

THE CAPITAL ASSET PRICING SIMULATOR

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Abstract

The characteristics of the New York Stock Exchange closely resemble those necessary for an efficient market: large numbers of participants, rapid dissemination of information, low transaction costs and easy accessibility. These equilibrating conditions suggest that investors can accept existing security prices as usefully correct. Thus, investors should employ a portfolio strategy which controls the risk of the portfolio, eliminates unnecessary risks, and minimizes operating and transaction costs. The Capital Asset Pricing Simulator demonstrates how portfolios consistent with the above objectives would have performed over time and identifies possible sources of extra returns. The Simulator has supported the Capital Market Theory, assisted in the development of a realistic management strategy, and provided demonstrative materials for marketing and training. Furthermore, it has led to the offering of new investment management services.

I. The Problem

Capital Market Theory concentrates on the mechanism by which prices of securities are set in an "Efficient Market" characterized by large numbers of participants, rapid dissemination of information and low transaction costs. In an efficient market it is difficult for any one participant to consistently outperform other participants, all of whom are striving to attain superior performance. This competition forces prices to adjust rapidly to the point where they accurately reflect investors' expectations. The theory proceeds under the assumption that market prices contain most, if not all, relevant information, and can serve as a base for portfolio composition strategies.

But stock prices are notably volatile, and the "correct price" may vary considerably from one day to the next. An incentive is required to induce investors to draw their funds out of savings accounts and subject their wealth to this potential fluctuation. A reward must be offered to compensate for bearing this risk; and, the higher the risk, the higher must be the expected reward.

Not all risks carry an expected compensation, however. Security exchanges provide investors with an opportunity to minimize through diversification certain of these risks, specifically those risks unique to an individual company or industry. Thus, this type of risk bears little, if any, expected compensation. The remaining risk which does bear expected compensation is that risk due to common factors affecting all securities, which cannot be diversified away. The relevant risk of an individual company, referred to as beta, is the extent to which it is affected by these common factors. The return on a portfolio is vastly more sensitive to the extent to which these common factors are present than it is to any other single factor.

Using measures of this risk, portfolios can be constructed which systematically undertake a controlled amount of risk. A controlled low risk portfolio would, for example, appeal to more conservative investors, whereas a higher risk portfolio could be constructed to satisfy more aggressive clients. All of these portfolios would feature accurate control of risk exposure to maximize systematic return in compensation for bearing risk.

The objective of the Capital Asset Pricing Simulator was to develop a practical method of constructing portfolios utilizing the principles of Capital Market Theory. The Simulator is a computer program designed to simulate portfolio strategies based on historical return data of individual stocks and their risk/return relationships and inter-relationships as hypothesized under Capital Market Theory. The conceptual methods of constructing risk/managed portfolios was formulated by consultants to Wells Fargo Bank.¹ The Capital Asset Pricing Simulator extended this basic research by:

1. Allowing more real-world assumptions - e.g., trading in round lots, brokerage commissions, leverage constraints, and inclusion of management fees and cash outflow requirements;
2. Operating as if stocks were actually bought and sold - e.g., dividends are re-invested by purchasing more stocks; and,
3. Allowing more flexibility in choosing a portfolio strategy. Areas of flexibility include: time interval between portfolio and individual stock reviews, handling of short sales, number of stocks held in the portfolio, adjustment of portfolio risk with leverage, and value of individual risk deciles.

II. The Application of Capital Asset Pricing Theory

The primary advantage claimed for Capital Asset Pricing Theory based strategies is control of the risk assumed in the portfolio. Risk control is advantageous as it permits the identification and removal of those risks which bear no compensation, and also allows more sophisticated investment strategies to be employed without losing control of the risk of the portfolio. Risk control allows the client to meaningfully express his investment objectives to the portfolio manager.

The risk of a portfolio is controlled relative to the risk of the market. Since the total risk of the market tends to change rather slowly over time, investors can use this standard to evaluate the proper degree of risks in their portfolios. Risk relative to the market is measured as the total relative risk which is defined as the standard deviation of the portfolio returns divided by the standard deviation of the market index. In the case of a perfectly diversified portfolio, this would be equivalent to the market sensitivity. The market index used as the standard is an index constructed of equal investment of all stocks comprising the New York Stock Exchange universe. (This stock index is approximately 15% more risky than the commonly used Standard & Poor's 500 Stock Index.)

The Capital Asset Pricing Model indicates that investors are compensated only for bearing those risks which cannot be diversified away (beta). Thus, investors are compensated not for the total risk of an individual security, but only for that portion which still remains after the security is held in a highly diversified portfolio. The degree of diversification of a portfolio is measured by its correlation coefficient. A portfolio whose rate of return moved in lock step with the market's rate of return would have a correlation coefficient of 1.0. Recent studies have indicated that widely diversified portfolios can attain correlation coefficients between .90 and .98. Very low (or very high) risk portfolios tend to have lower correlation coefficients than those portfolios of approximately market risk due to the industry concentration within certain risk ranges. The low risk groups, for example, tend to contain an inordinate number of utilities and international oils, and virtually exclude high risk, low capitalization industries such as electronics, aerospace and airlines.

If it were not for the finding that low risk portfolios generated larger than expected rates of return, this lowering of the correlation coefficient would imply that the rate of return should drop proportionate to the drop in correlation for portfolios with equal total relative risk. However, given the existence of the extra returns (explained below) on low risk portfolios, the correlation coefficient is not sufficient to determine the relative efficiency of a portfolio.

It does not appear that it is possible to increase the coefficient of correlation to a value higher than approximately .985, even for portfolios of securities selected from the total risk spectrum of the market. When we compare a portfolio to an index which maintains constant equal proportions in every security, we make comparisons to a standard which is unattainable as a portfolio without incurring large and useless transaction costs. As the value of securities held in our portfolio fluctuates, the proportions of the individual securities will change relative to each other and relative to their proportion in the market index used for purposes of comparison. Thus, a coefficient of correlation of 1.0 is not practically attainable and should not be viewed as a standard of perfect diversification.

The operating strategy is based upon the assumption that the prices of securities generated in the market can be taken to be usefully correct prices. It is therefore unnecessary to undertake in-depth analysis of the individual issues and to continually buy and sell stocks to "take advantage of special situations" as most current portfolio managers try to do. As mentioned above, even though the explicit costs of transactions are low for large trades (or nil if done on the fourth market), the implicit costs are quite high. Thus, the cost of operations for Capital Asset Pricing Theory based portfolios should be lower than available under current management practice due to low demands on the time of the portfolio manager, as well as low transaction costs generated by low turnover rates.

The studies performed by the consultants showed that a combination of low risk stocks and leverage produced results superior to portfolios composed of unlevered high risk stocks. The method proposed for utilizing this finding was to maintain a constant ratio of borrowed debt to paid-in equity capital. When the value of the portfolio rose, more debt would be acquired and invested in the same low risk stocks. When the market declined, stocks would be sold and the proceeds used to pay off some of the debt.

In summary, portfolios based on Capital Asset Pricing Theory claim the following advantages: control of total risk, elimination of those risks which do not bear expected compensation, low turnover, and extra returns generated by the use of leverage. Each of these variables is measured quantitatively. This allows the client, or the simulator user, to state his portfolio objectives (level of total risk, minimum level of diversification, and maximum level of turnover) and, therefore, gauge his expected portfolio performance relative to the return of the market. Later he can evaluate the portfolio manager accurately, in the context of the pre-defined portfolio objectives. Importantly, the objectives are stated in terms which the portfolio manager can actively control; no undeliverable promises of rates of return are made.

III. The Model

Using monthly New York Stock Exchange data from January 1931 through June of 1970, the computer program constructs portfolios and controls them in the manner specified by the input control parameters. An extensive performance analysis of the portfolios is performed during the simulation to give information on past stock/portfolio-risk/return relationships.

The unifying dimension of simulator design and usage is risk control: how can a portfolio manager control the risk of a portfolio, and how is the ultimate performance of that portfolio dependent upon the methods he uses for controlling risk? The risk control options available to a portfolio manager can be divided into three major categories:

1. Division of the universe into risk deciles. A quantitative measure of the risk of each individual security is calculated as of each month in the time span covered by the simulator. The coetaneous risk estimates are sorted so that individual securities are ranked from the most risky to the least risky. This sorted array is arbitrarily divided into risk deciles; securities within a risk decile are assumed to be equally risky, and securities excluded from the risk decile are assumed to have a different risk. Portfolio composition (and risk) are controlled by limiting (maximum and minimum) the proportion of portfolio value which is held in each decile.

2. Diversification. Diversification is controlled primarily by constraints on the number of securities held in the portfolio, and specifying maximum and minimum portions for an individual security.
3. Leverage. Leverage is adjusted to maintain a constant debt to equity ratio as specified by the control parameters. The amount of debt outstanding is adjusted to compensate for changes in equity.

Each simulation run produces a massive output for analysis including parameter specifications, traces of actions taken, status reports, and periodic performance evaluations. (See the appendix for a description and example of each report.) Analysis of this data and the data from comparable runs allows the user of the simulator to study and define alternative portfolio decision rules.

IV. Real-World Validity

Real-world operability and expected performance of Capital Market Theory based on portfolio strategies must be inferred from the results of portfolios simulated over historic time periods. Therefore, it is important that the simulation model accurately reflects the relevant aspects of the real-world. Criteria necessary for valid representation include:

1. All data employed by the simulator were actually available at the time of the decisions.
2. Actual decisions would have yielded the same results as the simulated decisions.
3. The simulator decision process accurately reflects the proposed strategy and does not invoke knowledge unavailable at the time.

The remainder of this section discusses the data, operating assumptions and decision rules utilized by the simulator in the context of the three criteria above. They will be justified with comparisons to the real-world and actual operating strategy where appropriate.

Data

The data base used by the simulator consists of monthly data from January 1931 to June 1970 for all NYSE common stocks. The decision point for purchases and sales occurs at the beginning of each month. At that time only the current prices and risk estimates (volatility of historical returns relative to market index returns) are utilized. The securities and portfolio are valued at the end of each month using the closing prices. Dividends are assumed to be received in the same month that the stock goes ex-dividend. This implies that the dividends are available for investment on average three weeks earlier than they would in a real operating environment. The effect is an upward bias of the simulator returns by 0.03%/year.

Operating Assumptions

The portfolio simulator assumes that an unlimited amount of any security can be bought or sold at the closing price on the last day of the month at commission rates equal to 1969 rates (including the New York State transfer tax). Under traditional portfolio practice (purchase of a specific security) this would be a very naive assumption because it ignores the implicit costs of the transaction, the movement in stock price caused by this transaction, which are estimated at 1% - 2%. However, since Capital Asset Pricing Theory based portfolios are relatively indifferent to a wide range of issues and in many instances the speed of execution, we

believe that it will be possible to avoid the implicit transaction costs even if the explicit costs cannot be eliminated.

The simulator adjusts the debt of a levered portfolio to the specified level at the start of each month. The interest is pre-paid at the commercial paper rate plus 1% per year.

Although any particular contribution/disbursement schedule can be simulated, in general the portfolio simulator assumes no cash flows. This would be consistent with the management of a closed-end fund but not necessarily with either a pension fund or an open-end fund. The effect of cash flows cannot be evaluated without knowing the flow schedule, portfolio risk, amount of debt and market trend. In practice, the portfolio manager will schedule expected cash receipts and disbursements, thus minimizing commissions costs of temporary investments. In addition, cash flows allow the portfolio manager to adjust the risk without generating additional commission costs.

Decision Process

When initializing the portfolio there are only two critical operations to be performed by the simulator user or in the real-world, the portfolio manager:

1. Given the objective risk level and the amount of leverage available, determine the risk of the underlying portfolio.
2. Determine which risk deciles of the security universe in which to invest in order to meet that specified risk level.

The first operation is simple: The risk of the underlying portfolio should be equal to the objective risk level divided by one plus the debt/equity ratio. The second operation is more difficult and also a potential source of bias since it is possible to invoke risk information not available at the simulated time. Fortunately, the risk of the underlying portfolio can be initialized by using the calculated risk of market segments from previous time periods.

To maintain the risk of a levered portfolio, the debt/equity ratio must be closely controlled since small changes in portfolio value (5% - 10% depending on the amount of leverage) can have considerable effect on the risk of the portfolio. Therefore, an actual portfolio would be evaluated as frequently as daily to insure that the debt/equity ratio was within reasonable limits. Daily control may increase the number of transactions somewhat (there would still be many days without any trades); however, any additional trading costs should more than offset by improved performance due to the closer control.

The simulator decision process selects issues to buy or sell in a manner which maintains a constant profile among risk deciles. That is, the simulator selects securities to buy from that risk decile which is most under-represented in the portfolio and sells from the risk deciles which are over-represented.

Constraints on individual securities are employed by the simulator to insure accurate risk estimates, to reduce costs, and to improve diversification. At the beginning of each month all issues for which data is not available because of delisting or merger are sold at the price as of the beginning of the previous month. The proceeds are assumed to be held in cash for that month. Commissions are paid only on the cash reinvested because it is assumed that with actual mergers, the new securities

would be held. Security positions valued at less than .1% of the portfolio are sold to simulate the real-world consideration of reducing custodial costs. Holdings in securities valued at more than a maximum limit set by the simulator user are reduced to improve diversification.

Are the simulations a valid track record for an operating portfolio strategy? The security universe and data to be employed in the actual portfolio management are comparable to those used by the simulator. The investment strategy is a refinement of the simulator decision process in that it makes use of additional information and more frequent observations to improve portfolio risk control. Furthermore, we believe that the operating assumptions are a reasonable approximation of the real-world. Therefore, in our opinion the simulations do constitute valid experience. However, this statement could be revised as more information from an actual operating environment is collected.

V. Technical Aspects

Program Design

The portfolio simulator was designed to accommodate generality (user options) in the specification of input parameters, as well as flexibility (program changes) in the modification of the decision process and the strategy assumptions. Unlike many simulators which attempt to mirror existing systems, the portfolio simulator was to test a model which was being developed concurrently. The details of the model had not been specified and would depend in part on the results from the simulator. Furthermore, the usage of the simulator was not well defined at the start. It was to be used for verification of the initial research, development of a new portfolio strategy, marketing, and research on other models. Each of these has different requirements on input, data handling, decision rules, and output. In addition, the real world investment restrictions and trading assumptions were not known with certainty. Many of the ideas proposed for the operating strategy are unfamiliar to the traditional investment process. Having had to make numerous changes in the simulator, the value of flexibility has been readily apparent. The importance of designing a simulator so that changes could be made easily and also tested cannot be emphasized enough.

It appears now that it might have been more efficient to build the simulator for specific runs and modify it for succeeding runs. Programming the simulator for generality was very time consuming, made testing more difficult, and made running less efficient. Many changes and additions have been made, and furthermore, many options (e.g., short sales) have not been used and are not expected to be used in the immediate future. An analysis of why we were designing the programs for flexibility would have made us realize the inconsistency on designing for generality as well.

Program Language & Computer

Fortran was selected for the programming language. Simulation languages did not appear advantageous because the model was too complex, not adaptable to a simulation language, and required output flexibility. Fortran appeared more efficient than general languages such as Cobol or PL1 for doing the mass calculations and array manipulation and more efficient than assembler for programming and making changes. Furthermore, the people who designed and programmed the system were trained in Fortran. The main difficulty with Fortran is the data input inefficiency.

The program was developed on the time-shared IBM 360/67 computer of Interactive Data Corporation in Waltham,

Massachusetts. After development it was converted to remote batch usage, and later to in-house 360/65 facilities.

Data Base Constructions

Constructing the data base was as difficult as programming the model. Starting with monthly price and return data for all companies on the NYSE from January 1926 through June 1970, a data base which included risk estimates for each company had to be built which could be handled efficiently by the simulator. The first step consisted of checking for errors, reconciling inconsistencies, and screening out any companies with bad data. A risk estimate (slope coefficient of the regression of the security's returns against the market using only data from a previous period) for each month for each company was then calculated and stored with the monthly prices, returns without dividends, and returns with dividends for the company. The third step was a massive transpose of the data from month within company to company within month. Then the companies were sorted each month on the basis of their risk estimate.

Resources

Constructing the data base and designing, programming, and debugging the simulator were done by a systems analyst and application analyst within one calendar year. The three steps required approximately nineteen man-months, thirty-two hours of CPU time, and 1000 hours of connect time. Almost half of this was spent constructing the data base. A simulation takes about ten minutes of CPU time to run on a 360/67; however, over half of the time is used to read in the data. Using the in-house 360/65 with the data base on disk, the CPU time for a twenty-five year simulation is reduced to approximately four minutes.

VI. Analysis

To demonstrate the advantages of the portfolio strategy and the use of the simulator in managing portfolios, the results of three simulation runs are presented. Each has a total relative risk objective of 1.0, i.e., risk equal to that of the market index; but utilizes different amounts of leverage. In Run #1 a portfolio of the lowest risk stocks was purchased and levered to market risk by borrowing \$.50 for every \$1 of equity; in Run #2, a medium-low risk portfolio was levered to market risk by borrowing \$.25 for every \$1 of equity; and in Run #3 a market risk portfolio was selected using no leverage. All three portfolios contain a large number of issues (average number of issues over 100) and try to maximize diversification by limiting the maximum value in each security (approximately 5% maximum for Run #1 and 2-1/2% maximum for Runs #2 and #3). The diversification and turnover objectives depend on the risk level of the underlying portfolio and the amount of leverage, respectively. For these three portfolios the minimum diversification (measured by the correlation coefficient) would be .92, .96, and .98 and the maximum turnover objectives would be 30%, 25%, and 15%. Of course, these are expected limits and in any run could be exceeded.

Risk Control

Table A summarizes the risk control for the three simulation runs. For each run the first column (beta) is the systematic risk for the five year sub-periods, the second column (r) is the correlation coefficient, and the third column (σ_p / σ_m) is the total relative risk.

If the portfolio returns of strategies #1 and #2 are sufficiently above those of the more highly diversified unlevered portfolio #3 we are willing to accept the

relatively low level of diversification. Note that this does not imply that this portfolio is riskier than the market; over time the levered portfolios have the same variability in returns as will the index,

Turnover

Table B shows the transactions incurred by the three strategies over the twenty-five year simulation period. Dollar turnover is defined as:

$$\frac{(\text{Dollar Purchases} + \text{Dollar Sales})/2}{\text{Average Equity}}$$

Issue turnover (Table C) is similar except the number of transactions or issues is used instead of dollar value. Notice that in order to control the total risk of the portfolio, the turnover increases as more leverage is used. The increased transaction costs of a leveraged strategy must be offset with higher returns.

Extra Returns on Low Risk Stocks

As expected, the returns on the two levered portfolios over the twenty-five year period were both statistically and economically superior to those which would have been predicted by the Capital Asset Pricing Theory. Statistical significance, measured by a t test on the intercept of the regression of portfolio returns on market returns, indicates at the 95% significance level that the extra returns could not have been produced by chance. Economic significance is best indicated by the growth of a \$1 invested in the levered low risk portfolios compared to the growth of a similar investment in the market index. \$1 invested in the levered low risk portfolio in July of 1945 would have compounded to \$33.75 by June of 1970. \$1 invested in the index would have grown to only \$15.97.

Tables D, E, and F below also contain several other measures of the return and/or the efficiency of this portfolio. Monthly excess return refers to the return achieved by the portfolio (or the market) above the risk-free rate, measured in the simulator run as the rate of return on three-month Treasury Bills. Over the twenty-five year period, the excess monthly return on #1 portfolio was one quarter of one percent per month greater than the market index.

When the average excess return is divided by the total risk (the standard deviation) of the portfolio or the market, the resulting number measures the efficiency of the portfolio. These numbers are given in the column headed Excess Returns/Risk. These numbers can be interpreted as follows: the .67 levered portfolio generated .23 units of return for each unit of risk undertaken, whereas the index returned only .18 units of return; the low risk portfolio is clearly preferred to the unlevered market risk portfolio.

During the period 1965 through 1970, the levered low risk portfolios did not yield an extra return. In fact, the returns on the levered low risk portfolios were lower than that on the return of the market, although by a statistically insignificant amount. This result is not due to the recent market performance, but due to the extraordinary performance of high risk stocks from the end of 1965 through the first few months of 1967. Over the last three years, the levered low risk portfolios would have outperformed the market. This example points out that the advantage of leveraging low risk stocks can be negative for any given year. Only over longer periods of time are we able to detect significant differences. (Five years of data are probably necessary before any meaningful performance results can be obtained.) Thus, Capital Asset Pricing Theory based portfolio strategies are inherently long term investment strategies.

VII. Value and Benefits Derived

Since the simulator was designed to serve multifarious objectives, the actual benefits derived from the designing and the usage of the simulator are diverse. Benefits have been derived in the areas of research, development, communications, and marketing.

The major benefit of the development of the simulator probably lies in the construction of a "track record" which, in our opinion, constitutes valid experience and is representative of results which can be expected in the future. Since the simulator is concerned with new portfolio strategies which have never been consciously executed in reality, there is no track record to demonstrate the efficacy of this method in actual portfolios. Therefore, the real-world operability of the expected performance must be inferred from the results of studies which manage simulated portfolios over historic time periods. Because the simulator makes realistic assumptions regarding such elements as commission costs, interest rates, re-investment of dividends, and buy/sell decision rules, the results of a simulator run are credible to the developers, management, and potential clients. The simulator effectively demonstrates the advantages of low transaction costs, control of risk, and superior returns obtainable through portfolio strategies utilizing the concepts of Capital Asset Pricing Theory.

As has been previously stated, the simulator was designed to add more realistic assumptions to theoretic development provided by our consultants. The initial runs of the simulator were used to provide an independent verification of the results obtained by the consultants. Confident then that an opportunity to improve portfolio performance existed as a result of the basic findings, we proceeded to test the model under current real-world assumptions and to develop an operating strategy.

The experience gained in the construction and utilization of this simulator allowed us to refine the operational strategies to maximize the benefits from this style of portfolio management. Operating manuals and operating programs, intended for the direct use of an operating portfolio manager, were developed partially through the experience gained through the simulator. Because the simulator uses monthly data and handles debt in a fairly naive manner, it is not of specific use in helping a portfolio manager to understand his day by day activities. Specific methods of handling taxes, trading strategies, debt maintenance and client relationships are primarily person to person inter-action problems, and thus are not incorporated into the portfolio simulator. However, the simulator does define the stock selection process, and indicates the amount of trading and leverage adjustments required to maintain a portfolio.

The simulator runs, and papers prepared summarizing and contrasting the results of the particular runs, have been extensively utilized as a means of educating users and clients of the merits of the portfolio strategy. In addition to management education, the "learning by doing" experiences construction in using the simulator added invaluable in-depth knowledge to the developers.

The simulator continues to be a valuable tool in the further research and development of refinements and extensions of the basic strategy. The simulator is continually being extended and modified to test these more advanced strategies.

VIII. Conclusion

Simulation models attempt to accurately represent the relevant aspects of the real-world. If this selective representation of reality is incorrect or incomplete, the simulated results may not be obtainable when the concepts are implemented. The simulator, which possesses a high degree of correspondence to reality, can be a variable tool to acquire insight into the complex decision processes. Careful use of simulations can provide more information in a matter of minutes than could be acquired in a whole lifetime of experience.

The utility of the simulator is directly related to the credibility the simulator has to those who are the intended users. Candid discussions of the design, results, and possible shortcomings increase the willingness of the user to accept the results. Open access to the use of the simulator assures him that he is not receiving a biased selection of output. Frequent discussions and well written non-technical topical papers provide assurance that the simulator is a useful and valuable tool in the management of change.

Appendix: Output Reports

In order to give a reader a better grasp of the simulator, we have inserted a sample of each of the six major output formats, along with a brief description of each. The sample reports were taken from a run which simulated the investment performance of a 25% levered to market risk portfolio from July 1945 to June 1970. The reports are presented in order of importance to the user.

Performance Regression (Report A)

This report summarizes the performance of the portfolio as compared (via regression) to the market. In general, this report shows the superiority of a levered low risk portfolio to an unlevered portfolio of comparable risk.

- Comparison of the mean rates of return (or alpha, the intercept of the regression of portfolio returns against market returns) shows the difference in compound monthly rates of return.
- Beta, the slope of the regression of portfolio returns on market returns, or a comparison of the standard deviations of returns, shows that the risk of the portfolio is approximately that of the market.
- The correlation coefficient shows the portfolio diversification.

Monthly Returns Report (B)

Compares the returns of the portfolio to the returns of the market on a month by month basis, as well as for each year and since the beginning of the simulation.

This report also presents a current balance sheet for the account. Note that leverage appears as a liability in approximately the 1:4 proportion to equity specified as a control parameter.

Transactions Summary Report (C)

Summarizes the number of transactions, the dollar value of transactions, commissions paid, etc. Turnover averages are discussed in Section VI of this paper.

Portfolio Holdings Report (D)

Lists the securities held as of a particular point in

time. Also printed is the dollar value of the holding as well as the current estimate of market sensitivity of the issue. Note the wide diversification across industries and riskiness.

Action Report (E)

Shows the specific transactions necessary to maintain risk control for a six month period. The reasons for transactions include adjustment of debt, reinvestment of dividends, de-listing of securities, and control limits on amount invested in one group or a single security.

Input Parameter Specifications (F)

Portfolio input specification allows wide generality of operating assumptions regarding:

- Time span of simulation
- Frequency of action
- Risk specification
- Decision rules
- Fees & cash outflows
- Risk group identification
- Report frequency

References

1. "The Capital Asset Pricing Model: Some Empirical Tests", by Myron Scholes, Fischer Black and Michael C. Jensen, forthcoming in Studies in the Theory of Capital Markets, M. C. Jensen, Editor, Praeger Publishers, 1971.

TABLE A
CONTROL OF RISK

Portfolio Period	.67 Risk Levered .5 (#1)			.8 Risk Levered .25 (#2)			1.0 Risk Unlevered (#3)		
	Beta	r	σ_p/σ_m	Beta	r	σ_p/σ_m	Beta	r	σ_p/σ_m
7/45-6/50	.99	.949	1.04	1.02	.98	1.04	.97	.99	.98
7/50-6/55	.89	.909	.98	.96	.96	1.00	1.01	.98	1.04
7/55-6/60	.93	.910	1.02	1.04	.97	1.07	.94	.98	.97
7/60-6/65	1.10	.941	1.17	1.04	.97	1.07	.93	.99	.94
7/65-6/70	.91	.927	.98	.90	.95	.94	.95	.98	.96
7/45-6/70	.97	.929	1.04	.99	.97	1.02	.96	.98	.98
Objective		.920	1.00		.96	1.00		.98	1.00

TABLE B
PORTFOLIO TURNOVER

Course of Action	1/2 (\$Purchases + \$Sales)/(Average Equity)		
	(1)	(2)	(3)
	.67 Risk Levered .5	.8 Risk Levered .25	1.0 Risk Unlevered
1. Risk Control Of Underlying Portfolio	0.0%	6.9%	0.0%
2. Control of Diversification	1.9	1.3	1.7
3. Company Going Off Exchange Or Merging	5.7	3.7	2.4
4. Control Of Debt	9.2	3.1	0.0
5. Investment Of Dividends + Interest From Cash - Interest From Debt	1.5	1.7	1.8
Total	18.3	16.7	5.9
Objective	< 30.0	< 25.0	< 15.0

TABLE C
PORTFOLIO TRANSACTIONS

	(1)	(2)	(3)
	.67 Risk Levered .5	.8 Risk Levered .25	1.0 Risk Unlevered
Average Number of Issues	190	152	139
Average Number of Transactions/Year	32	24	12
Issue Turnover	8.4%	7.9%	4.3%

TABLE D
.67 RISK LEVERED .5 PORTFOLIO RETURN ANALYSIS

Period	Value of \$1		Monthly Excess Return		Excess Return/Risk		Alpha of Portfolio	Significance (t value) of alpha
	Portf.	Mkt. Index	Portf.	Mkt. Index	Portf.	Mkt. Index		
7/45-6/50	2.05	1.47	.0128	.0072	.233	.135	.0058	2.54
7/50-6/55	3.00	2.83	.0178	.0168	.531	.492	.0028	1.38
7/55-6/60	2.33	1.73	.0124	.0074	.375	.228	.0055	3.02
7/60-6/65	2.07	1.66	.0108	.0068	.221	.163	.0033	1.52
7/65-6/70	1.14	1.34	-.0009	.0017	-.018	.031	-.0025	-.95
Total Period	33.75	15.97	.0106	.0080	.230	.181	.0029	2.86

*Value of \$1 invested at start of each five-year period.

TABLE E
.8 RISK LEVERED .25 PORTFOLIO RETURN ANALYSIS

Period	Value of \$1		Monthly Excess Return		Excess Return/Risk		Alpha of Portfolio	Significance (t value) of alpha
	Portf.	Mkt. Index	Portf.	Mkt. Index	Portf.	Mkt. Index		
7/45-6/50	1.57	1.47	.0085	.0072	.153	.135	.0011	.83
7/50-6/55	3.17	2.83	.0187	.0168	.550	.492	.0027	1.89
7/55-6/60	2.25	1.73	.0119	.0074	.343	.228	.0042	3.66
7/60-6/65	1.83	1.66	.0086	.0068	.192	.163	.0015	1.04
7/65-6/70	1.16	1.34	-.0008	.0017	-.017	.031	-.0024	-1.19
Total Period	23.87	15.97	.0094	.0080	.208	.181	.0015	2.20

TABLE F
1.0 RISK UNLEVERED PORTFOLIO RETURN ANALYSIS

Period	Value of \$1		Monthly Excess Return		Excess Return/Risk		Alpha of Portfolio	Significance (t value) of alpha
	Portf.	Mkt. Index	Portf.	Mkt. Index	Portf.	Mkt. Index		
7/45-6/50	1.46	1.47	.0071	.0072	.136	.135	.0001	.11
7/50-6/55	2.96	2.83	.0176	.0168	.497	.492	.0006	.52
7/55-6/60	1.68	1.73	.0068	.0074	.216	.228	-.0002	-.22
7/60-6/65	1.75	1.66	.0076	.0068	.194	.163	.0013	1.52
7/65-6/70	1.26	1.34	.0006	.0017	.011	.031	-.0010	-.77
Total Period	16.01	15.97	.0079	.0080	.184	.181	.0003	.60

PERFORMANCE REGRESSION (REPORT A)

EXCESS RETURN FOR MONTHS 7/45 - 6/70 MARKET (BETA = .8) PORTFOLIO LEVERED (D/E OF .25) JULY 1945 - JUNE 1970

ALPHA	VALUE	STD. ERR.	T VALUE
BETA	0.001515	0.000689	2.197846
	0.984854	0.015381	64.029802
MARKET RETURN	MEAN	STD. DEV.	MEAN/SD
PORTFOLIO RETURN	0.007974	0.044161	0.180569
	0.009368	0.045045	0.207967
NUMBER OF OBSERVATIONS	= 300		
CORRELATION COEFFICIENT	= 0.965525		
STANDARD ERROR OF RESIDUALS	= 0.011745		
MEAN ABSOLUTE DEVIATION	= 0.008902		
DURBIN WATSON STATISTIC	= 1.900939		

MONTHLY RETURNS REPORT (B)

RETURNS FOR MONTHS 7/69 - 6/70 MARKET (BETA = .8) PORTFOLIO LEVERED (D/E OF .25) JULY 1945 - JUNE 1970

MONTH	PORTFOLIO	MARKET	90 DAY BILL	COMM. PAPER	BORROW RATE
7/69	0.901717	0.920165	1.006048	1.007567	1.008331
8/69	1.045497	1.050304	1.006039	1.007275	1.008040
9/69	0.974075	0.983586	1.006138	1.007412	1.008177
10/69	1.068516	1.077228	1.006057	1.007485	1.008249
11/69	0.950859	0.952522	1.006283	1.007393	1.008159
12/69	0.953977	0.953666	1.006800	1.007743	1.008505
1/70	0.937607	0.943692	1.006854	1.007688	1.008451
2/70	1.061021	1.051723	1.006183	1.007476	1.008241
3/70	1.002811	0.990283	1.005733	1.007275	1.008040
4/70	0.881087	0.870245	1.005616	1.007028	1.007795
5/70	0.916026	0.911857	1.005913	1.007183	1.007950
6/70	0.952151	0.922101	1.005769	1.007155	1.007922
INTERVAL:					
TOTAL	0.682364	0.668975	1.075954	1.092374	1.102371
A MEAN	0.970445	0.968948	1.006119	1.007390	1.008155
G MEAN	0.968653	0.967054	1.006119	1.007390	1.008155
TO DATE:					
TOTAL	23.869364	15.971273	1.971970	2.300101	2.925957
A MEAN	1.011635	1.010241	1.002267	1.002782	1.003586
G MEAN	1.010631	1.009279	1.002266	1.002780	1.003585

BALANCE SHEET AT END OF MONTH 6/70

ASSETS		LIABILITIES	
CASH AVAIL	114093.48	DEBT	6254982.67
CASH SHRT W/HLD	0.0	SHORT	0.0
LDNG	30010805.57	EQUITY	23869916.39
TOTAL	30124899.06	TOTAL	30124899.06

TRANSACTIONS SUMMARY REPORT (C)

REPORT FOR MONTHS 7/69- 6/70

MARKET (BETA = .8) PORTFOLIO LEVERED (D/E OF .25)

JULY 1945 - JUNE 1970

	REPORT INTERVAL TOTALS	REPORT INTERVAL MO. AVERAGE	MO. AVERAGE TO DATE
COMMISSIONS			
DUE TO CASH OR DEBT CHANGES	14611.86	1217.65	499.64
DUE TO GROUP ADJUSTMENT GT MAX	5273.32	439.44	518.22
DUE TO GROUP ADJUSTMENT LT MIN	0.0	0.0	0.0
SECURITIES GT MAX VALUE	1463.02	121.92	27.99
SECURITIES LT MIN VALUE	161.31	13.44	5.82
LIMIT ORDER VALUE EFFECT			
DUE TO CASH OR DEBT CHANGES	0.0	0.0	0.0
DUE TO GROUP ADJUSTMENT GT MAX	0.0	0.0	0.0
DUE TO GROUP ADJUSTMENT LT MIN	0.0	0.0	0.0
SECURITIES GT MAX VALUE	0.0	0.0	0.0
SECURITIES LT MIN VALUE	0.0	0.0	0.0
DIVIDENDS	1509032.62	125752.69	45675.76
INTEREST			
PAYBLE ON DEBT	760537.01	63378.08	13887.91
RECVBLE FROM CASH AVAIL	9791.83	815.99	248.36
PAYBLE ON CASH FROM SHRTS	0.0	0.0	0.0
RECVBLE FROM SHORT POST	0.0	0.0	0.0
VALUE OF TRANSACTIONS			
DUE TO CASH OR DEBT CHANGES	4268276.12	355689.62	140461.44
DUE TO GROUP ADJUSTMENT GT MAX	2289728.44	190810.69	129999.31
DUE TO GROUP ADJUSTMENT LT MIN	0.0	0.0	0.0
SECURITIES GT MAX VALUE	524024.56	43668.71	8305.49
SECURITIES LT MIN VALUE	178531.36	14877.61	3597.49
SECURITIES GOING OFF EXCH	635119.69	52926.64	34901.88
NUMBER OF TRANSACTIONS			
DUE TO CASH OR DEBT CHANGES	19	1.58	1.27
DUE TO GROUP ADJUSTMENT GT MAX	5	0.42	0.95
DUE TO GROUP ADJUSTMENT LT MIN	0	0.0	0.0
SECURITIES GT MAX VALUE	1	0.08	0.03
SECURITIES LT MIN VALUE	5	0.42	0.18
SECURITIES GOING OFF EXCH	3	0.25	0.22
SECURITIES WENT BROKE	0	0.0	0.0
MANAGEMENT FEES	0.0	0.0	0.0
CASH OUTFLOW	0.0	0.0	0.0
PORTFOLIO SIZE			
LONG		184.92	152.35
SHORT		0.0	0.0
PORTFOLIO VALUES			
EQUITY		31115712.00	11233239.00
DEBT		7763087.00	2804838.00
CASH AVAILABLE		133789.37	69755.12
LONG		38679968.00	13953417.00
SHORT		0.0	0.0

GROUP 1		PORTFOLIO HOLDINGS REPORT (D)			
LONG					
AVCO CORPORATION	11954.91 1.34	TEXAS INSTRUMENTS, INC	4312.42 1.41		
GROUP 2					
LONG					
AMERICAN ENCAUSTIC TILIN	18849.98 1.19	HCA INDUSTRIES INC	12421.74 1.20	MUNSGHEAR INC	9624.80 1.15
BELDING HEMINWAY CO	15124.79 1.16	HOLLY SUGAR CORP	13399.81 1.11	NATIONAL CASTINGS CO,	5774.77 1.23
BUCYRUS-ERIE CO	17760.71 1.14	INTL MINING CORP	2474.81 1.25	PEPSICO INC	6387.42 1.09
GEN SIGNAL CORP	20349.96 1.18	JONES LAUGHLIN STEEL	13229.70 1.29	REPUBLIC STEEL CORP	21839.62 1.27
GROUP 3					
LONG					
BLACK & DECKER MFG CO	10599.84 1.00	DOEHLER JARVIS CORP	14999.86 1.05	MONSANTO CO	13918.72 1.03
BLISS & LAUGHLIN, INC	18599.69 0.93	FLORESHEIM SHOE CO	12149.88 0.94	NASHVILLE CHATTANOOGA &	18999.96 0.89
BOND INDUSTRIES	7799.86 0.94	GILLETTE CO	14849.58 1.01	NEW YORK SHIPBUILDING CO	18699.90 0.94
CHEMWAY CORP	13174.87 0.89	GREEN BAY & WESTERN RAIL	9874.77 0.93	PHELPS DODGE	14699.55 0.99
CHRYSLER CORP	14324.66 0.98	LEHN & FINK PRODUCTS	17324.95 0.92	UAL INC	15060.29 0.97
CLEVITE CORPORATION	15449.65 0.90	LOUIS & NASHVILLE RR	11099.76 0.91	WARREN PETROLEUM CORP	14399.83 0.94
CROWN CORK & SEAL	11476.07 1.05				
GROUP 4					
LONG					
AIR REDUCTION CO	14874.83 0.82	EASTERN AIR LINES	13999.89 0.87	LIBBEY-OWENS-FORD CO	21599.51 0.87
BEATRICE FOODS CO	23999.52 0.80	FIBREBOARD CORP	9224.88 0.85	NATL CASH REGISTER	18768.61 0.80
BROWN SHOE CO, INC	25349.47 0.81	GEN MOTORS	17499.61 0.84	OWENS-ILLINOIS INC	12649.76 0.85
CHICKASHA COTTON OIL	14399.74 0.81	GEN ELECTRIC CO	22687.26 0.80	THOMAS STEEL CO	22687.43 0.85
COLUMBIA BROADCASTING	11599.74 0.86	GOODRICH, B. F. CO	18049.89 0.86	U S SMELTING	10349.79 0.89
COMBUSTION ENGINEERING	15749.66 0.88	HONEYWELL INCORPORATED	22049.74 0.84	UNDERWOOD CORP.	12299.75 0.87
CUNEO PRESS, INC	11275.14 0.86	KENNECOTT COPPER	21999.86 0.83		
GROUP 5					
LONG					
ABBOTT LABORATORIES	19934.43 0.73	FREEMPORT SULPHUR CO	23199.85 0.78	MOBIL OIL CORP	24020.73 0.72
AMER HOME PROD	18013.34 0.69	GEN CIGAR CO INC	14624.94 0.73	PHILLIPS PETROLEUM	25559.81 0.78
ATLAS CHEMICAL IND	11399.74 0.76	HOWMET CORP	13873.96 0.73	RAYBESTOS-MANHATTAN	14999.67 0.76
BARBER OIL	16699.61 0.76	INLAND STEEL CO	24299.55 0.76	SEARS, ROEBUCK & CO	17799.63 0.71
CONSOLIDATED LAUNDRIES C	24599.88 0.73	JEWEL CO	24499.49 0.79	SPENCER KELLOGG AND SONS	16799.92 0.77
CONT STEEL	21599.77 0.73	LEHIGH COAL & NAVIGATION	12824.94 0.73	ST JOE MINERAL CORP	18374.88 0.69
DIAMOND INTERNATIONAL	22499.60 0.69	LONE STAR CEMENT CORP	20774.64 0.78	STAND OIL OHIO	24581.39 0.77
FMC CORPORATION	15677.72 0.78				
GROUP 6					
LONG					
CHESAPEAKE & OHIO RY	8737.33 0.55	NATL BISCUIT CO	20624.55 0.66	SCOTT PAPER CO	26857.12 0.54
GEN AMER TRANSPORT	13762.19 0.67	PATINO MINES & ENTERPRIS	7899.83 0.55	SO CALIFORNIA EDISON	16499.59 0.57
GEN FOODS CORP	19699.62 0.58	PENN GLASS SAND	22137.45 0.56	TALCOTT NATIONAL INC	20924.62 0.67
H L GREEN CO., INC.	22799.60 0.64	PET INC	18749.58 0.68	TEXAS GULF SULPHUR CO	28199.55 0.55
KROGER CO	23999.48 0.64	PHILIP MORRIS, INC	20910.30 0.67	UNION CARBIDE CORP	27149.46 0.67
LINK-BELT CO	23899.43 0.68	QUAKER STATE OIL	18393.56 0.64	UNIT FRUIT CO	35399.22 0.57
MORRELL & CO	16499.72 0.61	RELIABLE STORES CORP	16499.71 0.69	WALDORF SYSTEM	13887.24 0.57
MURPHY, G.C. CO	17799.97 0.56				
GROUP 7					
LONG					
AGUIRRE CO	13799.76 0.35	GEN MILLS INC	15899.68 0.36	PILLSBURY CO	21700.00 0.53
AMER SHIP BLDG	20374.59 0.40	HOUSEHOLD FINANCE CORP	22769.92 0.40	POTOMAC ELECTRIC POWER	21937.50 0.16
AMER TEL & TEL	15703.02 0.25	IDAHO POWER CO	17437.43 0.36	REYNOLDS R J INDS	17562.43 0.47
AMERICAN SUMATRA TOBACCO	18499.90 0.53	INTERCO	15399.67 0.45	TRANS UNION CORP	15749.70 0.52
CHILE COPPER COMPANY	20999.98 0.53	NORFOLK & WESTERN RY	17949.73 0.45	WHEELING & LAKE ERIE	22999.50 0.11
EL PASO NATURAL GAS	31557.89 0.53				

ACTION REPORT (E)

MARKET (BETA = .8) PORTFOLIO LEVERED (D/E OF .25) JULY 1945 - JUNE 1970

			PRICE	NEW VAL	BETA	GRP NR	REASON
7/59	DIVEST	19498.83	GREEN BAY & WESTERN RAIL	195,000	0.0	0.526	6 SEC OFF EXCH
8/59	INVEST	140562.50	NATL SERVICE IND	21,625	140562.50	0.419	6 CASH/DHT CHG
	DIVEST	13234.00	SO PUERTO RICO SUGAR	22,375	0.0	0.338	7 SEC LT MIN
9/59	DIVEST	208798.06	WHITE MOTOR CORP	58,000	0.0	0.880	3 GRP ADJ; MAX
	INVEST	140625.00	BELL INTERCONTINENTAL	15,625	140625.00	0.761	4 GRP ADJ; MAX
	INVEST	136400.00	CANADA DRY CORP	22,000	136400.00	0.729	4 GRP ADJ; MAX
	DIVEST	12429.33	GOEBEL BREWING	3,125	0.0	1.241	2 SEC LT MIN
10/59	DIVEST	64193.47	UNITED CARBON	69,250	0.0	1.481	1 CASH/DHT CHG
	INVEST	15800.00	GARRETT CORP	39,500	47934.37	1.409	1 CASH/DHT CHG
	DIVEST	20679.28	MAHONING COAL RAILROAD C	440,000	0.0	0.536	6 SEC OFF EXCH
	DIVEST	12599.21	BOND INDUSTRIES	22,375	0.0	0.677	5 SEC LT MIN
11/59	DIVEST	195747.44	BATES MFG CO	11,250	0.0	0.970	3 GRP ADJ; MAX
	INVEST	131175.00	CHESAPEAKE CORP OF VA	39,750	131175.00	1.172	2 GRP ADJ; MAX
	INVEST	130950.00	TEXTRON, INC	24,250	130950.00	1.121	2 GRP ADJ; MAX
	DIVEST	12799.42	SPENCER KELLOGG AND SONS	16,875	0.0	0.999	3 SEC LT MIN
	DIVEST	209624.75	MOTOROLA INC	128,000	161246.50	0.848	4 SEC GT MAX
12/59	INVEST	133875.00	INDIANAPOLIS POW & LT	39,375	133875.00	0.417	6 CASH/DHT CHG
	INVEST	133875.00	TOLEDO EDISON CO	15,750	133875.00	0.422	6 CASH/DHT CHG

INPUT PARAMETER SPECIFICATIONS (F)

PORTFOLIO SIMULATION IDENTIFICATION:

MARKET (BETA = .8) PORTFOLIO LEVERED (D/E OF .25) JULY 1945 - JUNE 1970

1-START DATE: 4507
 2-LAST DATE: 7006
 3-SEED NUMBER: 5587.
 4-GROUP READJUSTMENT INTERVAL: 1
 5-DEBT READJUSTMENT INTERVAL: 1
 6-LONG READJUSTMENT INTERVAL: 1
 7-SHORT READJUSTMENT INTERVAL: 0
 8-REPORT INTERVAL: 12
 9-PORTFOLIO/GROUP STATISTIC INTERVAL: 12
 10-REGRESSION ANALYSIS INTERVAL: 60
 11-DESIRED STANDARD DEVIATION (RELATIVE TO MARKET): 0.0
 12-STARTING DEBT TO EQUITY RATIO: 0.250
 13-PORTFOLIO STD. DEV. CALCULATION = DEBT/EQUITY ADJUSTMENT INTERVAL: 0
 14-PORTFOLIO SIZE OPTION: 3
 15-OPTION VALUE: 0.100
 16-SIZE RESTRICTION PARAMETER: 0
 17-CONSTANT (YRLY%) ADDED TO COMMERCIAL PAPER RATE TO DETERMINE BORROWING RATE: 1.000
 18-SHORT TO EQUITY RATIO: 0.0
 19-PROP. OF SHORT PROCEEDS AVAIL. FOR INVESTMENT: 0.0
 20-INTEREST RATE (YRLY%) PAYABLE ON SHORT PROCEEDS RECEIVED: 0.0
 21-INTEREST RECEIVABLE ON SHORT PROCEEDS WITHHELD PARAMETER: 0
 22-ROUND LOT SIZE: 100
 23-CASH INVESTMENT PARAMETER: 1
 24-COMMISSION PARAMETER: 1
 25-MAX CASH AFTER INVESTMENT (% OF PORTFOLIO VALUE): 1, 1
 26-INDIVIDUAL SECURITY LIMIT CHECK INTERVAL: 1
 27-MIN SEC VALUE (% OF PORTFOLIO VALUE): 0.100
 28-MAX SEC VALUE (% OF PORTFOLIO VALUE): 2.500
 29-STARTING EQUITY: 100000.00
 30-LIMIT ORDER EFFECT (DIFFERENCE IN PRICE): 0.0
 31-MANAGEMENT FEE AND OUTFLOW INTERVAL: 0
 32-FIXED MANAGEMENT FEE: 0.0
 33-VARIABLE MANAGEMENT FEE (% OF EQUITY): 0.0
 34-FIXED OUTFLOW: 0.0
 35-VARIABLE OUTFLOW (% OF EQUITY): 0.0

GROUP DESCRIPTION (SECURITIES RANKED BY BETA = HIGH TO LOW):

GROUP	PROP OF MARKET	GROUP LONG TO TOTAL	LONG DESIRED	LONG MAX	GROUP SHORT TO TOTAL	SHORT DESIRED	SHORT MAX
1	0.200	0.0	0.0	0.100	0.0	0.0	0.0
2	0.200	0.050	0.100	0.150	0.0	0.0	0.0
3	0.200	0.050	0.100	0.150	0.0	0.0	0.0
4	0.100	0.100	0.200	0.300	0.0	0.0	0.0
5	0.100	0.100	0.200	0.300	0.0	0.0	0.0
6	0.100	0.100	0.200	0.300	0.0	0.0	0.0
7	0.100	0.100	0.200	0.300	0.0	0.0	0.0
FIRST MONTH	175	LAST MONTH	474	EQUITY	100000.00		