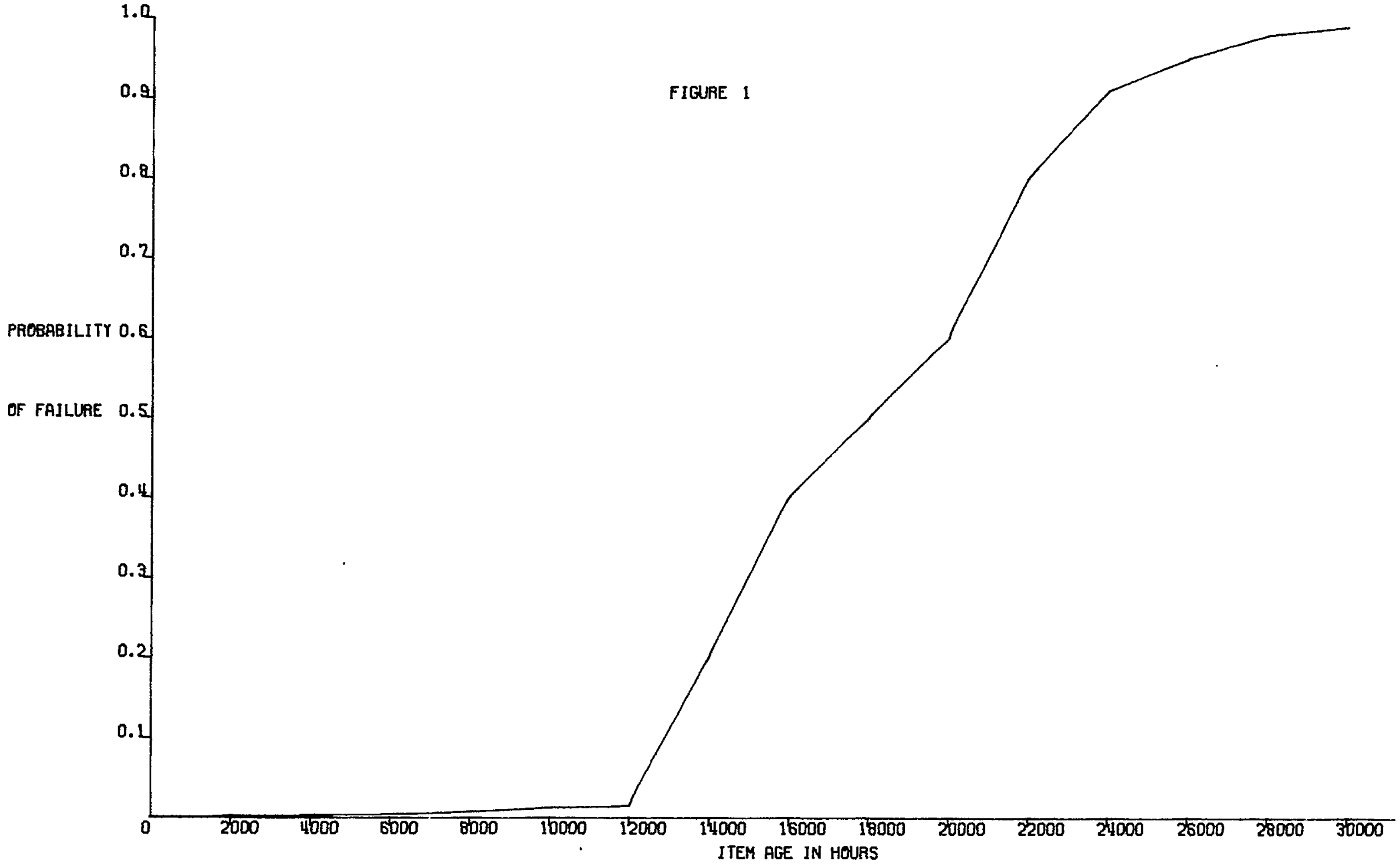
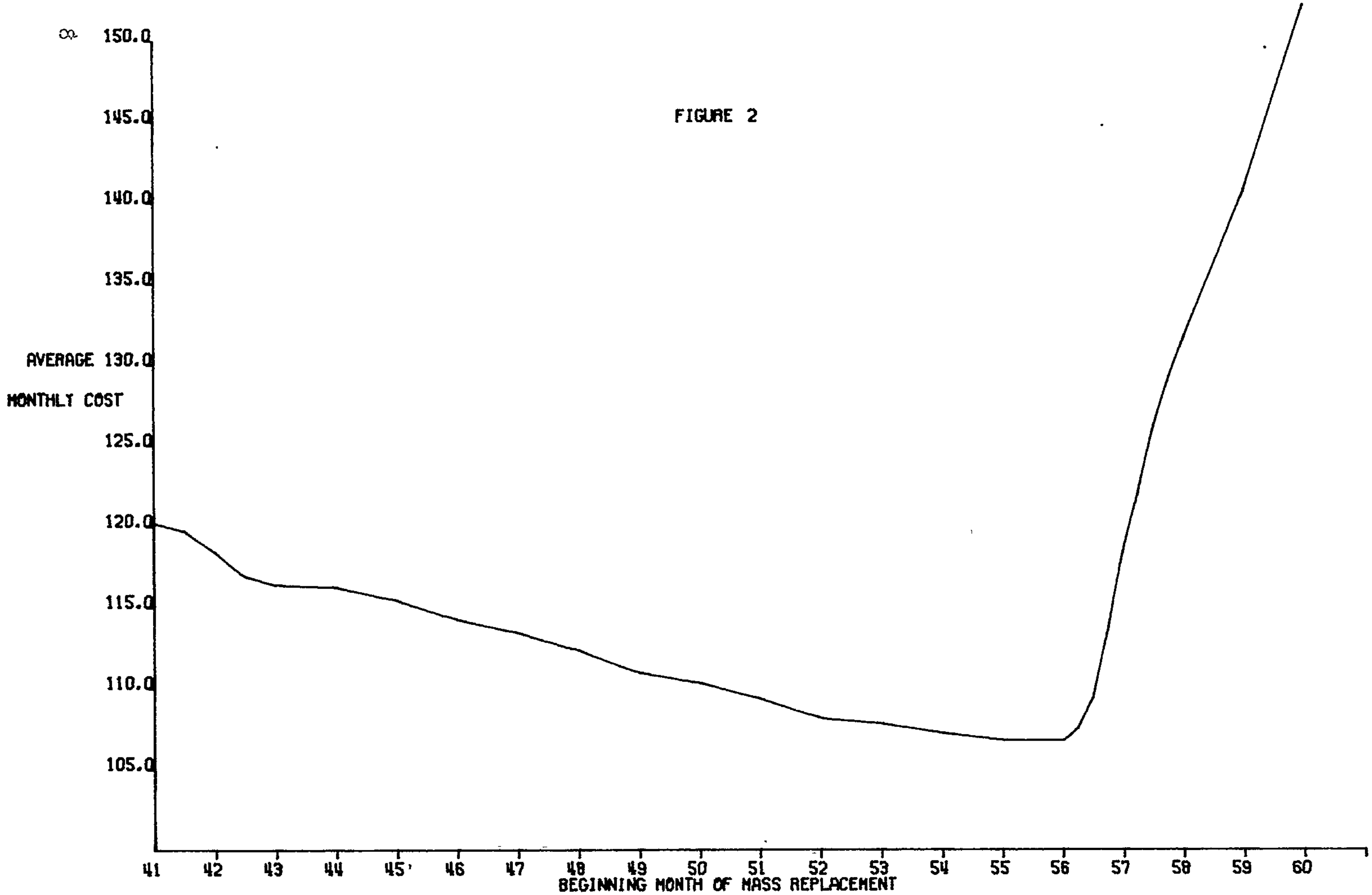

PROBABILITY
OF FAILURE
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
20000
22000
24000
26000
28000
30000
ITEM AGE IN HOURS

FIGURE 2


AVERAGE
MONTHLY COST
150.0
145.0
140.0
135.0
130.0
125.0
120.0
115.0
110.0
105.0
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
BEGINNING MONTH OF MASS REPLACEMENT

of individuals beginning life at age zero (l_o) remains constant. Note that the size of the increment of calendar time (dt) is taken to be equivalent to the size of the age interval (dx). Typical examples of mortality rates are shown in Figure 1.

2.1 PRACTICAL CONSIDERATIONS

The idealized situation described in the previous paragraph is almost never found in studies of real world populations. In some cases, all individuals in the population are "born" simultaneously. If mortality rates at early ages are relatively low, this "cluster" of individuals will move through the higher age intervals as a group. Age zero replacements of failed individuals may reach a peak when the "cluster" reaches higher age intervals where higher mortality rates prevail. It should be pointed out here that, for many populations, e.g., human beings, aircraft engines, etc., there is usually a high initial mortality rate, a "dip" in the rate, and increasing rates with age.

2.2 POPULATION AGING

In the previous paragraph several practical considerations were mentioned which make necessary the use of a more realistic conceptual framework for actual computations. Because the population is not stationary, the age distribution of the population keeps changing. For the remainder of this paper, the portion of the population exposed to the risk of failure in each age interval (x to x + dx) is denoted by S_x. Real world age distributions were measured and used as initial conditions in studies reported in this paper. Another phenomenon which is sometimes significant enough to warrant increased computational complexity is that of differences in aging rates. For example, aircraft engines are exposed to the risk of failure primarily when they are operating. For engines, the unit of age is normally taken to be 20 flying hours. However, different populations of engines have different utilization rates and individuals within groups may vary significantly with respect to rate of usage. In the first of the two studies reported here, the variation in the daily operation times of individual items was represented.

3. PURPOSE OF STUDIES

The remainder of this paper describes two actuarial studies which are examples of the application of combined simulation techniques. The first study compared the costs of two equipment replacement methods to determine the one which would be more economical. The second compared the costs of two methods for accomplishing aircraft engine repair.

4. REPLACEMENT METHOD STUDY

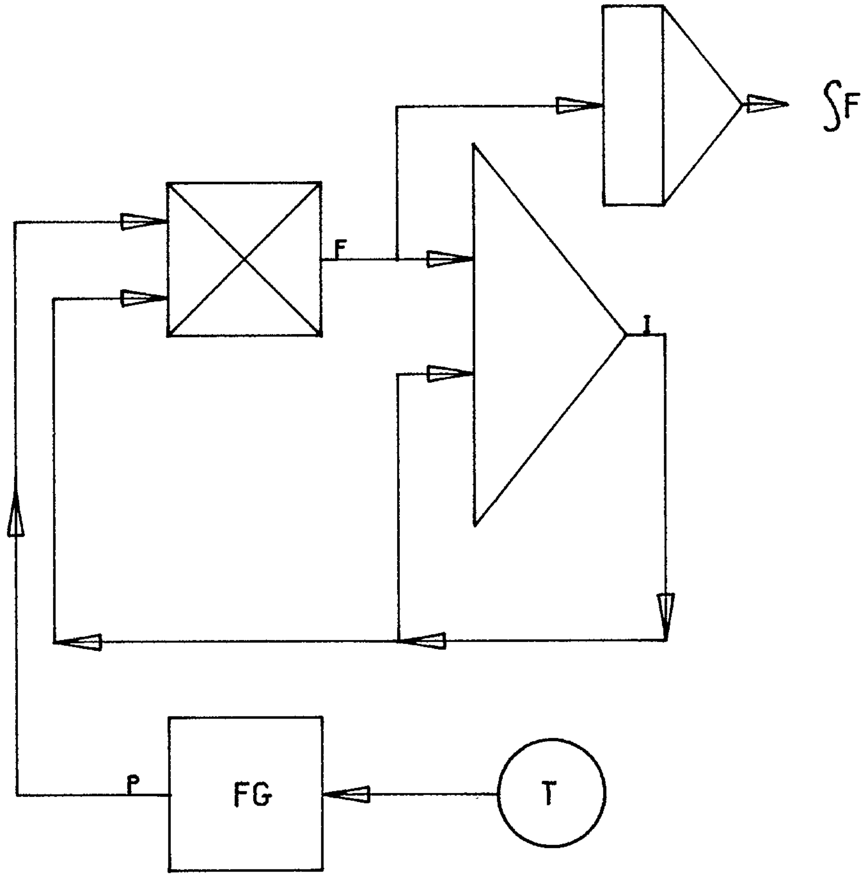
The first study concerned a population of 3480 equipment items subject to failure, with no possibility for repair, only replacement. One-half of the items are in operation 12 hours per day; four-tenths of them operate 14 hours per day, and one-tenth operate from 15 to 24 hours per day. All items (ΣS_x) begin the first day of operation at age zero. For a period of five years after this first day, the population was simulated under two separate and distinct maintenance systems. One of these was the practice formerly in use in the real world, "as-required item replacement," which occurs after each item failure. The other was a proposed change in maintenance practice, namely, a combination of "as-required" and "mass periodic" replacement. Under the policy of mass replacement, the unit is replaced at times KT (K = 1, 2, 3, ...) and at failure, where T is the mass replacement interval.

The former practice involved the replacement of failed items by new items on a demand basis. The prerequisite for implementing the latter practice is the estimation of an economical mass replacement interval. At the expiration of this interval (during which demand replacement is accomplished), mass replacement takes place. Since this mass replacement takes items out of service which have not yet failed, some discretion is usually used by the replacement technician. He makes a visual inspection of each item, and based on the results of this, he saves a percentage of the removed items. Those saved are considered to have more potential life remaining than the others. The percentage of items saved is equivalent to the estimated number of replacement items required during the replacement interval and the saved items are used for this purpose.

The estimation of an economical calendar time interval for mass replacement should be based on a sound appraisal of the replacement costs of both periodic mass replacement and the "as-required replacement" accomplished between mass replacements. To determine the most economical interval, the study described here utilized the results of a single simulation of the population with items replaced solely on an "as-required" basis. The results of the simulation were processed with a computational algorithm which measured the variation in system costs as the length of the mass replacement interval was varied (by one month increments) from a minimum of 41 months to a maximum of 60 months. The results of applying this algorithm are shown in Figure 2. From this graph of average monthly cost by replacement period length we see that a 56 month period between mass replacements is the most economical. The computation was continued at this point to compare the cost of mass replacement ($11,952) to cost of demand replacement ($39,461) over two mass replacement periods (112 months). This comparison showed that mass replacement costs $2947 per year less than demand replacement when implemented as described. Costs included both cost of new replacement items and the cost of labor for replacement.

The technique used in modeling the item failure process is represented in Figure 3 through the

FIGURE 3.



T - TIME
FG - FUNCTION GENERATOR
I - INSTALLED ITEMS
P - PROBABILITY OF FAILURE
F - FAILED ITEMS
$\int F$ - TOTAL FAILED ITEMS FOR PERIOD

FIGURE 4.

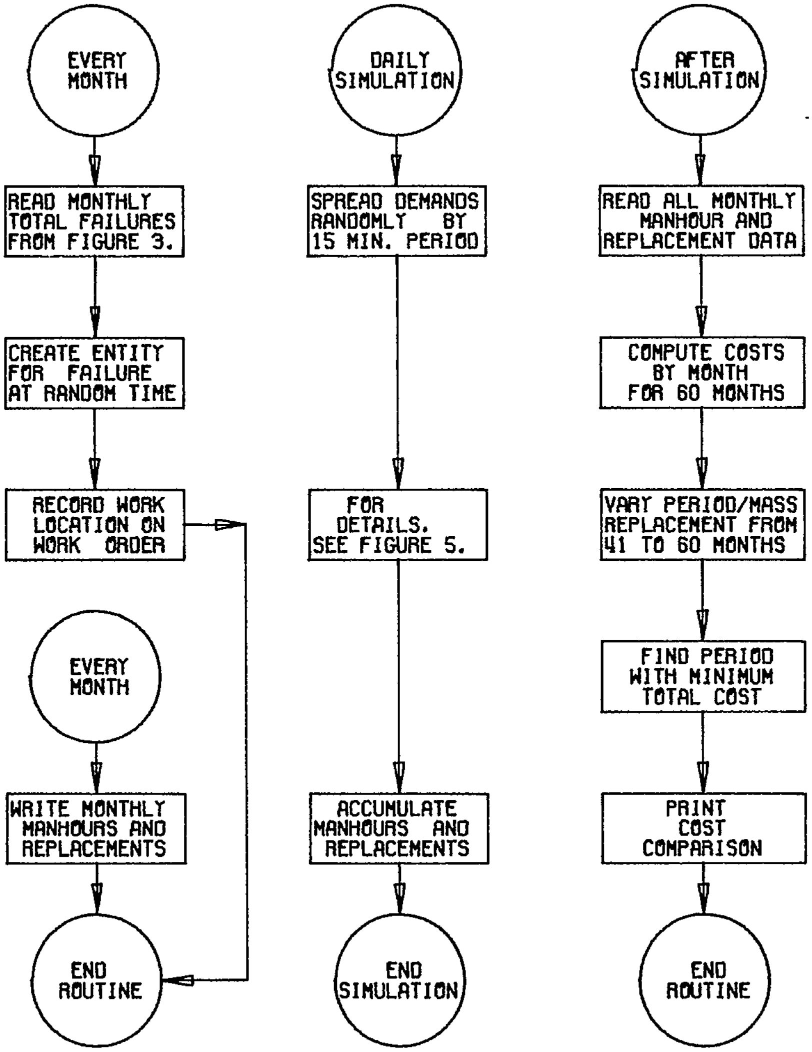
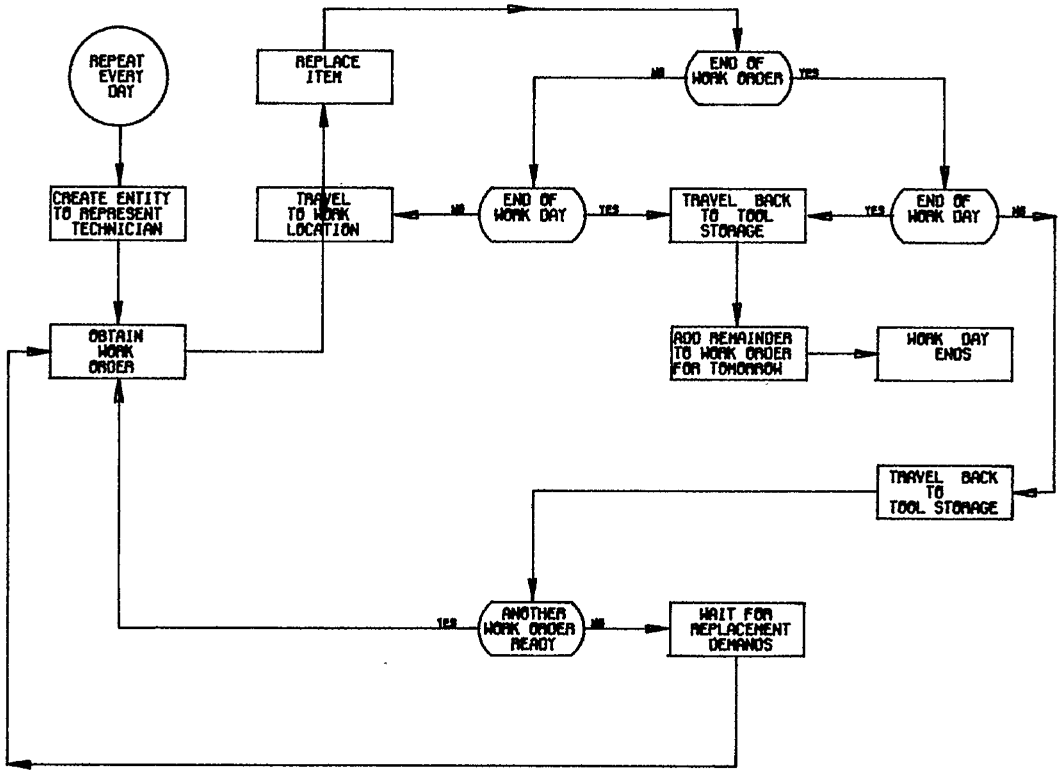

EVERY MONTH
READ MONTHLY TOTAL FAILURES FROM FIGURE 3.
CREATE ENTITY FOR FAILURE AT RANDOM TIME
RECORD WORK LOCATION ON WORK ORDER
EVERY MONTH
WRITE MONTHLY MANHOURS AND REPLACEMENTS
END ROUTINE
DAILY SIMULATION
SPREAD DEMANDS RANDOMLY BY 15 MIN. PERIOD
FOR DETAILS. SEE FIGURE 5.
ACCUMULATE MANHOURS AND REPLACEMENTS
END SIMULATION
AFTER SIMULATION
READ ALL MONTHLY MANHOUR AND REPLACEMENT DATA
COMPUTE COSTS BY MONTH FOR 60 MONTHS
VARY PERIOD/MASS REPLACEMENT FROM 41 TO 60 MONTHS
FIND PERIOD WITH MINIMUM TOTAL COST
PRINT COST COMPARISON
END ROUTINE

FIGURE 5.


REPEAT EVERY DAY
CREATE ENTITY TO REPRESENT TECHNICIAN
OBTAIN WORK ORDER
REPLACE ITEM
TRAVEL TO WORK LOCATION
END OF WORK DAY
NO
YES
END OF WORK ORDER
NO
YES
TRAVEL BACK TO TOOL STORAGE
YES
END OF WORK DAY
NO
ADD REMAINDER TO WORK ORDER FOR TOMORROW
WORK DAY ENDS
TRAVEL BACK TO TOOL STORAGE
YES
ANOTHER WORK ORDER READY
NO
WAIT FOR REPLACEMENT DEMANDS

use of standard continuous system block diagramming symbols. The number of failures is converted from floating point to integer representation and communicated monthly from the continuous process to the discrete process (Figure 4). These failures are spread randomly over the month and as these failure times are reached, an entity is created to represent each failed item. In the real world, as failures occur and are discovered, telephone calls from the failure locations to the technician dispatching office result in addition of the replacement request to a work order. As the technician leaves the central tool storage location each morning, he takes the accumulated work order with him and travels to as many locations as time permits. If he completes the work order before the end of the work day, he returns to the central office, where a new list of work locations may have been constructed in the mean time which he can use for his next tour. If there is no new list, he can perform other duties. If he does not complete the work called for by a list, he travels back to the office and adds the excess work locations to the work order for the next work day. In the simulation (Figure 4), the failed item entities merely add their work location to the current list at the proper time. As shown in Figure 5, another entity (one for each day) represents the technician, and the routing of this entity through the model represents his work and travel activities. After completion of the simulation of demand replacement, an analysis program used the failure and man-hour data which had been written monthly and retained on computer tape. These data were used to compute the cost of demand replacement and the cost of combined demand and mass replacement. By first computing the average monthly cost for a mass replacement period of 41 months, then increasing the trial period up to 60 months, the minimum cost period of 56 months was found. The cost for this period was used in making the final cost comparison of "on demand only replacement" and "demand plus mass replacement." The minimization of travel time is obviously the reason for "demand plus mass replacement" being the more economical method.

5. AIRCRAFT ENGINE REPAIR STUDY

The second study measured simulated failure, repair, and related activity of a population of 942 installed aircraft engines (ΣS_x) with two possibilities for repair location, at base (on site) and at depot (remote location). The base repair is less extensive, less expensive, and does not return the engine age to zero. The depot repair approximates a remanufacture of the engine, with more new or reconditioned parts used as replacements, and more demanding quality control checks. This process, provided it is a major overhaul, results in a return of engine age to zero. Only one base and one depot were studied although the generalized model was designed to handle more than one base. Initial conditions included the distribution of installed engines by age at simulated time zero and the number of spare engines at base and depot required to meet operational requirements. An arbitrary criterion of mandatory replacement of a failed engine by a repair engine within eight hours after failure was applied to determine the proper quantity of spares. All spares were specified to be initially at age zero. The primary purpose of the continuous system simulation module was to measure failure quantities and the subsequent input to both types of repair processes. The primary objective of using the discrete event simulation was to measure the number of spare engines needed for each base repair return rate postulated. The measurement was done on an iterative trial basis. The Air Force has developed a self-adaptive model which continually corrects each quantity of spares until the proper level is reached according to specified performance measures. However, the method is not presented here due to its complexity and reliance on specially developed computer techniques. The base repair return rate is the proportion of total failed turbojet engines returned to service by the base maintenance facility. The determination of the most economical return rate policy was the objective of this study. Such a policy is implemented by establishing a target return rate and using this target to influence all planning and operational decisions. Costs used in determining the most economical return rate (in Figure 6) included both spare and repair expenses and were computed manually. The time period for the simulation and cost accumulation was ten years at a uniform engine utilization rate.

The aircraft engine failure process model involves concepts familiar to continuous system simulation. The process is monitored every ten days by the discrete event model. The number of failures is truncated from floating point to integer representation in the monitoring and interface logic and then this quantity is spread randomly throughout each ten day time span. As each of these times is reached, an entity is created, representing a failed aircraft engine. Each failed engine is repaired either at the local base or at an overhaul depot some distance away.

The economics of the problem have been simplified in this example since operations at only one base are represented. Ordinarily, when more than one base is involved, there is an imbalance of training, tooling, and personnel cost between one depot repair facility and many base repair shops. This imbalance was not present and was therefore not accounted for here. Engine repair and spare procurement costs were considered. An engine overhaul is much more expensive than a base repair. However, more spares are required to support a depot overhaul operation due to the distance involved. Simulation results included measurements of the number of repairs completed during a simulated period of ten years and the number of spare engines required to keep the maximum time interval between failure and replacement less than eight hours. Thus, by multiplying these measurements by the appropriate

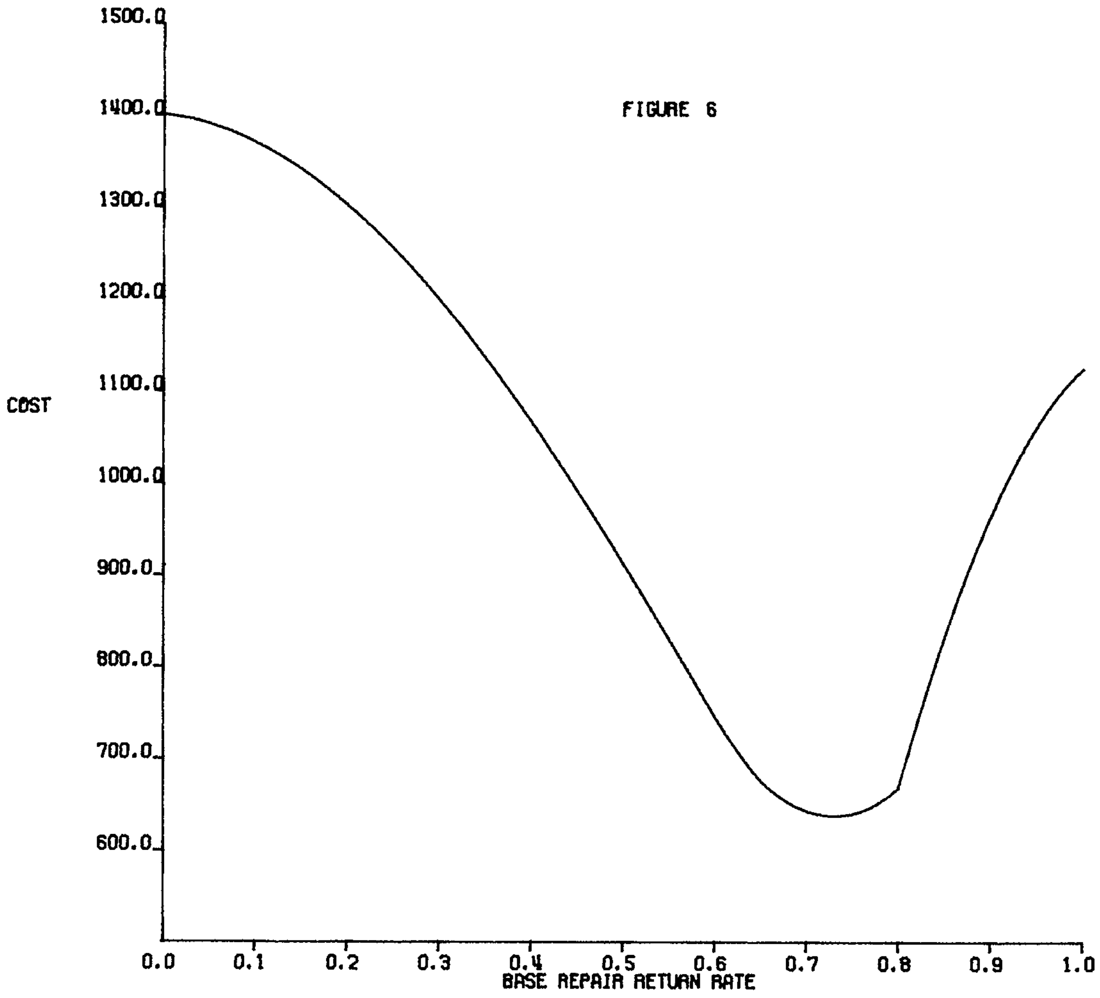


FIGURE 6

1500.0
1400.0
1300.0
1200.0
1100.0
1000.0
900.0
800.0
700.0
600.0
COST
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
BASE REPAIR RETURN RATE

repair and spare costs and accumulating, the engine portion of the weapon system life cycle cost was computed. Numerous simulation runs were made, each with a different base repair return rate. The graph of total cost versus return rate (Figure 6) indicates that a return rate of .73 is most economical for the particular type, model, and series engine population studied.

This study does not reflect the most recently developed improvements in Air Force engine logistics modeling. These incorporate the concept of a maximum allowable age for engines. Upon reaching this age, the installed engine must be removed and overhauled, regardless of its apparent condition. The impact of this policy is that the return rate is not constant for engines of all ages. The replacement of an age-constant return rate by variable return rates by age interval improves the accuracy of model results substantially although it makes the determination of a proper overall average return rate somewhat more difficult. This in turn raises the question of what maximum allowable age is proper, considering economics, flight safety, and other important factors. Investigations of this type are recommended for future simulation studies.

6. SUMMARY

For each of these studies, the continuous system model (and in one case the cost analysis module) was written in FORTRAN and the discrete event logic was GPSS. However, the only reason for selecting these languages was convenience and ease of debugging.

For both studies, input was taken from failure rate and cost data published in Air Force documents. As for verification of model logic, in both cases the author drew upon decades of prior work and study in this field. With the exception of over-simplified logic adopted for ease of presentation (which have been duly noted in this paper), the logic of these models is considered to be proper, complete, and accurate.

Resources expended on the maintenance method model and the maintenance location model were 174 hours and 193 hours, respectively, of the time of one operations research analyst. Computer time used was 14 hours (UNIVAC 1107) for the first study and 13 hours (IBM 7044/7094) for the second.

7. CONCLUSIONS

The type of model organization documented here is particularly well suited to the study of extremely large populations. In order to expedite debugging, the continuous model and the discrete model may be run separately with a minimum of modifications to preserve coupling requirements.

Although these studies concentrated on exploring economical maintenance practices and policies, they illustrate generalized concepts and indicate the broad potential for using composite continuous system and discrete event simulation. Other applications with obvious value to be gained by using this approach include the planning of new process control systems, chemical manufacturing systems, etc.

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9. BIOGRAPHY

Harold G. Hixson was born in Dover, Ohio, in 1935. He received the B. S. degree in mathematics from Otterbein in 1957. Since that time, first as an Air Force Lieutenant and later as a Civil Service employee, he has held several positions as an Actuary (Property), Mathematician, and subsequently, as an Operations Research Analyst in the Air Force Logistics Command at Wright-Patterson Air Force Base in Ohio. His current position involves work with advanced simulation methods and research. He is a member of Simulation Councils, Inc., and currently is serving as the Manager of the SHARE Systems Simulation Project.