SIMULATION VIA TIME-PARTITIONED LINEAR PROGRAMMING: A GROUND AND SURFACE WATER ALLOCATION MODEL FOR THE GALLATIN VALLEY OF MONTANA

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ABSTRACT

Simulation of the total water resources system was used to demonstrate that ground-water development is the solution to water shortage problems in the Gallatin Valley. Systems analysis was used to identify the components of the model. Physical system response was approximately linear, so the IBM 1620-1311 linear programming code was used.

Each of 30 years was partitioned into two periods: water surplus and water deficit. Water was allocated among competing uses: agricultural, municipal, industrial, and recreational; and between surface and subsurface storage. Outflow was simulated to approximately $\pm$ 20 percent of gaged outflow, not exceeding probable experimental error.

INTRODUCTION

The Gallatin Valley, comprising a rich farming and grazing area along bottom lands of the East and West Gallatin Rivers, is located in Southwestern Montana. The valley is a basin surrounded by mountains and filled with Tertiary period lake beds which are partially overlain by younger alluvial gravels. A U.S. Geological Survey report [4] estimates that 1,250,000 acre-feet of ground water is stored within 100 feet of the land surface and that this reservoir can supply supplemental needs over several years of drought.

Although the Gallatin Valley is a water exporting area, critical surface water shortages characterize the peak of the irrigation season. A recent survey showed that 53 percent of the farmers reporting need additional water, many having their supplies shut off by July 1 to 15. Ironically, the lower reaches of the Valley are plagued by a rising water table which has water-logged thousands of acres of valuable farmland, caused by flood irrigation practices in the upper reaches.

The holders of early water rights are located at the water surplus, lower end of the Valley. Any scheme that would overcome shortages experienced by holders of later rights at the upper end of the Valley that does not tap under-ground reserves, can only aggravate the water-logging problem at the lower end.

Consequently, a strong case exists for cooperation between these two classes of irrigators- "lower enders" trading surface water diversion to "upper enders" for help in defraying the cost of pumping ground water. Recreationists are potential third-party cooperators in that a 12-mile stretch of the West Gallatin River, an excellent fishing stream, becomes dewatered over 90 percent for 3 to 8 weeks each year. The problem is clearly one of resource management- that of making the right quantity of water available at the right place at the right time. If under-ground reserves are used to supplement surface flows, water shortages could be alleviated.

A plan by the U.S. Bureau of Reclamation [5] called for the development of 193 irrigation wells to be located along existing ditches, and pumping that would maintain flows normally diverted from the river. As a result, stream supplies could be diverted onto lands not having access to ground water.

This paper describes a computer simulation of ideas advanced by the Bureau's plan. First, the dynamic characteristics of the present water resources system were modeled and calibrated to simulate historical outflow from the Valley. Next, the response of the system to ground-water pumping was simulated. The simulation demonstrated the capability of supplementing surface supplies from ground-water reserves over an extended period of drouth at the expense of only slightly decreased flows during the recovery period, the non-necessity of artificial recharge, and the average lowering of the water table. A detailed description [1] of the work is available at Montana State University.

DEFINITION OF THE SYSTEM

BOUNDARIES

Boundaries of the Gallatin Valley water resources system were selected to coincide with the U.S. Geological Survey's Gallatin Valley hydrologic unit [4]. The valley floor is about 25 miles long, 20 miles wide, and has an area of about 540 square

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miles. The principal streams entering the area are the West Gallatin River, Hyalite Creek, East Gallatin River, and Bridger Creek. There is only one outlet- the Gallatin River.

TIME UNIT

Since an annual time unit does not permit differentiating between annual highs and lows, the year was divided into two periods: (1) a water surplus period, May and June, and (2) a water deficit period, consisting of the months July through April.

COMPONENTS

Components of the water system (see Figure 1) were selected with the purpose of maintaining generality and application to hydrologic units other than the Gallatin Valley. The blocks represent various lumped quantities of water flow or storage per time period, for example, a surface inflow of 300,000 acre-feet during period 1. Consumption is any depletion activity which removes water from the system during an annual cycle. An example is the transfer of water by phreatophytes (well-plants) from sub-surface storage (the ground-water basin) to surface storage (interstage between sub-surface and supra-surface) in the stem, branches, and leaves; and subsequent evapotranspiration to supra-surface storage (the atmosphere). Supra-surface storage was represented by the outputs, precipitation and rainmaking, and one input, consumption. Recharge is any activity, natural or artificial, transferring water from surface storage to sub-surface storage. Conversely, discharge is any activity such as springs or seepage that transfers water from subsurface storage to surface storage. Finally, surface storage is any surface impoundment of water: channel, reservoir, etc. No reservoirs have been developed in Gallatin Valley.

WATER USES

Agricultural use includes all evaporation and transpiration losses from crop lands and natural vegetation plus evaporation from water surfaces and unvegetated fields. In addition there is infiltration (deep percolation, or recharge) and runoff (return flow).

Municipal use also entails consumption, infiltration, and runoff.

Industrial use has but token representation and also involves consumption, infiltration, and runoff.

Recreational use is implicitly represented in the form of undeveloped flow or surface-flow residue and consists of three components: export (surface outflow), consumption (evaporation loss), and infiltration.

DATA BASE

Streamflow and precipitation records for the 30-year period from 1931 through 1960, containing record dry years 1934 and 1935, and record wet year 1948, were selected. Sub-surface outflow is negligible, and sub-surface inflow is approximately 1,300 acre-feet per year. Average annual ground-water discharge is estimated to be 240,000 acre-feet. Gallatin Valley contains 345,000 acres of which 32 percent are under irrigation. Annual agricultural consumptive use is about 2 acre-feet per acre. Water diversion and return flow figures are available for the City of Bozeman. Phreatophyte loss is estimated to be upwards of 30,000 acre-feet per year, and stream infiltration loss up to 10 percent of stream flow.

Primary data are data that are derived directly from field observations. The above mentioned data do not constitute a complete data base but are all the data that were available. At this point, many persons would discount the potential utility of a simulation model on the assumption that the data base is inadequate. They believe that extensive statistical data must first be collected. Exactly the opposite is true. The model comes first, identifying what the actual data requirements are. Another fallacy is to omit constants and functional relationships if unmeasured or unmeasurable. To omit these items is equivalent to saying they have zero effect- probably the only value that is known to be wrong!

Certainly a mathematical model should be based upon the best information that is readily available, but the design of a model should not be postponed until all the pertinent parameters have been accurately measured. Once the general qualitative nature of a particular phenomenon is present in a model and approximately correct) often within a factor of 2 is close enough), the model can usually be adjusted to obtain a desired value by changing system parameters without moving these parameters outside the range of their values in the actual system. Furthermore, there are usually several parameters, any one of which can be adjusted.

The data base is completed by generating "secondary" data via equations of the model; i.e., secondary data constitute a portion of the output of the model. The accuracy of secondary data depends upon (1) the accuracy of the primary data, and (2) the validity of the expressions chosen to relate primary and secondary data. Accuracy of secondary data is adequate if the model is capable of simulating historical records to within acceptable limits.

MODEL ANALYSIS

In formulating a model of a system, one should rely less exclusively upon statistics and primary data but rather make better use of knowledge about the system. The model maker who derives relationships from statistical analysis will

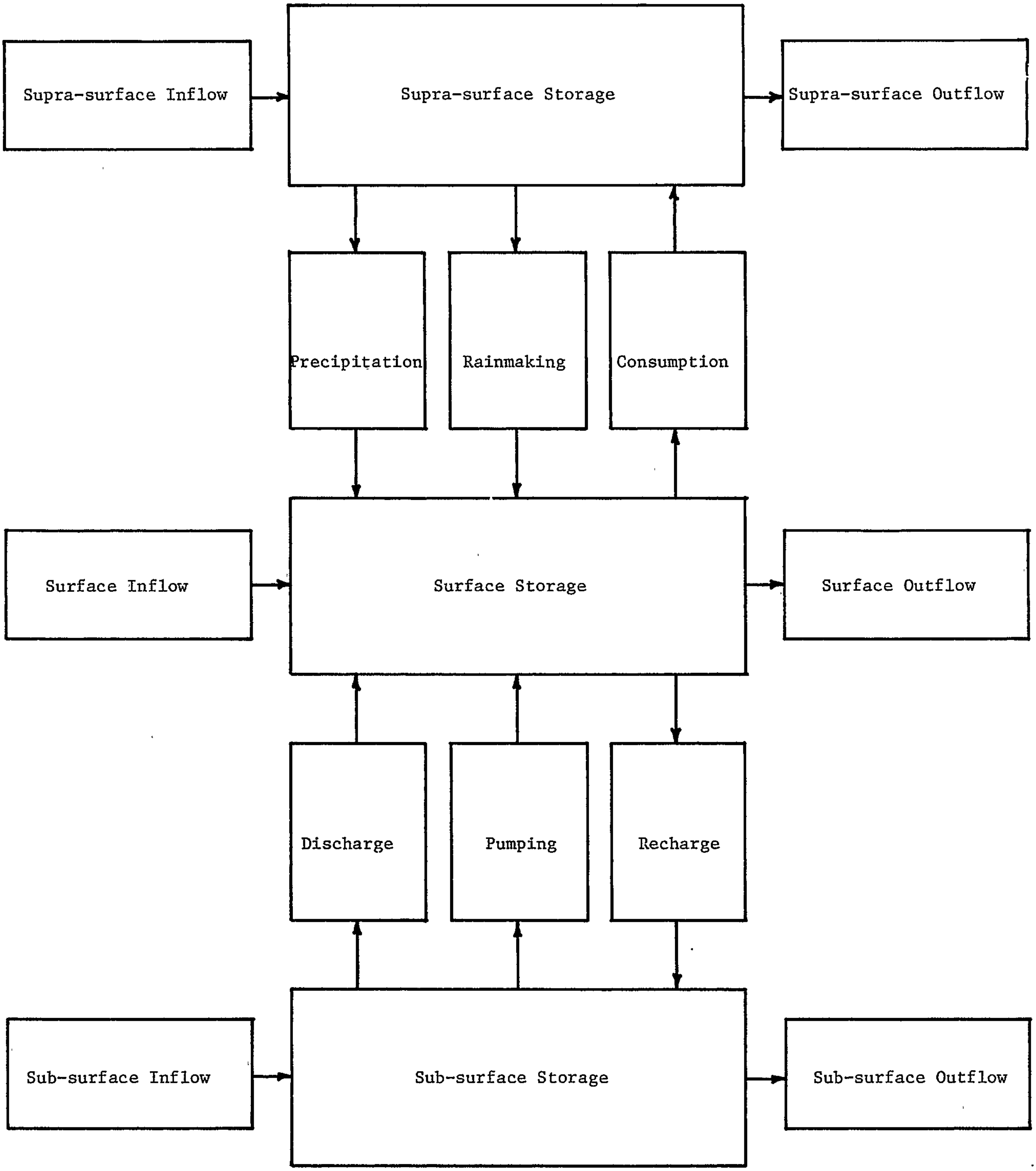

Figure 1. Components of the Water System

end up with coefficients which are abstract empirical results not identifiable with particular features of the real system.

However, the input-output data of the preceding section was used to statistically test the hypothesis that the Gallatin Valley water resources system can be represented by a linear model. The following identifications were made to facilitate computer solution for linear regression coefficients relating input to output:

X1 = stream inflow during period 1
X2 = stream inflow during period 2
X3 = precipitation during period 1
X4 = precipitation during period 2
X5 = stream outflow during period 1
X6 = stream outflow during period 2

Equations for the linear regression model were

$$X5 = -148{,}822 + 1.023\ X1 + 0.293\ X3$$
$$X6 = -51{,}615 + 1.092\ X2 + 0.261\ X4$$

with R squares of 0.95 and 0.90, respectively. All coefficients were significant at the 0.01 level except for X3 and X4 which were significant at 0.05. Thus 95 percent of the variability of X5 and 90 percent of the variability of X6 was explained by the linear regression model with probability 95 percent or higher and provided the justification for linear systems analysis.

Note that physical meaning for the coefficients is totally absent in the simple regression model, and also that in no way could this model simulate the pumping of ground water. The subsurface subsystem is not even represented!

In view of the need for an operationally feasible means of total system simulation, the linear programming algorithm was selected for its speed and efficiency. In adopting linear programming to the requirements imposed by the system, three main problems were solved:

1. How to communicate between period 1 and period 2. The objective function spanned an entire year but the constraints were segmented into two periods, thus necessitating the carry over of water "inventories" from one period to the next.

2. How to communicate between successive data years. The data base spanned a 30-year period, thus requiring the carry over from one year to the next plus the sequential insertion of input data.

3. How to interact surface and ground-water flows. Surface supplies and ground supplies are linked hydraulically, thus requiring linkage within the model.

PROCEDURE

In constructing the model, all relative components of the system (see Figure 1) were parameterized and their interrelationships characterized by linear expressions. The judicious selection of model detail was predicated upon the following working assumption by Forrester [3]: Given that if all the necessary components are adequately described and properly interrelated, the model system cannot behave other than as it should. The converse is not true; an endless variety of invalid components and structures can exist to give the same apparent system behaviour.

The water system was broken down into components, analyzed, and represented in equation form containing 72 parameters. These parameters included secondary data, coefficients, and limits, excluding primary data. If only three values within permissible range were assigned to each parameter, say a low, medium, and high value, there would be a total of about 6 million billion billion billion (the unrestricted permutation of 72 things taken 3 at a time) different solutions. Quite clearly, the inspection of so many alternatives is impossible. The approach which proved successful was to assign numerical values to the parameters based upon prior information and one's best judgment, and to work with average values attributed to a hypothetical, average year.

Inputs (streamflow and precipitation) were averaged over 30 years, and the parameters were adjusted so that computed outflow coincided with averaged outflow for the same 30 years. Once adjusted, a 30-year run was made, annual period 1, period 2 discrepancies noted, parameters readjusted, and the whole process repeated until final balance was obtained. The process was equivalent to performing a sensitivity analysis and it soon became apparent which parameters must be adjusted first.

The ground-water subsystem provided time delays which were very essential in matching period-to-period outflows. Although ground-water levels varied from period-to-period, the inclusion of this subsystem of known trend permitted dynamic stabilization of the entire system over the total period. For initial values, an average year was assumed to immediately precede the first year of record.

COMPUTER PROCESSING IN THREE PHASES

Using the IBM 1620-1311 Linear Programming Application Program, the model equations were coded for computer processing. Processing was performed in three phases: (1) a testing phase which required a high degree of interaction between computer, analyst, and model; (2) a validation phase which consisted of parameter adjustment to simulate historical outflow; and (3) a simulation phase which generated informative data concerning the integrated use of ground and surface water.

The testing phase gave rise to proposed form and structure of the model. Particular concern was given to maintaining internal consistency, as well as solvability by the revised simplex algorithm.

The percentage of total computer time consumed by each of the three phases was 28, 44, and 28, respectively.

MODEL VARIABLES

The following list gives period-1 variables followed by parentheses containing period-2 variables having the same definition. Those variables which are determined by solution of the model are called "endogenous" variables, and those variables that are fixed prior to the solution process are called "exogenous" variables. Each exogenous variable is indicated by an asterisk following its definition.

SURFACE

X1 (X36) = inflow*
X2 (X37) = undiverted flow
X3 (X38) = water exported
X4 (X39) = evaporation loss from undiverted flow
X5 (X40) = infiltration loss
X6 (X41) = supply
X7 (X42) = initial channel storage*
X8 (X43) = terminal channel storage

SUB-SURFACE

X9 (X44) = rejected recharge
X10 (X45) = inflow*
X11 (X46) = ground-water discharge to surface
X12 (X47) = ground water exported
X13 (X48) = water pumped to the surface
X14 (X49) = natural recharge
X15 (X50) = artificial recharge
X16 (X51) = initial available capacity (*period 1 only)
X17 (X52) = terminal available capacity
X18 (X53) = discharge constant*

SUPRA-SURFACE

X19 (X54) = natural precipitation*
X20 (X55) = effective precipitation
X21 (X56) = loss to deep percolation
X22 (X57) = precipitation runoff
X23 (X58) = induced precipitation*
X24 (X59) = phreatophyte loss
X25 (X60) = total water consumption

AGRICULTURAL-USE

X26 (X61) = interbasin transfer (defines shortages)
X27 (X62) = diversion
X28 (X63) = consumption
X29 (X64) = infiltration

MUNICIPAL-USE

X30 (X65) = diversion
X31 (X66) = consumption
X32 (X67) = infiltration

INDUSTRIAL-USE

X33 (X68) = diversion
X34 (X69) = consumption
X35 (X70) = infiltration

OBJECTIVE FUNCTION

The objective function spans the activities of a single year.

$$2\ X13 - 2\ X15 - 500\ X26 + 4\ X27 + 75\ X30 + 150\ X33 + 4\ X48 - 2\ X50 - 500\ X61 + 6\ X62 + 75\ X65 + 150\ X68 \quad (0)$$

The coefficients indicate approximate value-in-use in dollars per acre-foot. However, sensitivity analysis showed that the model will perform equally well with coefficients that indicate rank or priority ratings instead of dollar values. A dummy interbasin transfer activity was provided so that any irrigation shortages might be computed. The dummy activity was assigned minimum priority via a "rank" of minus 500.

ROW CONSTRAINTS

The row constraints were obtained during phases 1 and 2 of computer processing. Period-1 constraints are identical to period-2 constraints except for the designation of variables, values of time-variant coefficients, and structure of constraints 29 and 58. Exogenous values were input a year at a time by changing right hand sides and by internal transfer between periods.

PERIOD 1

$$X1 = X1(n) \quad (1)$$

provides the means for inputting annual surface inflow data.

$$X1 - X6 + X7 + X11 + X13 + X19 + X23 = 0 \quad (2)$$

is the surface supply component equation.

$$X3 + X4 - X6 + X8 + 0.25\ X13 + X14 + X15 + X20 + 0.5\ X25 + 0.25\ X27 + X31 + X34 = 0 \quad (3)$$

is the surface subsystem balance equation.

$$- X10 + X11 + X12 + X13 - X14 - X15 + X16 - X17 + 0.5\ X24 = 0 \quad (4)$$

is the sub-surface subsystem balance equation.

$$- X1 + X3 - X7 + X8 - X10 + X12 + X16 - X17 - X19 - X23 + X25 = 0 \quad (5)$$

is the total system balance equation.

$$X5 - X9 - X14 + X20 + X29 + X32 + X35 = 0 \quad (6)$$

is the natural recharge component equation.

$$X4 + 0.25\ X13 + X20 + X24 - X25 + 0.25\ X27 + X31 + X34 = 0 \quad (7)$$

is the total water consumption component equation.

$$X1 - X7 - 0.3\ X11 + X15 - 0.5\ X22 + X27 \leq 0 \quad (8)$$

is the agricultural diversion constraint.

$$X7 = X43(n-1) \quad (9)$$

provides the means for inputting terminal channel storage from period 2 of the previous year.

$$0.014\ X3 - X8 = 0 \quad (10)$$

defines terminal channel storage.

$0.8\ X2 - X3 = 0$ (11)
defines surface water exported.

$0.1\ X2 - X4 = 0$ (12)
defines evaporation loss from undiverted flow.

$0.1\ X2 - X5 = 0$ (13)
defines infiltration loss from undiverted flow.

$X16 = X52(n-1)$ (14)
provides the means for inputting terminal available capacity from period 2 of the previous year.

$X11 + 0.09(X16 + X17) - X18 = 0$ (15)
defines ground-water discharge to the surface.

$0.0\ X11 - X12 = 0$ (16)
defines ground-water exported.

$X23 = X23(n)$ (17)
provides the means for inputting induced precipitation, in this case zero.

$X19 = X19(n)$ (18)
provides the means for inputting natural precipitation.

$0.65\ X19 - X20 + 0.65\ X23 = 0$ (19)
defines effective precipitation, and as used here means that 65 percent of precipitation is available to the root zone.

$0.12\ X19 - X21 + 0.12\ X23 = 0$ (20)
defines precipitation loss to deep percolation.

$0.23\ X19 - X22 + 0.23\ X23 = 0$ (21)
defines precipitation runoff.

$0.25\ X13 + 0.32\ X20 + 0.25(X26 + X27) - X28 = 0$ (22)
defines agricultural consumption.

$0.25(X13 + X27) - X29 = 0$ (23)
defines agricultural infiltration.

$0.2\ X30 - X31 = 0$ (24)
defines municipal consumption.

$0.1\ X30 - X32 = 0$ (25)
defines municipal infiltration.

$0.2\ X33 - X34 = 0$ (26)
defines industrial consumption.

$0.5\ X33 - X35 = 0$ (27)
defines industrial infiltration.

$0.5\ X1 - X3 + 0.5\ X22 \leq 0$ (28)
provides for a minimum outflow constraint imposed by a hypothetical, regional water commissioner.

$- X3 + 0.2\ X38 \leq 0$ (29)
provides a flow-balance constraint between periods 1 and 2 in recognition of recreational use.

PERIOD 2

$X36 = X36(n)$ (30)

$X36 - X41 + X42 + X46 + X48 + X54 + X48 = 0$ (31)

$X38 + X39 - X41 + X43 + 0.28\ X48 + X49 + X50 + X55 + 0.5\ X59 + 0.28\ X62 + X66 + X69 = 0$ (32)

$- X45 + X46 + X47 + X48 - X49 - X50 + X51 - X53 + 0.5\ X59 = 0$ (33)

$-X36 + X38 - X42 + X43 - X45 + X47 + X51 - X52 - X54 - X58 + X60 = 0$ (34)

$X40 - X44 - X49 + X56 + X64 + X67 + X70 = 0$ (35)

$X39 + 0.28\ X48 + X55 + X59 - X60 + 0.28\ X62 + X66 + X69 = 0$ (36)

$- X36 - X42 - 0.3\ X46 + X50 - 0.5\ X57 + X62 \leq 0$ (37)

$X8 - X42 = 0$ (38)
provides the means for carrying over terminal channel storage from period 1.

$0.00281\ X38 - X43 = 0$ (39)

$0.92\ X37 - X38 = 0$ (40)

$0.03\ X37 - X39 = 0$ (41)

$0.05\ X37 - X40 = 0$ (42)

$X17 - X51 = 0$ (43)
provides the means for carrying over terminal available capacity from period 1.

$X46 + 0.45(X51 + X52) - X53 = 0$ (44)

$0.0\ X46 - X47 = 0$ (45)

$X58 = 0$ (46)

$X54 = X54(n)$ (47)

$0.35\ X54 - X55 + 0.35\ X58 = 0$ (48)
where 0.35 is the effective precipitation yield.

$0.06\ X54 - X56 + 0.06\ X58 = 0$ (49)

$0.59\ X54 - X57 + 0.59\ X58 = 0$ (50)

$0.28\ X48 + 0.32\ X55 + 0.28(X61 + X62) - X63 = 0$ (51)

$0.22(X48 + X62) - X64 = 0$ (52)

$0.15\ X64 - X66 = 0$ (53)

$0.15\ X65 - X67 = 0$ (54)

$0.2\ X68 - X69 = 0$ (55)

$0.4\ X68 - X70 = 0$ (56)

$0.8\ X36 - X38 + 0.8\ X57 \leq 0$ (57)

$X52 \leq 200{,}000$ (58)
provides a threshold draw-down constraint imposed by a hypothetical, district, water board.

COLUMN BOUNDS

The following table indicates the additional constraints placed on activities in the form of column bounds for period 1 and for period 2. Values are in acre-feet.

Variable	Lower Bound	Upper Bound
X10 (X45)*	300(300)*	1,000(1,000)*
X13 (X48)	-	80,000(150,000)
X15 (X50)	-	150,000(100,000)
X18 (X53)	49,735(248,675)	49,735(248,675)
X24 (X59)	12,000(18,000)	12,000(18,000)
X28 (X63)	60,000(139,860)	73,200(170,940)
X31 (X66)	200(600)	240(750)
X34 (X69)	80(400)	100(500)

*Parentheses pertain to period 2.

VALIDATION RESULTS

Because the system parameters were adjusted to provide the closest possible agreement by comparison to surface outflow for a hypothetical, average year, the model was tested for response to each of the 30 years of record. Period-1 outflow averaged 0.72 percent high, and period-2, 0.16 percent low. Ground-water discharge averaged 3.85 percent high. No pumping or artificial recharge activities were included; and, the only limitation was placed on agricultural diversion, a physical constraint determined by availability of supply.

Ninety percent probability limits were computed for the error measure (simulated outflow - measured outflow)/(measured outflow), assuming a normal distribution with mean and standard deviation estimated from the 30-year record. The probable error limits were (-16.6%, 21.3%) for period 1, (-15.4%, 20.8%) for period 2, and (-17.0%, 21.2%) for the data year.

The Figure 2 plot demonstrates a strong linear correlation between gaged annual outflow and simulated annual outflow. However, the results also show a tendency to over predict for dry years in the 1930's but to under predict for some wet years. Ground-water discharge rates and consumptive-use percentages might next have been adjusted to provide closer agreement to annual outflows. However, since probable experimental error was $\pm$ 20 percent, little could be claimed for further adjustment, and the present model was asserted to be valid for the purpose of simulating ground-water pumping.

SIMULATION RESULTS

During this third (and final) phase, the model was operated with all activities and constraints included. Any (agricultural) water shortage that had existed during any period of the historical record was alleviated by a ground-water pumping activity. In addition, several more low streamflow years were supplemented by pumping to meet minimums imposed by the constraints.

The amounts of ground water pumped were based on irrigation delivery requirements of the present inefficient delivery system. The average annual ground water pumped was 57,321 acre-feet which was much less than the Bureau of Reclamation's estimate of 92,300 acre-feet based on 80 percent effective precipitation and 5 dry years. The maximum simulated requirement was 125,325 acre-feet in 1935. Only once was supplemental water required in period 1- 23,151 acre-feet in 1934. Pumping was required in all but seven years.

A long-run effect (several years) was that pumping noticeably decreased ground-water discharge, a major component of surface flow. However, during a single period of high pumping activity, the amount removed in this manner more than made up the surface loss due to reduced ground-water discharge.

For four consecutive years (1935-1938) the threshold draw-down was reached, yet only once were agricultural requirements supplied at the lower bound (period 1, 1934). Ground-water discharge began to recover at the end of the drouth period in 1927 (refer to Figure 3, center and bottom graphs) continuing on through the early 40's, even though pumping was required until 1944, and was completed when the two records converged in 1948. Again, water pumped in period 2 of 1949 resulted in reduced discharge until ground-water levels returned to normal about three years later. Recovery was short lived for moderate amounts of ground water were pumped again causing the records to diverge. By 1957, the water table (not shown) stabilized well above the draw-down threshold level, and even though pumping averaged 71,287 acre-feet between 1953 and 1960, was rapidly recovering by 1960.

Year-by-year values of the objective function for the two records were subtracted and the difference, or extra value, created by the regulated pumping activity is shown in the top graph of Figure 3. Extra value is positive in all years except 1938, 1941-1943, 1945, and 1955. These six negative values average $10,730 and resulted from pumping to maintain minimum streamflow.

The artificial recharge activity was zero for all but three years (1935, 1936, 1937) following the record dry year, 1934, of the 30-year span. Maximum recharge was 18,402 acre-feet in 1935, trailing off to 13,355 in 1936, and 6,671 in 1937.

Although ground-water discharge showed a remarkable tendency to recover after prolonged periods of pumping, available ground-water capacity at the end of period 1 showed an average decrease of 38,104, thus resulting in an average lowering of the water table.

CONCLUSIONS

The simulation of ground-water pumping demonstrated the dynamic characteristics of the system: its capability of supplementing surface supplies over an extended period of drouth at the expense of only slightly decreased flow during the recovery period; no serious need of artificial recharge at the present level of irrigation development; and average lowering of the water table. Thus, mathematical evidence shows that ground-water pumping is an attractive solution to the double problem of water shortages and rising water tables.

Other items highlighted by the model were (1) simulation of historical streamflows, (2) that the Bureau of Reclamation's pumping estimate is maximum, (3) simulated average ground-water discharge in agreement with U.S. Geological Survey data, (4) derivation of consumptive use coefficients, and (5) a methodology for optimal annual allocation of water resources over a given span of years.

Of perhaps greatest importance is the general relevance of this approach to the simulation of water resource systems. At the present time, this methodology is being used in water planning by the State of Montana [2].

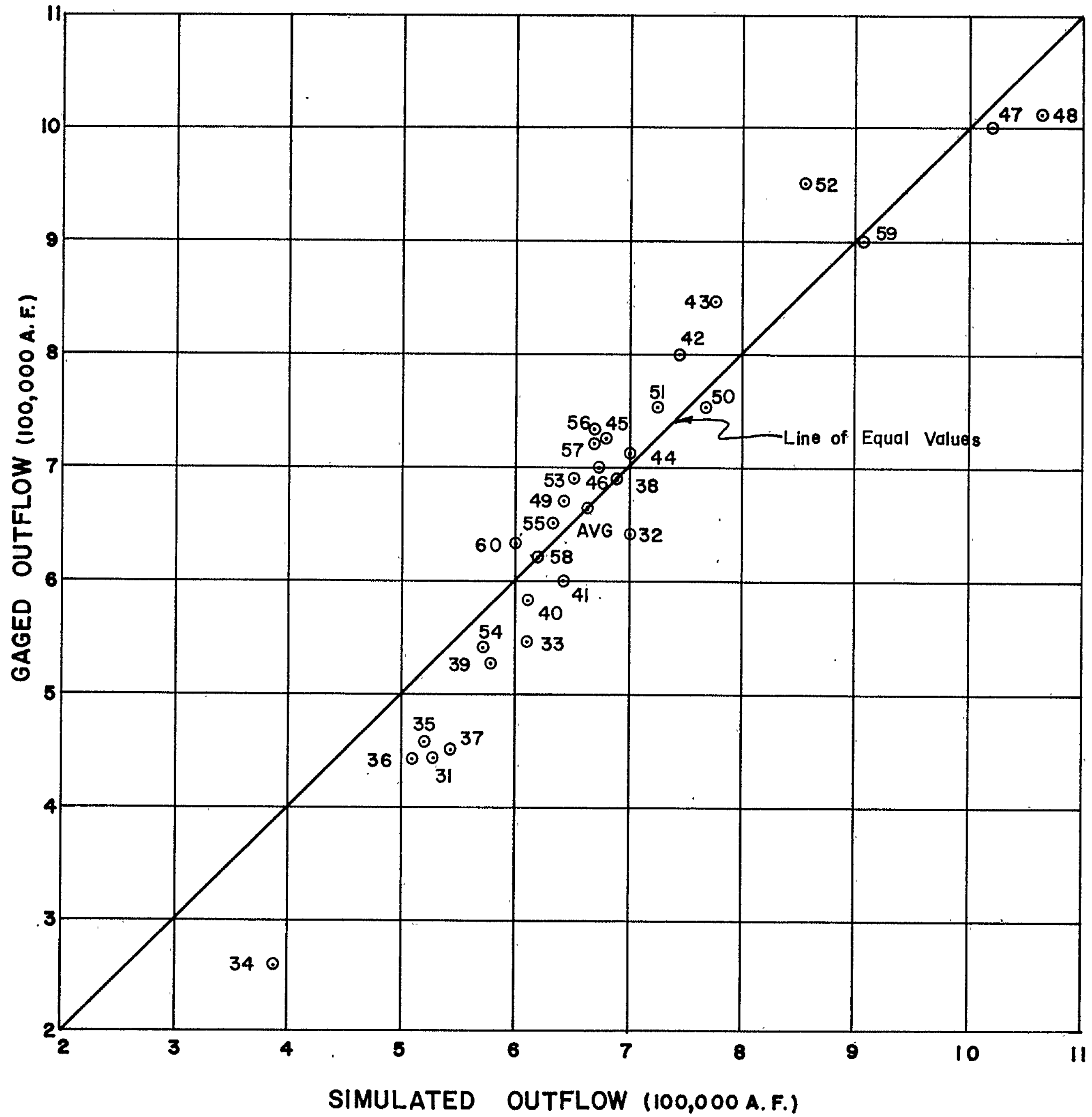

Figure 2. Gaged Outflow versus Simulated Outflow

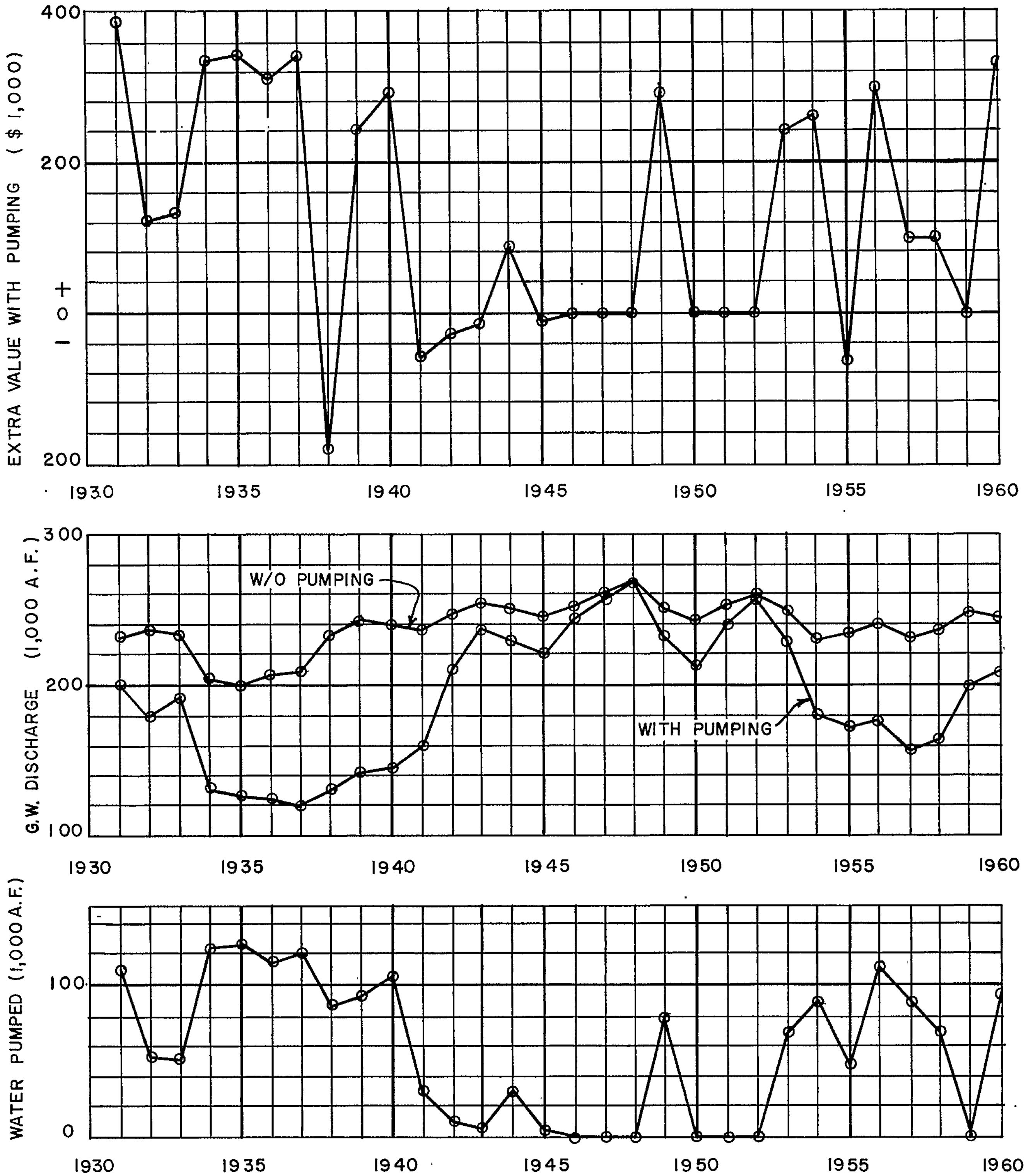

Figure 3. Top: Extra Value vs Year
Center: Ground-Water Discharge vs Year
Bottom: Water Pumped vs Year

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