A SIMULATION STUDY OF A MULTI-CHANNEL QUEUEING SYSTEM IN THE HOSPITAL ENVIRONMENT

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Abstract

This paper deals with the problem of analyzing the medical examinations of employees in a company managed hospital facility consisting of the following work stations, (1) receptionist, (2) rest rooms, (3) personal history, (4) sight screener, (5) x-ray, (6) laboratory, (7) electrocardiogram, (8) doctors, and (9) immunizations. The medical examination system is analyzed through the use of a GPSS simulation model for which the queue information is developed in three phases -- (1) data collection, (2) estimation of probability distribution forms, and (3) model format development.

Each of the above work stations requires one operator with the exception of the doctor's facility for which there are four full time doctors. Considering the personnel and function requirements of each work station, recommendations are made, as a result of the simulation study, for a more efficient utilization of these facilities.

1. INTRODUCTION OF THE PROBLEM

The systems of today are becoming more complex as the state of the art in all fields continues to expand. With this increase in complexity, the need for analyzing systems is imperative.

As the population of the world continues to grow, its hospitals must service more patients each year. Therein lies the problem of analyzing all the resources of a hospital to satisfy the ever increasing demand placed on it. This difficulty was experienced by the author of this paper while working at a local company.

Because of a growing demand for this company's product, they had the task of increasing their employment from approximately 3,000 to 9,000 employees in a minimal amount of time. Each new recruit must receive a physical, and each employee of the company must receive an annual physical.

This plant supports its own hospital facility which is responsible for giving both annual and new recruit physicals. Considering the increased demands on the hospital with respect to the number of physicals required, it was requested that some tool be developed with which to analyze the entire medical facility's utilization.

Advantage can be taken of the fact that a hospital is a system with the same basic characteristics of any system. By allowing the system to make its own operating decisions, the hospital may be affected as any other system in that some decisions may improve while other decisions may be detrimental to the over-all system.

No system is perfect; therefore, changes must be made in a hospital to improve such facets as personnel and facility utilization, staffing levels, and material inventories. Being a complex system, a most desirable method of analyzing certain aspects of a hospital is by Monte Carol simulation. This approach will allow a hospital to function normally while proposed changes are scrutinized through the use of a simulation model.

2. COLLECTION OF DATA

For the hospital system under consideration, the type of information needed is the amount of time required to process a physical examination. The best estimate of this time parameter is obtained by collecting data for each stage of the physical processing system. There are nine such stages in this system consisting of (1) receptionist, (2) rest room, (3) personal history, (4) sight screener, (5) x-ray, (6) laboratory, (7) electrocardiogram, (8) doctor, and (9) immunizations.

There were several important factors associated with each patient that must be reflected in the data; (1) the sex of the patient receiving a physical examination, (2) whether the patient was a new recruit or an annual employee, and (3) the amount of time every patient spent in each of the nine facilities. This type of data was collected over a three month period, and from this data, service time was calculated as the difference in the time a patient enters a facility and the time he leaves the facility.

A requirement for resolving the collected information is to fit a distribution function to the service time data. The data were analyzed and histograms were plotted for each facility required in a physical examination. The abscissa for each histogram reflects the service time, covering the range from 1 to 26 minutes, for a particular facility while the ordinate reflects the number of patients, ranging from 0 to 76 persons, that required that amount of time.

As was seen from the histograms plotted, several different distribution patterns resulted. Because of this situation, a general statistical distribution is needed which can fit a number of service time patterns. Such a distribution was introduced by Weibull [9] in 1951 and is appropriately called the Weibull distribution. This distribution was chosen because of its flexibility in that it represents the non-negative sector of any distribution; therefore, the parameters of only one type of distribution must be estimated in order to describe the service time patterns of each work station. This was not an attempt to force the data to fit a particular distribution, but since the Weibull distribution is so flexible, it provides an excellent fit for any pattern of data.

The Weibull distribution is a three parameter distribution which has the following cumulative distribution function.

$$F(x) = 1 - \exp\left(-\left(\frac{x-\gamma}{\eta}\right)^{\beta}\right) \qquad \text{(eq. 1)}$$

for $x \geq \gamma,\ \beta \geq 0,\ \eta \geq 0$

and

$F(x) = 0$

for $x \leq \gamma$

where $F(x) = P[X \leq x]$

and

γ is the location parameter

η is the scale parameter

β is the shape parameter.

The capability of the Weibull distribution to fit many probability distribution patterns may be attributed to its three parameters, which are the Shape (β), the Scale (η), and the Location (γ). Because the function of β is to determine the general shape of the curve, it is the most important parameter. Ranging from 0 to ∞, the shape parameter allows the distribution to display certain characteristics. At $\beta = 0$ the distribution curve is at its most concave inward shape, and for $\beta = 1$, the curve is the negative exponential distribution. For β greater than 1, the distribution begins to assume an increasing convex shape until at $\beta = 3.5$ the curve is that of a normal distribution.

As their names would indicate, the location and scale parameters determine the position and spread of the Weibull distribution. The location parameter (γ) is an indication of the first point where a non-zero probability of occurrence exists. Its value is determined by the difference in time zero and the time of the first occurrence of patients taking that amount of time in the facility. If the least amount of time that patients spend in a certain facility is two minutes, then γ would be two for that facility. Incrementing the location parameter will only shift the curve; therefore, its shape is undisturbed. The location parameter performs a function analagous to the mean (μ) of the normal distribution; by the same comparison, the scale parameter is analogous to the standard deviation (σ) of the normal distribution. Taking on any value greater than zero, the scale parameter (η) merely contracts or expands the curve of the distribution.

Estimation of the Weibull distribution parameters is performed by the method of Maximum Likelihood, and is described by Hoel [6]. The steps in the derivation of the parameter functions are presented in Section 2.1.

The estimation procedure lends itself well to computer programming; therefore, a general FORTRAN computer program for estimating Weibull distribution parameters was developed. Another function of this program is to provide a cumulative probability function for each of the hospital's work stations. The cumulative probability function is obtained by entering the estimated parameters into the Weibull Cumulative Density Function.

2.1 Estimating the Weibull Distribution Parameters

By taking the first derivative of the Cumulative Distribution Function, the Weibull Probability Density Function results:

$$f(x) = \left(\frac{\beta}{\eta}\right)\left(\frac{x-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} \qquad \text{(eq. 2)}$$

Let $\alpha = \eta^{\beta}$ and x_i be the value of each observation in a random sample of size n. The probability density function then becomes:

$$f(x) = \frac{(x_i-\gamma)^{\beta-1}}{\alpha} e^{-\frac{(x_i-\gamma)^{\beta}}{\alpha}}$$

Thus the likelihood function is given by:

$$L(x_1,x_2,\ldots,x_n;\beta,\alpha,\gamma) = \prod_{i=1}^{n} \frac{(x_i-\gamma)^{\beta-1}}{\alpha} e^{-\frac{(x_i-\gamma)^{\beta}}{\alpha}}$$

and

$$\ln L = n(\ln\beta - \ln\alpha) + (\beta-1)\sum_{i=1}^{n}\ln(x_i-\gamma) - \frac{1}{\alpha}\sum_{i=1}^{n}(x_i-\gamma)^{\beta}$$

differentiating with respect to α:

$$\frac{\partial \ln L}{\partial \alpha} = -\frac{n}{\alpha} + \frac{1}{\alpha^2}\sum_{i=1}^{n}(x_i-\gamma)^{\beta}$$

setting $\partial \ln L/\partial\alpha$ equal to 0 and rearranging:

$$\alpha = \frac{\sum_{i=1}^{n}(x_i-\gamma)^{\beta}}{n} \qquad \text{(eq. 4)}$$

Taking $\partial \ln L/\partial\beta$ setting it equal to 0, and substituting equation 4 yields:

$$\frac{n}{\beta} + \sum_{i=1}^{n}\ln(x_i-\gamma) - \frac{n\sum_{i=1}^{n}(x_i-\gamma)^{\beta}\ln(x_i-\gamma)}{\sum_{i=1}^{n}(x_i-\gamma)^{\beta}} = 0$$

By rearranging, finding a common denominator, and reciprocating, the following open form exists:

$$\beta = \frac{n\sum_{i=1}^{n}(x_i-\gamma)^{\beta}}{n\sum_{i=1}^{n}(x_i-\gamma)^{\beta}\ln(x_i-\gamma) - \sum_{i=1}^{n}\ln(x_i-\gamma)\sum_{i=1}^{n}(x_i-\gamma)^{\beta}}$$

After an estimate of β is obtained through an iterative process, then η can be estimated by rearranging equation 3 to yield

$$\hat{\eta} = (\alpha)^{1/\beta}$$

The location parameter need not be estimated because it represents the first point where a non-zero probability of occurrence exists.

3. SIMULATION MODEL DEVELOPMENT

A question must be answered before actual model development can commence; that is, "What is the sequencing of work stations for a physical examination?" In order to define the sequence differences in the present system and subsequent trial systems, network diagrams are provided in Figures 1-5. From studying these diagrams it may be noted that the sequencing pattern for new recruits and annual employees differs. The purpose for these differences is to most efficiently utilize the hospital's facilities.

In this model, the terms patients, arrivals, and transactions will be used synonymously. Generating transactions or arrivals is the first step in designing the model. In this study, the arrival distribution pattern was controlled by assuming a normal distribution; therefore, this distribution was used in generating arrivals for the model.

Now that both sequencing patterns and service time distributions have been established, the simulation model can be developed. Several computer languages are available which could be used to describe this type of system; however, General Purpose Systems Simulator III (GPSS) was chosen because of its efficiency, compactness, ability to gather statistics automatically, and the author's familiarity with it.

Arrivals, in GPSS, are originated by the means of a GENERATE statement or block, and each arrival must contain one of the following attributes:

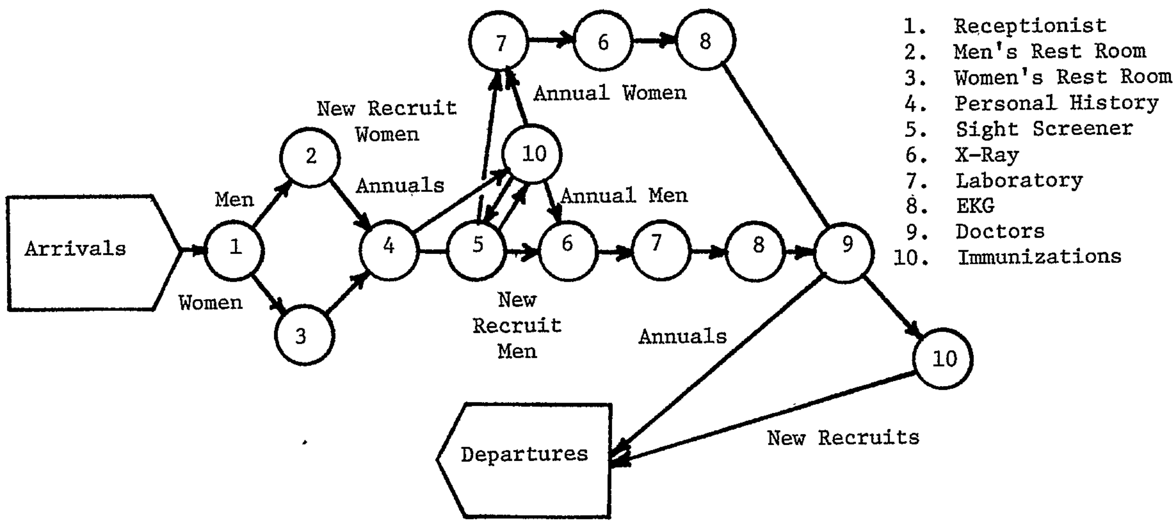


Figure 1. Simulation I
Original Sequence

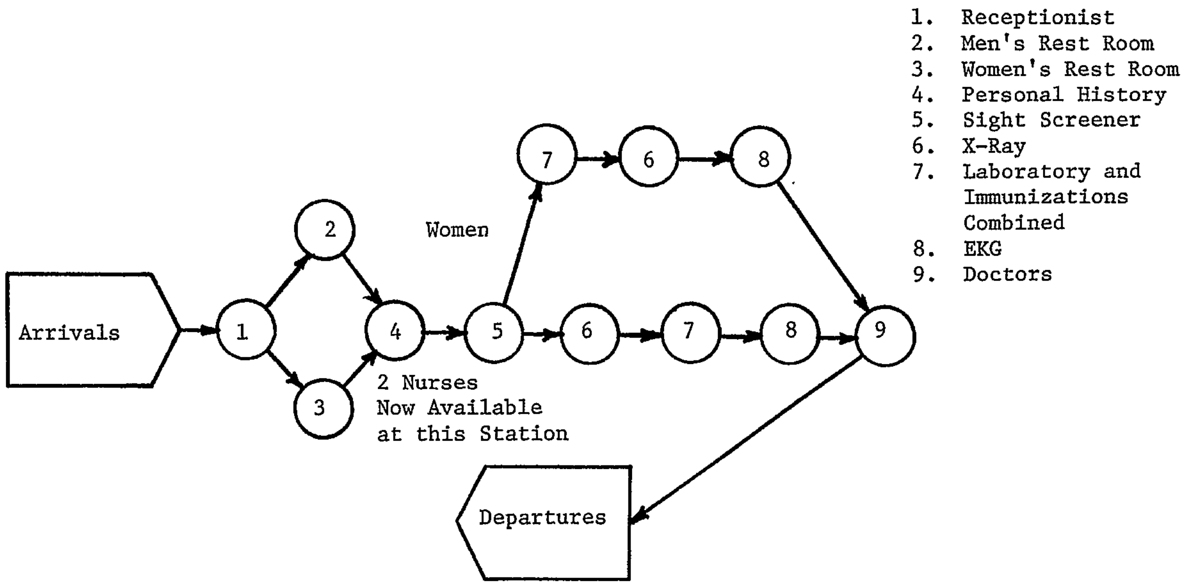


Figure 2. Simulation II

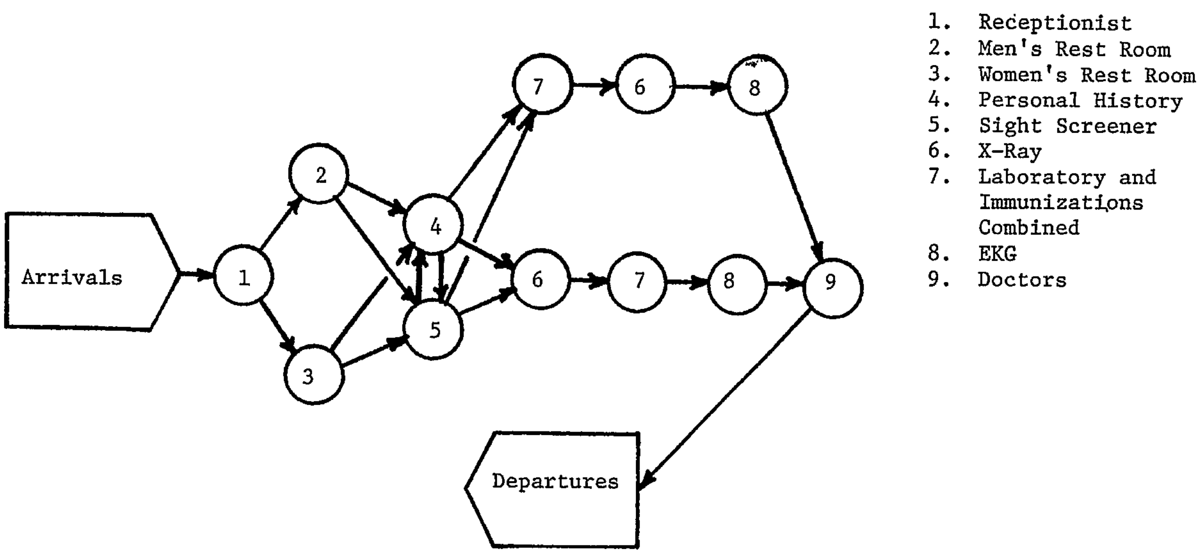
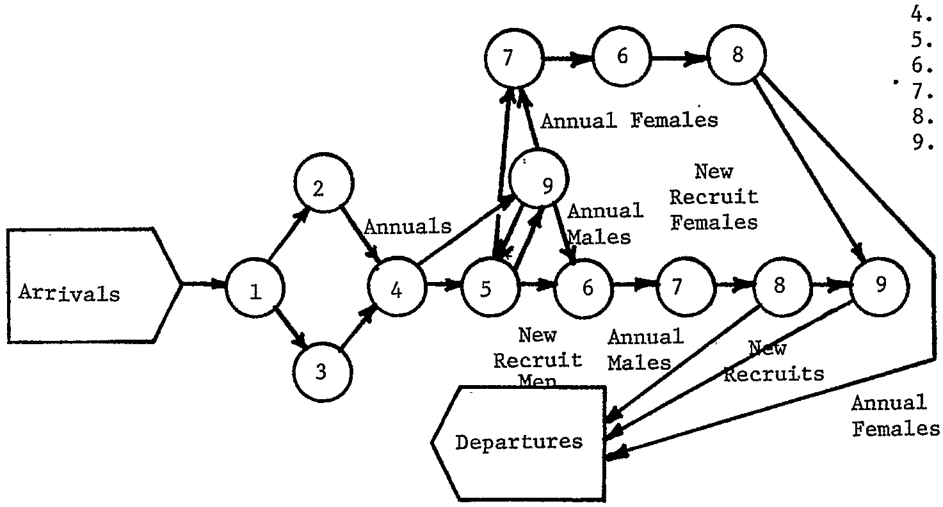
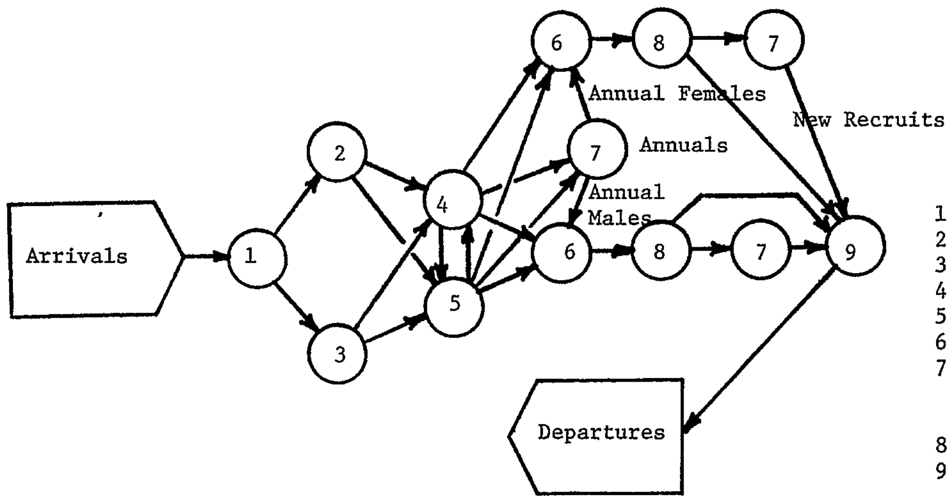


Figure 3. Simulation III



1. Receptionist
2. Men's Rest Room
3. Women's Rest Room
4. Personal History
5. Sight Screener
6. X-Ray
7. Laboratory
8. EKG
9. Doctors and Immunizations Combined

Figure 4. Simulation IV



1. Receptionist
2. Men's Rest Room
3. Women's Rest Room
4. Personal History
5. Sight Screener
6. X-Ray
7. Laboratory and Immunizations Combined
8. EKG
9. Doctors

Figure 5. Simulation V

(1) male, new recruit, (2) female, new recruit, (3) male, annual employee, and (4) female, annual employee. The data collected revealed that 85.6% of the arrivals were male and 73% of the arrivals were annual employees. Through the use of parameters, one of the above attributes was established for every generated transaction. This attribute remains with each transaction throughout his physical examination by the means of an ASSIGN block. After the parameters have been assigned, the patient flows through an examination sequence that is prescribed by his attributes.

Because a service facility may be occupied when a transaction arrives, a waiting line must be provided for by the use of QUEUE block. The transaction must wait in the QUEUE block until the work station is no longer occupied to capacity. At that time the transaction is able to depart the queue and seize the facility.

The amount of time a transaction spends in a facility depends on the ADVANCE block which is modified by a function. This amount of time is called service time and follows the Weibull distribution patterns developed previously for each work station.

If a service facility has a capacity of more than one patient, it is considered a storage by GPSS with the number of transactions it can service at one time as its capacity. The GPSS blocks that accompany a storage are QUEUE, GATE, DEPART, ENTER, ADVANCE, and LEAVE.

Another GPSS block must be provided that allows transactions to branch to service facilities other than the one immediately succeeding. There are two such blocks used in this model, the TEST block and the TRANSFER block. In this model, the test block examines the attribute parameters of each transaction and transfers the transactions accordingly; the transfer block transfers every transaction entering that block according to the conditions specified within the block.

Statistics are automatically kept by the GPSS program and may be displayed in the output by the use of a TABULATE block accompanied by its table definitions. If the statistics are to be meaningful, a frame of reference must be established throughout the simulation model. In this particular model, all time units were expressed in minutes.

Once the model has been properly constructed by the use of GPSS blocks, the hospital's physical examination system may be studied by specifying the amount of time (chosen in units of the frame of reference) it is to be simulated. For this study, the simulation period is 40 days which was performed on an IBM Model 50 system in less than 5 minutes.

4. RESULTS OF CURRENT PROCEDURES

After simulating the hospital for a period of 40 days, it was determined that the utilization in several of the hospital's facilities was too great. Displaying a high utilization is not a desirable characteristic of any work station because in the system being studied, this type of situation denotes a delay in the flow of patients. Simulation of the original sequence indicated that the highly utilized facilities were the personal history, the sight screener, and the electrocardiogram (EKG) facilities. Therefore, the point of attack for systems study were these low service rate work stations. Although many approaches could have been taken toward studying this system, it is the purpose of the author to provide a model with which to manipulate the system and to analyze the effects of the manipulation without disturbing the true system.

One technique of increasing the service rate at each facility is to increase the machinery and hospital personnel at that station. Consequently, the utilization would decrease in these stations, assuming a constant arrival rate, and service time would be evenly balanced throughout the hospital. However, equipment and personnel are both expensive commodities; therefore, it was desired that only the present system components enter the simulation model.

In order to increase the service rate of a highly utilized facility, it was thought that an over-abundance of either equipment or personnel in one of the lower utilized facilities might be shifted to a more highly utilized work station. By taking this approach, system flow becomes more uniform and a balancing effect occurs on the entire system. The lower utilized facilities were (1) receptionist, (2) men's rest room, (3) women's rest room, (4) x-ray, (5) laboratory, (6) electrocardiogram, (7) doctors, and (8) immunizations. Each of these work stations had only one piece of equipment and one staff member with which to perform its respective function. An exception was the doctor's facility from which four doctors practiced.

As a result of the staffing and equipment levels, it was thought that a combining of service stations would give rise to a more efficient physical examination system. The criteria for measuring efficiency was the following:
(1) Balancing the utilization of the work stations so that overall system utilization increases or remains fixed,
(2) Decreasing the average waiting time per transaction for the total system,
(3) Decreasing the average maximum queue length for the overall system,
(4) Decreasing the average service time per transaction for the total system.

5. RESULTS OF SIMULATION

Changes in the present system were scrutinized with the aid of a simulation model. Four unique system changes were investigated, and Table I is a summary of the results obtained through simulation.

It can be seen from Table I and Figures 6-9 that Simulation III yields the most efficient result considering the criteria for measuring efficiency. This proposal not only allowed a nurse to be more fully utilized, but indicated that at least one of the four doctors could be released from the work station to perform other duties within the hospital. This proposal also allows the required number of people to be recruited for the plant. As a result, the simulation model provides a tool by which the utilization of the entire medical facility can be analyzed and measured without destroying the present system.

Although the criteria made in formulating the proposals of this problem was that new equipment and personnel would not be considered, the simulation model is not limited by this criteria. With a few minor alterations in the GPSS model, other facility capabilities may be introduced into the system by changing the equipment and personnel at each work station. It is because of its inherent flexibility that this model may be adapted to any physical examination processing system. Merely by changing work station capabilities and the system flow sequence, this simulation model may be used to analyze and reflect the results of any proposed changes to similar systems. A model of this type may also be incorporated into a much larger model that simulates other hospital systems; thus, the simulation of an entire hospital would not be infeasible.

In conclusion, the simulation model is a very useful tool in analyzing large complex systems. It is most important that the input data and the simulation model represent the true system; therefore, care must be taken in data collection and development of the model. If this essential care is taken, then a technique will be provided with which large complex systems may be studied without disturbing the present system.

Table I. Summary of Simulation Results

System Proposal	Average Utilization	Average Service Time/Transaction (min.)	Average Maximum Queue Length	Average Waiting Time/Transaction (min.)
Simulation I	0.427	55.117	3.000	17.491
Simulation II	0.433	55.088	3.333	18.637
Simulation III	0.433	55.145	2.778	16.092
Simulation IV	0.412	55.420	3.667	15.932
Simulation V	0.430	54.965	3.111	18.323

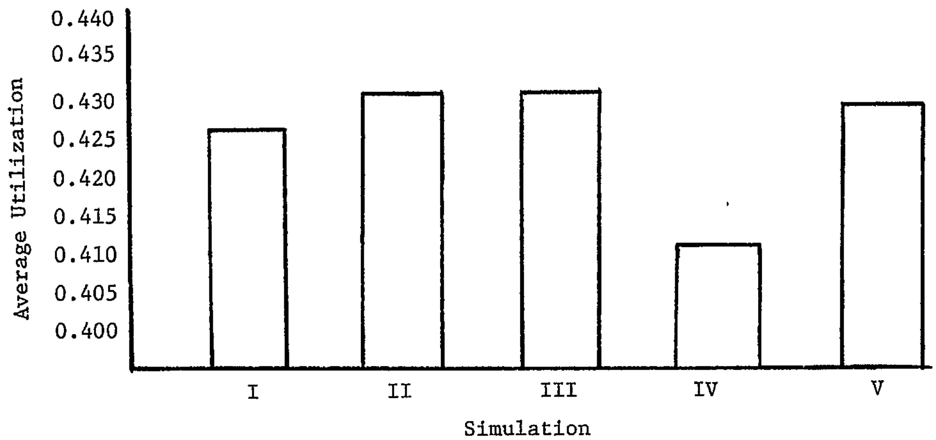


Figure 6. Average Utilization

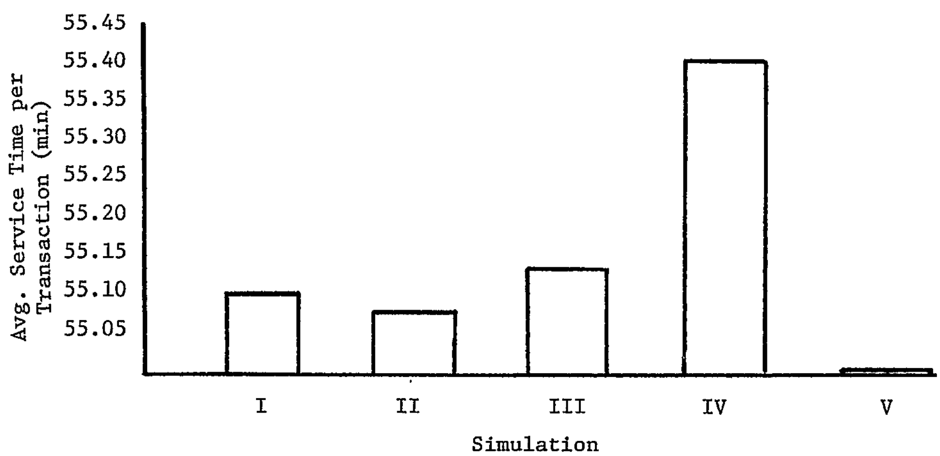


Figure 7. Average Service Time/Transaction

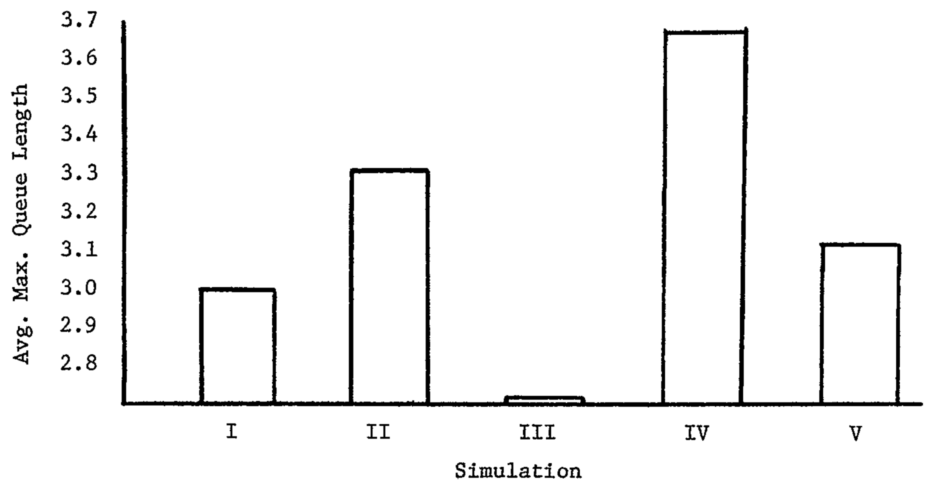


Figure 8. Average Maximum Queue Length

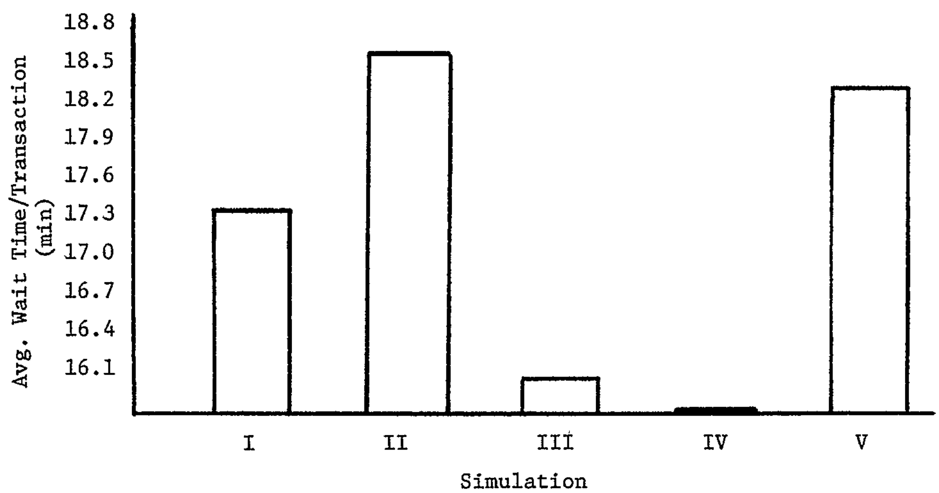


Figure 9. Average Waiting Time/Transaction

6. REFERENCES

[1] Bartscht, Karl G. "Hospital Staffing Methodologies - Concepts, Development, and Uses," Hospital Industrial Engineering, Proceedings 1967, pp. 119-128.

[2] Cohen, Clifford A. "Maximum Likelihood Estimation in the Weibull Distribution Based on Complete and on Censored Samples," Technometrics, Volume 19, November 2, November 1962, pp. 17-26.

[3] __________ and L. W. Falls. "Estimation of Parameters in Compound Weibull Distribution," Nasa Technical Note, TND-4033, Washington, D. C., July 1967.

[4] Colley, John L., Jerome B. Hallan, and Arnold H. Packer. "A Simulation Model of a Saturated Medical System," Hospital Industrial Engineering, Proceedings 1967, pp. 138-155.

[5] Guarino, David J. A Mathematical Model for Scheduling Preventive Maintenance, M.S. Thesis, Hercules Incorporated, Radford, Virginia, August, 1967.

[6] Hoel, Paul G. Introduction to Mathematical Statistics. John Wiley and Sons, New York, New York, Third Edition, 1965.

[7] International Business Machines Corporation, General Purpose Systems Simulator III, Technical Publications Department, White Plains, New York, 1965.

[8] Shycon, Harvey N. and Richard B. Maffei. "Simulation - Tool for Better Distribution," Harvard Business Review, November-December 1960, pp. 65-75.

[9] Weibull, W. "A Statistical Distribution Function of Wide Applicability," Journal of Applied Mechanics, Volume 7, November 3, September 1951.

7. BIOGRAPHY

The author was born on August 29, 1944 in Sampson, New York. In September, 1962 he enrolled at the Virginia Polytechnic Institute. Requirements for the Bachelor of Science degree in Industrial Engineering were completed in June, 1967.

During the summer of 1966, the author worked as an engineering assistant at Hercules Incorporated in Radford, Virginia. After June, 1967, he began full time employment with this company as an Industrial Engineer, and at the same time, began graduate studies in the field of Industrial Engineering. In October, 1968 the author joined the Computing Center staff at Virginia Polytechnic Institute in the capacity of a Systems Analyst. After receiving his Master of Science degree in Industrial Engineering in June, 1969, he became Manager of Information Services where he is presently serving.

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