

## EVALUATING JOB SHOP SIMULATION RESULTS

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### Abstract

The problem of predicting the effectiveness of simulation results as applied to job shop production is discussed. A measure of effectiveness is developed which allows both absolute and relative evaluation of different scheduling techniques and the conditions for profit maximization under a customer service constraint are presented. The development is based upon a model of the shop process which assumes random scheduling behavior and which serves as the reference point for evaluating schedules.

### I. Introduction

The problem of scheduling production is the central element of the production control problem for the job shop. Production performance is almost entirely determined by the quality of schedules produced. Research and applications studies on the problem have, almost entirely, been either simulations or simulation-based. The use of simulation is mandated by the complexity of the problem and the lack of an algorithm for finding solutions. The importance

of the scheduling problem is widely recognized by members of both the academic and the industrial community.

The solution to the job shop scheduling problem has its ultimate value in application. The scheduling rule or procedure, which is the solution, may be used in either a manual or computer based system to effect maximal production performance. The objectives served by the production control function in the job shop, through the schedule, are: (1) profit; (2) customer

service; (3) reduced investment in work in process inventories; and (4) improved utilization of capacity. Conway, Maxwell, and Miller [1] discuss these objectives, pointing out that direct costs are irrelevant to the scheduling decision and that only indirect costs form the proper basis for evaluating schedules.

The search for the best scheduling rule has gone on for many years. The work that has been done may be placed in two broad categories: "Academic Research" and "Industrial Applications". Much of the progress made in one category is of little value in the other. Results and findings are not seen as transferrable and, as a result, are not transferred. The academic studies are devoted to problems too highly abstracted and bearing little relation to those found in industry. Industrial applications are too specialized, conforming to the special and possibly unique needs of the company which developed the application. In addition, results and details of system structure are often viewed as proprietary and are not publicized. Reports of effectiveness, when made available, are often without meaning when taken out of context. Thus, when one company can claim to have reduced lateness by a half or a third through scheduling, another company has no guarantee that the same, or a similar, system will produce the same result for them.

## II. The Problem of Measurement

A major source of difficulty in assessing the transferability of results between two industrial applications or between academia and the industrial community is the lack of a good and generally acceptable measure for evaluating schedule quality or performance. The problem is compounded by the fact that academic studies are typically couched in purely technical terms (flow time is the most common measure) with no reference to economic criteria, and that industrial applications are described primarily in terms of the change in economic performance achieved. A means of constructing a true cost effectiveness measure for changes in schedule quality must be available if scheduling studies and applications are to effect general changes in manufacturing performance. The cost of a scheduling study or application cannot be determined in the absence of information about costs of computation, programming and manpower. The effectiveness of a scheduling rule or system, in application, can be found both in absolute terms and in terms of the improvement over current procedures.

The requirements for the measure of effectiveness, or benefit, are neither simple nor easily satisfied. Although many requirements of varying importance may be stated, there are four which are of primary importance:

- (1) Schedule performance must be measurable in both economic and technical terms. In addition, the conversion from an economic

to a technical measure, and vice versa, must be easily accomplished.

- (2) The effectiveness measure must be general and widely applicable. It must not be tied to unique or special economic or technical characteristics.
- (3) The measure must be capable of being easily understood by managers, both in terms of technical and economic performance.
- (4) The measure must be applicable to both actual and simulated performance. That is, it must not require information not readily available in a job shop.

The last of these requirements can be of particular importance, especially since the motivation to undertake a scheduling study or application most often arises from an evaluation of the effectiveness of procedures currently in use.

Every scheduling procedure or production control system, whether manual or computer based, presumably exhibits some of the characteristics of intelligent, goal seeking behavior. Under this premise, the performance under any such system should exceed that available from a completely random system; i.e., one in which scheduling decisions are made on a random basis. In the following sections, the measure of effectiveness will be developed and it will be applied to a random system so that a baseline for comparison will be available.

Job shop scheduling simulations, whether abstract or applied, do not lead to normative results in the sense that optimal schedules or scheduling rules can be specified. This, however, should not imply that a normative, profit maximizing, criteria should not be used as a basis of comparison for simulation results. Normative measures have not been either developed or applied in job shop research or application and progress toward the ultimate solution to the "job shop problem" has suffered as a result.

### III. The Job Shop Process

Job Shop production is difficult to characterize because of the variety and complexity of products produced, both to customer order and to inventory. No measure of output, other than a generalized measure of products produced per unit time, is available. Inputs are similarly difficult to characterize since they may be unique to each product. (This is particularly true for materials.) A generalized model of the job shop process must, for these reasons, be aggregated and stated in terms of cost and resource flows. This does not present great difficulties since our evaluation of the job shop process is, primarily, couched in terms of indirect resources and their costs. Since it is the direct resources required by products and the processing requirements for individual products that gives the job shop problem its complexity, we can avoid much of this complexity and the problems it creates through aggregation.

The job shop process is amenable to analysis under the structure of Figure 1.

The job shop process, as represented by Figure 1, is that of taking the inputs machinery, labor and supervision (in the required proportions) as the productive resource "capacity". To capacity is added the input materials and capacity is transformed into the resource "work in process." The resource work in process is then transformed into units of completed products which are represented in revenue terms. This model of the job shop process is unique, perhaps, since only capacity and work in process are viewed as productive resources (which have productivities). The inputs machinery, labor, supervision, and materials are not individually productive. In this model, demand occurs at a rate  $\lambda$  (in products per unit time) and the average processing requirement for products is  $P$  (in units of time per product). The resources are  $M$ , the number of machines (or points of processing capability), and  $N$  the number of products in process. The productivities of the resources are  $p_f$ , the utilization

of capacity, defined as

$$p_f = \frac{\lambda P}{M}$$

and  $p_d$ , the ratio of mean processing time to mean flow time ( $F$ ) for products<sup>1</sup>,

$$p_d = \frac{P}{F}$$

Assuming that the job shop system is a queueing system and that steady state conditions obtain, then  $N = F$  and  $p_d$  may be written as

$$p_d = \frac{\lambda P}{N}$$

At steady state,  $Np_d = Mp_f$  and output,  $X$  (in products per unit time) may be written as

$$X = \frac{Np_d}{P} = \frac{Mp_f}{P}$$

<sup>1</sup>

Technically, the productivities are  $p_f/P$  and  $p_d/P$ , but the denominators cancel in all cases and are ignored. Both  $p_f$  and  $p_d$  are between zero and one under steady conditions.

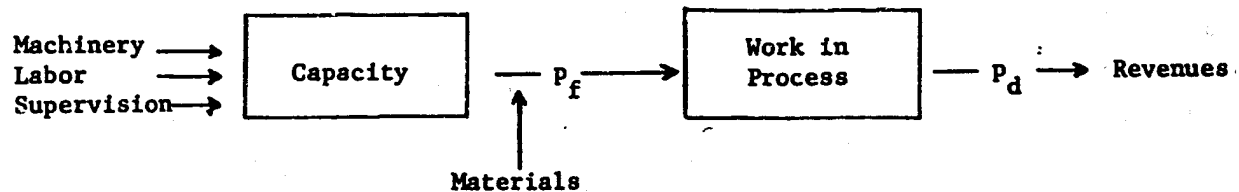


Figure 1

The Job Shop Process

#### IV. Measuring Effectiveness

Control over the job shop process under this model may be effected by controlling  $M$ . Under the assumptions stated, control is exercised so that profit is maximized subject to a constraint on customer service, stated as a mean flow time requirement,  $\bar{F}$ . The point at which maximum profit occurs is that at which the ratio of the marginal productivity to marginal cost is equal across all resources.<sup>1</sup> To compute these ratios, knowledge of marginal cost must be available. These are simply the incremental costs of having additional units of work in process and additional units of capacity.<sup>2</sup> It should be noted that since both  $N$  and  $M$  are time averaged, the increment costed need not be integer and, in the case of  $M$ , a new machine need not be purchased if overtime is available. Letting  $C_d$  and  $C_f$  be the marginal costs of incremental work in process and capacity, respectively, profit is maximized when

$$\frac{p_d}{C_d} = \frac{p_f}{C_f}$$

1

See Samuelson [5], p. 60.

2

The production system described by this model is stochastic and steady-state is assumed. Therefore, the best, and only, estimate of marginal quantities is their time average.

This may also be written as  $MC_f = NC_d$ . The optimal operating configuration is found in the following way. First  $\bar{N}$  is calculated from  $\bar{N} = \lambda \bar{F}$ . The optimal value of  $M$  is then computed as  $\bar{M} = \bar{N} C_d / C_f$ . From  $\bar{M}$  and  $\bar{N}$ , both  $p_f$  and  $p_d$  may be found.

These relations may be used to evaluate simulation results in a straightforward manner. Let us first assume that the simulation is conducted for potential application of a scheduling rule to a shop with known cost data. It should be noted in passing that, under a manual control system acting to maximize profits in the manner described above, it is extremely difficult to maintain the optimal operating configuration. It should not be assumed, for purposes of comparison, that a manual system achieves this state since the average effects of a set of individual control decisions, made serially, cannot always be controlled very well. Continuing the example, it is clear that the ratio  $p_d/p_f$  must be equal to  $C_d/C_f$  at the optimal operating point. The value of  $\bar{p}_d$  (under the random assumption) is also known from  $\bar{F}$ . On a graph of  $p_f$  versus  $p_d$ , the ratio  $C_d/C_f$  may be drawn as a line through the zero point of the graph with positive slope. The intersection of this line with the line represented by  $\bar{p}_d$  is the profit maximizing operating configuration. This graph is illustrated in Figure 2.

Simulation results, for a utilization of  $\bar{p}_f$ , should fall above the intersection of the  $\bar{p}_d$

and  $\bar{p}_f$  lines. If they do not, then the scheduling rules tested do not perform as well as the random rule. The simulation results for the load/capacity configuration represented by  $\bar{p}_f$  may be plotted on the  $\bar{p}_f$  line (the symbol "(x)" represents these on the graph). None of these represent optimal profit performance since none of them falls on the  $C_d/C_f$  line. Further simulations must be run until a result is found which falls on  $C_d/C_f$  line at the highest value of  $p_f$  attainable. It is noteworthy that this result may not be the minimum average flow time result for the load/capacity ratio tested. It may also occur that the  $\bar{p}_d$  line may not intersect the  $C_d/C_f$  line, particularly in cases when the value of  $C_d/C_f$  is small. When this occurs, there is no operating configuration or scheduling technique which will satisfy both the customer service constraint ( $\bar{F}$ ) and the profit maximizing condition. In this case the management of the job shop has some (possibly) difficult choices to make.

Simulation studies conducted in the absence of economic information, obviously, cannot specify an optimal scheduling technique for any job shop with a customer service constraint. In the absence of economic information, performance of the several scheduling techniques tested must be represented by lines on the  $p_f, p_d$  graph and the determination of optimality must be deferred. The performance lines for a set of techniques will, in general, start in the

upper left corner of the graph and slope downward to the right. These lines, particularly those representing a mixed scheduling technique (e.g., truncated SOT), may not necessarily be linear - or even continuous. A set of simulation results might be presented in the fashion illustrated in Figure 3.

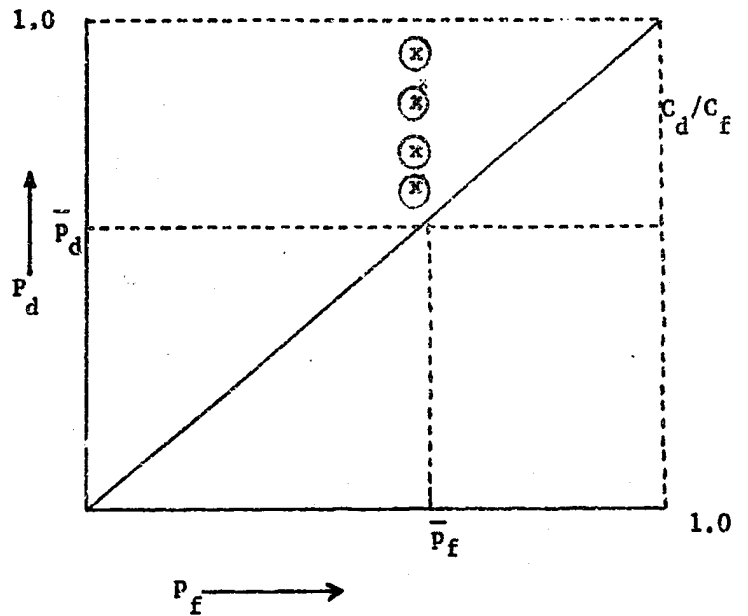


Figure 2

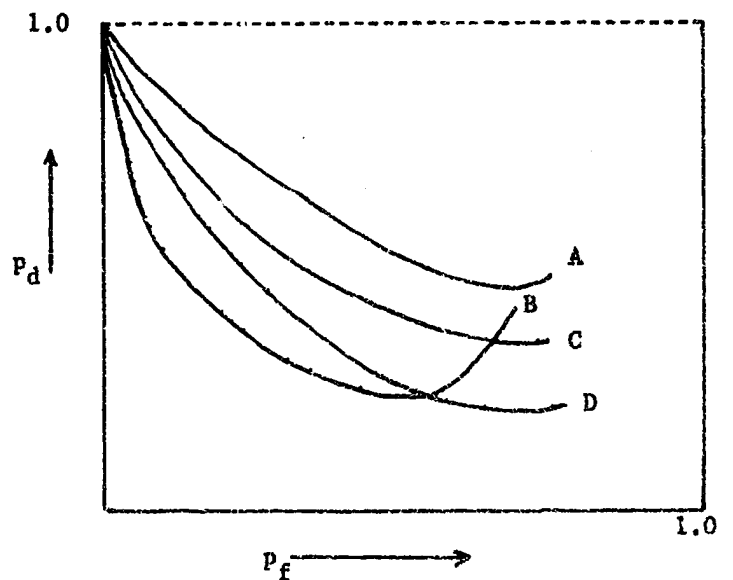


Figure 3

## V. Data Acquisition

As might be expected, the primary data problems which arise in evaluating the effectiveness of simulation studies are not related to the simulation or its results. The only precautions which need be taken are those of insuring that steady state is achieved and that actual resultant values are found for  $P$ ,  $\lambda$ , and  $N$ . The major difficulties arise in determining current shop performance (so that a point of comparison is obtained) and measuring the economic factors.

Since it cannot (or should not) be assumed that the performance of the control procedure in operation in the shop conforms to the structure developed under the assumptions of the random model, both technical (productivity) and economic measures should be made. There are several sources for this data, including the shop floor, the production control function and the financial statements of the firm. These sources may not prove to be consistent upon cross checking (which is highly recommended), creating problems which must be solved by closer examination of records or "by assumption."

Not the first, but probably the most significant problem that will be encountered is that of defining a job. A job, which may be several customer orders or a part of a single order, is a piece, or set of pieces, which follows a single route through the shop. The

route need not be simple, but must be followed for the entire job. The balance of the technical data can be found by using one or more of the sources mentioned above. Information about the rate of demand or shipment (presuming steady state) can be obtained from the production controller and the accounting office. Information about mean processing time is available from sampling production control records and from supervisors on the shop floor (after determining the fraction of jobs processed in each work center). Data on capacity is available in the production control office and from observation on the shop floor. The number of jobs in process is most directly found by inspection and may be confirmed by the production controller. The average flow time may be found by sampling production control records and compared to the calculated value.

Economic information may be more difficult to obtain, depending upon the nature and availability of accounting and financial data. The unit marginal cost of work in process must, in the absence of specific information about the "next" jobs, be assumed to be equal to the average cost. Further, the average in-process job can be assumed to be half completed. It is reasonable in most cases to assume that all materials are committed when the job is started and that processing costs are directly proportional to processing time. Processing costs are the costs of direct labor and machine time that can be

directly attributed to the job. It is important that these costs be direct, not allocated. The means for obtaining machine cost per unit of processing time will be discussed next.

The average unit cost of capacity is determined in the following way. First, the annual cost of supervision is determined and divided by the number of units of capacity. Next, the total annual depreciation and interest for productive machinery is calculated and divided by the number of units of capacity. Finally, the annual direct labor payroll is divided by the units of capacity. These are summed and further divided by productive days in the year to obtain the average daily cost of capacity. The average cost of capacity is not equal to the marginal cost of capacity. The machine cost component of capacity cost is zero at the margin if overtime or undertime production is available. The direct labor and supervisory components can likewise vary depending upon whether or not over or undertime is available or whether modifications to the size of the workforce must be made. Determination of the marginal cost of capacity is largely situation specific and, for the most part, depends upon the availability of overtime production.

The determination of work in process and capacity costs is not difficult in concept. In practice, some difficulties may be encountered

because individual costs must be aggregated in a way not consistent with standard accounting practice. Hillier [3] discusses a similar cost finding problem and his paper is recommended to those who wish to pursue this problem further.

#### VI. Summary

The procedures for determining the effectiveness of job shop scheduling simulations meet all of the criteria stated in Section II above. The measures developed allow both relative and absolute comparisons for every case in which economic information is available. They increase both the effectiveness and desirability of job scheduling simulations because they allow the interpretation of simulation results in either a technical or an economic context. The measures lead directly to a statement of the conditions for profit maximization and bring all of us with an interest in this fascinating and complex problem a step closer to its ultimate solution.



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