A SIMULATION OF A CONSTRAINED SECURITIES MARKET

Paul J. Swanson, Jr.[1] and Robert J. Graham[2]

[1]University of Cincinnati

[2]Thomas More College

ABSTRACT

This paper presents a simulated environment of (1) a pure securities market and (2) a market containing a specialist constrained by selected trading rules of the New York Stock Exchange. The purpose of this paper is to consider the price stability attributes of order service, price variance and market efficiency (as defined) in these two short-run (stable) stochastic environments.

After defining the simulation models, the paper examines the influence of the specialist in reducing order service time and the effect of these activities on the specialist's profit and liquidity positions. Market efficiency and price variance are examined by comparing the basic statistical characteristics of the resulting "time" series of prices produced by the two models in both the time and frequency domain.

It is concluded that the specialist dramatically reduces order service time and price variance but that these are achieved only with some loss in market efficiency. It is also suggested that market efficiency is preserved by the existence of both the specialist and institutional constraints on his activities.

INTRODUCTION

In a market characterized by orders with random quantities, prices, and arrival times, it is often suggested that a "mechanism" is needed, usually in the form of a speculator or specialist, that will provide price "stability" by acting against the market. However, the concept of "stability" has assumed several interpretations in the literature. In a theoretic model, Baumol (1, 24-34) has demonstrated that the activity of a specialist can result in a beneficial dampening of short-run price fluctuations. In a more general context, the market attribute of stability in financial folklore is generally defined in terms of continuity, orderly processing of orders, as well as a minimum incremental price change as suggested by Francis (3, 37).

However, the concept of stability needs to encompass more than the achievement of some minimum price variance and order service time. It is suggested by this paper that an efficient market will generate a series of prices which will reflect the diverse judgmental distributions, in Lintner's terms (8, 254-269) of the participants in the market. In terms of the activities of a specialist, this suggests that the basic statistical characteristics of the resulting time series of prices will be preserved in the presence of a specialist. The extent of the similarity of the price series in both the time and frequency domain can be used, therefore, as a measure of the specialists' effect on market "efficiency."

It is the purpose of this paper to consider the price stability attributes of order service, price variance and market efficiency in a short-run (stable) stochastic market. The stationary market will be examined under two separate environments. The first model (Model I) will view the market in its "pure" state without any attempt to artificially match demand and supply. Model II will impose a specialist upon the "pure" state who will operate in the market against the crowd and at his own arbitrary price. Selected operational procedures and trading rules of the New York Stock Exchange are imposed upon the specialist in this model. After defining these models, the paper then considers the attributes of price stability in the context of the generated trading activity and sequence of transaction prices for each model.

TRADING CHARACTERISTICS OF THE STOCHASTIC MARKETS

The markets of Model I and II are defined around Gaussian prices and random (uniform) order type and size. Selected operational procedures and trading rules of the New York Stock Exchange form the basis for the operational heuristics of both models. Since Model II is a more complex version of Model I, these definitions, procedures, and the trading rules of the Exchange will be cited in the earliest model in which they appear and will be assumed to carry over to the next model unless specifically corrected.

The types of orders allowed in both Model I and II are the market and limit orders with stop orders allowed in Model II. Market orders are unsigned orders (unspecified price) but limit orders have bounded prices associated with them as explained in detail in Leffler and Farwell (6, 169-172). These limit prices are generated from a normal distribution ($\bar{x}$=0, var=1) around an average price of $30.00 per share. The resulting prices define a trading range from $27.50 to $33.00 in 25¢ increments for both models.

Order attributes of size, type, and buy or sell are generated from a three digit random number (uniform distribution) as discussed in West (11, 115-126) with an equal proportion of buy and sell orders. Although the Exchange does not keep records of the incoming market/limit order mix, an order proportion of 80% market and 20% limit for both models appears to conform to a previous Exchange study (11). The assignment pattern for order size in trading units of one hundred share lots is defined as one trading unit (40% of orders), 2 trading units (30%), 3 trading units (20%) and 4 trading units (10%). The depository for unfilled limit orders (and stop orders in Model II) is called the "book" whether a specialist exists in the market or not. The unfilled market orders are filed in a special queue.

OPERATIONAL DEFINITIONS OF MODEL I

In Model I, an incoming market order first examines the file of uncompleted market orders. If any opposite market order exists, a transaction takes place, at the previous transaction price. If a part of the market order remains unserviced it seeks the entire opposite side of the book. If any opposite limit order resides in the book, transactions will take place at the limit price until the market or limit order is filled. If the market order remains unfilled, it is filed in its proper sequence in the market order queue.

Incoming limit orders first "look" at the book for possible transaction with an opposite limit order at the incoming limit price or better. Failing to transact with other limit orders, the new limit then investigates the file of uncompleted market orders and transacts at the limit price. If the limit order is still unsatisfied after these operations, the order is finally filed in the book at its limit price. Thus, the market and limit orders in Model I are "good until cancelled" (15, 112) and both take their place in the proper queue based upon their arrival time (i.e., FIFO accounting) (15, 112).

INSTITUTIONAL CONSTRAINTS IN MODEL II

The types of orders allowed in Model II are the market order (80%), limit orders (14%), stop orders (5%), and "missed market information" (1%) with the proportion of order size remaining the same as Model I. The stop order is a deferred market order and is elected for execution only after a transaction has occured at its stop price (6, 172-175). The missed market information represents a floor transaction which occurs beyond the specialist's quote range. He must guarantee to buy or sell during the trading day (defined as the next set of transactions) all book entries that could have transacted at the floor price (15, 113-114).

The specialist in Model II is further constrained by being unable to (1) elect a stop order (15, 127) or enter a stop order for his account (10, 2707), (2) sell short if the transaction price would be below the last different transaction price (10, 2707), (3) buy at a price above the last sale (10, 2701-5) or sell at a price below the last sale price (10, 2702), or (4) transact for his account if he holds an unexecuted market order on the opposite side of the market (15, 126-127).

In this model, the specialist will transact within the highest bid and lowest ask price (his bid-ask quote) as represented by the limit orders in his book (15, 114) subject to the above rules and to his own limitation of buying at or below $29.75 and selling at or above $30.00. The fixed price criteria is included in this model in order to give the specialist a degree of profit motive. The specialist will buy for his own account at or below and sell at or above the previous sale price subject to all the above rules. These transactions are considered to "improve" the market (15, 115).

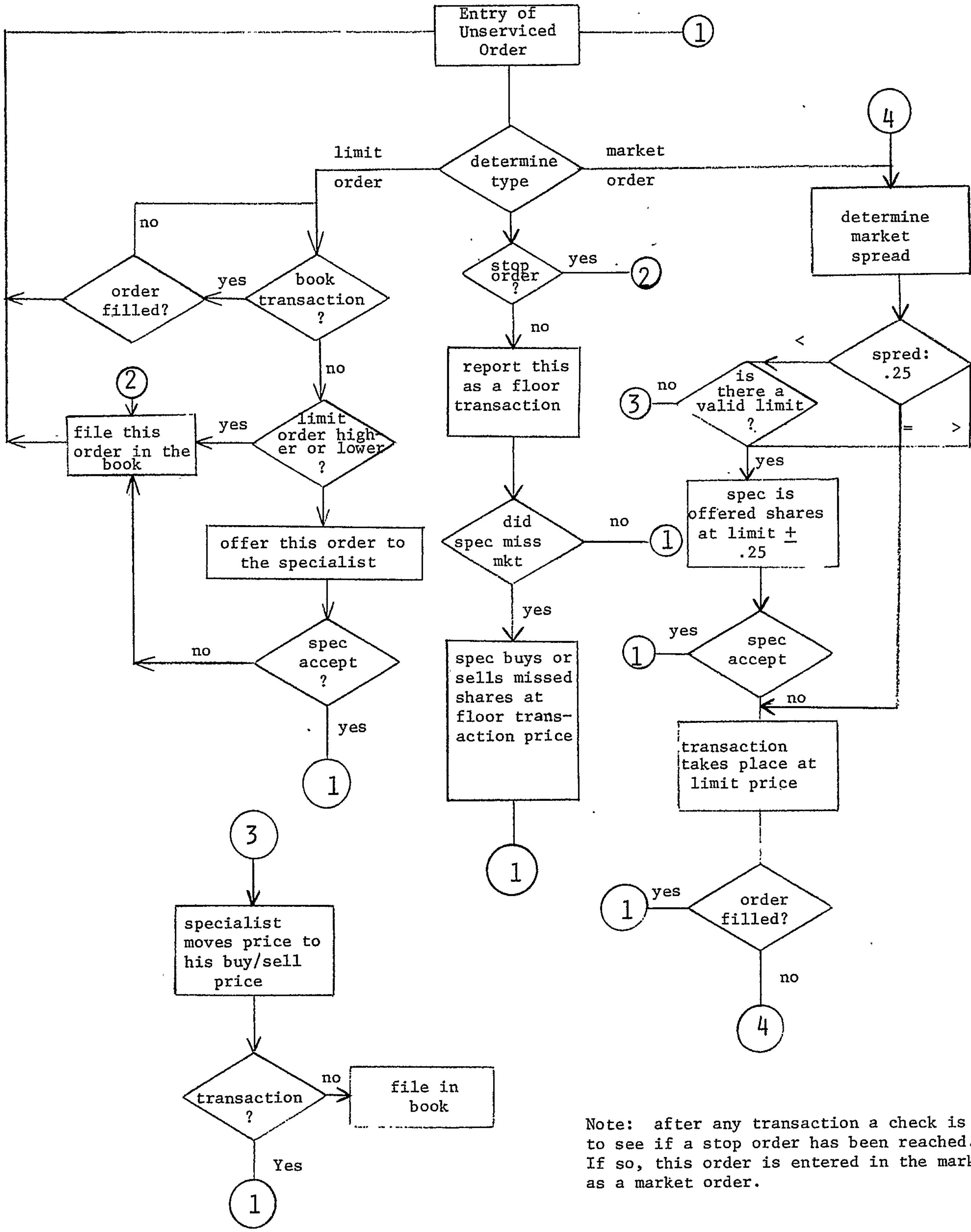
In general, it can be seen that the overall purpose of the institutional rules imposed upon his actions is to prevent him from moving the price to his most preferred position. Subject to his liquidity constraint (explained below), the specialist must operate against the crowd.

OPERATIONAL DEFINITION OF MODEL II

In Model II, an incoming market order first interrogates the market order queue for a transaction. Any unserviced part of the order is then offered next to the specialist as shown in the flow diagram (Figure 1). The specialist responds with the current bid/ask quote and will enter the market only if he can transact between the bid/ask price (improve the market). Thus, the quote spread must be greater than .25 (the smallest price increment) before the specialist can enter the market. If the specialist is prevented from entering the market because of the spread, the market order will transact with the nearest limit order. If the book is clean on the opposite side, the specialist will enter and transact with the market order at the previous price if that price is (i) compatable with his decision rule, (ii) his quote, and (iii) the transaction does not violate the preceding trading rules. If the specialist is prevented from entering the market, the market order is filed in the unfilled market queue.

An incoming limit order first "looks" at the book for an allowable opposite order with which to match at its limit or better. If none are found, the limit is then offered to the specialist at its limit price. The limit is filed in the book at its limit price when the specialist does not or cannot take the order.

Stop orders are entered immediately into the book. They are ignored during the handling of incoming market or limit orders since a transaction must take place at the stop price before they are



Note: after any transaction a check is made to see if a stop order has been reached. If so, this order is entered in the market as a market order.

FIGURE 1
Flow Chart of Model II

elected. After every transaction, however, stop orders are interrogated to determine if the previous transaction has elected them. Elected stop orders become market orders and are treated as above. The specialist also interrogates both the book and the market queue after every transaction in order to locate orders he may now transact but was prevented by the rules from previously accepting.

MARKET ABILITY TO SERVICE ORDERS

In order to analyze both the trading activity and the resulting time sequence of prices, both models were allowed to generate 1000 transactions regardless of the number of orders or potential volume needed. Potential volume is defined as the number of shares that would clear the market is all incoming buy and sell orders were completely matched.

In viewing the results of the trading activity as shown in Exhibit I, it is noted that Model II has

EXHIBIT I
Trading Activities

Model	Potential Volume	% of Potential Volume Cleared	Activity of Specialist: % of Traded Vol Matched	Activity of Specialist: % of Transactions
I	1404	48%		
II	1074	88%	88%	84%

cleared a larger proportion of its potential volume (88%) and required fewer incoming orders (1074) to generate 1000 transactions that Model I (48% and 1404, respectively). In the process of greatly reducing queue time for the average order, the specialist participated in 84% of the transactions and accounted for 88% of the matched volume.

However, the ability of a specialist to continue operating in a market is qualified by the amount of liquidity necessary to perform his functions and the extent of profitability. To examine these possible constraints, liquidity was defined as the total funds less accumulated profit, if any, required to maintain a long or short position. It should be noted that the Exchange requires as an initial minimum capital requirement for a specialist a net liquid asset position of $500,000 or 25% of the position requirements as stated in the rules, which ever is greater (10, 2704-2705). As seen in Exhibit 2 the largest capital requirement for the specialist, net of accumulated profits (before taxes, was $87,575.00.

Even though the specialist misses the market 1% of the time, the book had so few executable orders at any point in simulation time (a "thin" market), that the missed markets caused the specialist no losses. In general, the missed markets either

EXHIBIT 2
Specialist Liquidity and Profitability

Model	Liquidity: Maximum	Liquidity: At End	Profitability: Maximum	Profitability: At End
II	87,575. (29 unit short)	29,750. (1 unit short)	92,750	92,750

did not affect the specialist's profitability or liquidity or <u>increased</u> it since this occurrence allowed him access to orders that he normally would be prevented from executing. Thus a liquidity constraint was never an issue in the simulation runs of Model II.

Profitability was also not a constraint in Model II but became a monotonically increasing function of time. Thus, the ending profit represented the maximum accumulated profits as shown in Exhibit 2. However, any attempts to measure profitability as a function of time would not be meaningful in a simulation context because of the arbitrary relationship between simulation time and calendar time.

In summary, the trading activity from the simulation models suggest strongly that the efficiency of a short-run stochastic market in clearing its potential volume is considerably enhanced by the existence of a specialist.

ANALYSIS OF TIME SERIES

As discussed at the beginning of this paper, the analysis of price stability must include the determination of the basic characteristics of the resulting price series and the likely preservation of these characteristics after the introduction of the specialist. This analysis will include considerations in both the time and frequency domain.

The order mix of 80% market and 20% limit (as adjusted in Model II) causes the price series in both models to exhibit bursts of transactions at the same price followed by a random movement to the next price. Since market orders are unsigned orders and cause the sequence of continuous prices, the movement of the market to different price levels is caused by the execution of the limit orders in Model I and the limit orders or the activity of the specialist in Model II.

If the operation of the market is stable, we would expect that the frequency distribution of prices would approximate the Gaussian distribution which generated the prices. However the existence of a large proportion of market orders would argue against this proposition. At the least we should expect that the two generated price series should be approximately similar in shape. Exhibit 3 presents the price distribution over the trading range. It is noted at the outset that Model I does not appear symetric about the mean price of $30.00 but seems skewed to right. Model II displays a slightly greater skewness to the right, an apparent truncation in the upper range of its distribution and suggests the possibility of

EXHIBIT 3
Original Data Distribution

Model	27.50	27.75	28.00	28.25	28.50	28.75
I	8	6	23	3	22	29
II	0	0	3	1	19	8
Model	**29.00**	**29.25**	**29.50**	**29.75**	**30.00**	**30.25**
I	37	125	84	91	126	99
II	98	80	141	128	123	104
Model	**30.50**	**30.75**	**31.00**	**31.25**	**31.50**	**31.75**
I	115	46	54	40	30	48
II	102	123	23	0	34	13
Model	**32.00**	**32.25**	**32.50**	**32.75**	**33.00**	
I	4	0	4	2	4	
II	0	0	0	0	0	

N=1000

bi-modality. Of interest in Model II is the absence of reported prices at the extreme ends of the trading range. Apparently the activity of the specialist will truncate the tails of the price distribution and thereby reduce the trading range. We would expect to find a lower variance in the prices as a result.

In order to determine if the two price series could have been drawn from the same population, the Kolmogorov-Smirnov two sample test was used on both a two-tailed and one-tailed basis. For a two-tailed test, the procedure, as discussed by Siegel (12, 127-136) computes the maximum absolute difference (D_{max}) between two observed frequency distributions. The critical value for this test at the .001 significance level ($n_1=n_2=1000$) is a maximum absolute difference of .08. Since the computed absolute D_{max} is .11, it is apparent that the two price series could not likely have been drawn from the same population.

It is of interest to determine if the specialist causes prices to be lower than those of the "pure" market. Siegel (12, 131) has noted that the signed D_{max} can be used in a one-tailed test since

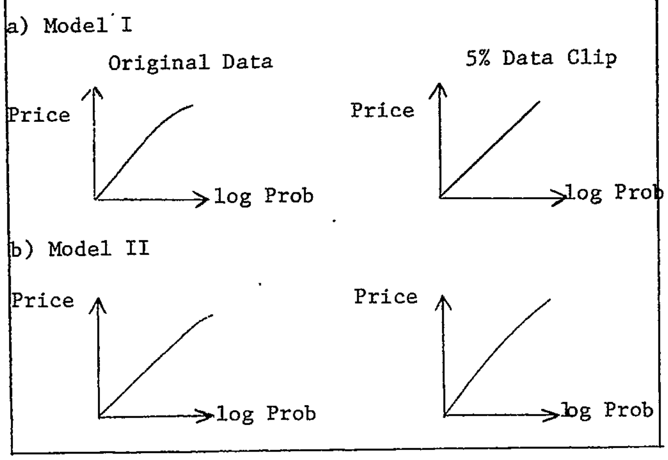
$$x^2 = 4D^2_{max} \frac{n_1 n_2}{n_1+n_2} \qquad (1)$$

has a sampling distribution approximately chi-square with 2 degrees of freedom. Since the critical value of x^2 at the .001 level is 13.8 (df=2) and the computed x^2=27.0 using equation (1), it is concluded that the prices generated by Model II are stochastically lower than Model I. Because both models share the same mean price ($30.00), this result further supports the existence of a lower variance in Model II.

In order to perform further tests on the two time series, it is necessary to determine if either or both series could reasonably be considered to be a realization of some Caussian process. The sequence of tests that were applied to these price series are briefly described below. To increase the efficiency of the tests, the price series from Model I and Model II were first randomized before the tests were applied.

The price data for both models were first used to construct a cumulative frequency distribution which was then plotted on a semi-log scale. The resulting graph is shown in Exhibit 4 and both

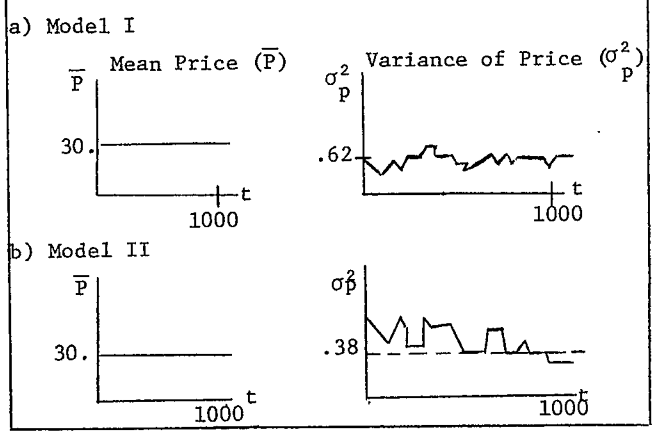
EXHIBIT 4
Semi-log Cumulative Distribution



series display the now familiar "S" curve suggesting the likelihood of infinite variance. The data was then clipped 2.5% from each end of the frequency distribution and the values eliminated were replaced by the mean of the remaining elements as suggested by Granger and Orr (4, 281). This procedure resulted in an approximately 5% data clip on both series. The resulting cumulative frequency distributions are shown also in Exhibit 4 and appear to be suitably linear.

After this transformation, the mean and variance were computed on the randomized clipped data for successively larger sequences of prices all starting from the same base, a procedure often called the "converging variance test" (4, 276-277). The results of this test are displayed in Exhibit 5

EXHIBIT 5
Convergence of Mean and Variance After Data Clips
(Randomized Data)



which show that the means of both time series are extremely stable. Both series display a tendency to dampen down after a large number of observations, a characteristic of a population with finite variance. This was not true with the original data which tended to occilate with large N ($1<N\leq 1000$) for both series. We conclude from this analysis taht the approximately 5% data clip has produced in both time series a stable mean and ample evidence of a finite population variance.

As shown in Exhibit 5, Model I produces a variance around .62 which was true in the original data. However, the data clip lowered the variance in Model II from about .49 to around .38. This was probably caused by the elimination of the high number of prices in the upper frequency. However, the variance as computed in the converging variance test for Model I always exceeded that of Model II for all sequences of N in both the original and clipped data. Thus, the lower variance characteristic of Model II in the original data is preserved in the clipped data.

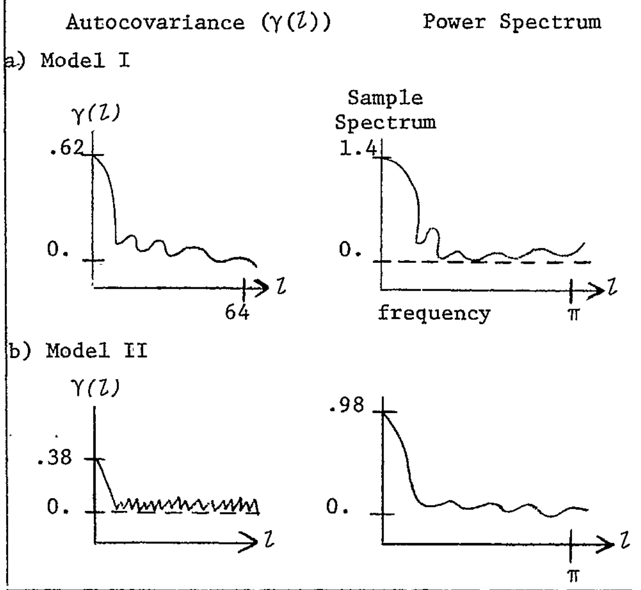
Another procedure suggested by Fama (2, 60-69) requires the calculation of the characteristic exponent α for a stable Paretian distribution. An exponent in the range $1<\alpha<2$ would be consistent with his finding (2, 65-67) while an exponent $\alpha=2$ would strongly suggest a Gaussian process. Using a base N=600 and utilizing the sample converging variance, the results (not shown) for Model I displayed an α probably slightly higher than 2 although its behavior was highly erratic and for Model II tended to move around 2.

In summary, there is sufficient reason to believe that the clipped data possesses a finite variance in both models. Granger and Orr (4, 282-283) have shown that spectral analysis is sufficiently robust to be useful for series possessing the same basic characteristics as the clipped data, namely, stability in the running means and variances and a reasonably linear cumulative frequency distribution plotted on semi-log scale.

The purpose of the spectral and autocovariance analysis is to determine if the specialist in Model II imposed or removed any periodicities in the basic structure of the time series. Any change in the structure of the serial dependence of the time series would constitute prima facie evidence that the specialist is interfering with the efficiency of the market. Since the "pure" market in Model I interferes with this transferral of information due to the 80/20 order mix, the following analysis will concern itself only with the difference between the series in Model I and Model II and not with a purely theoretic model.

The autocovariance ($\gamma(l)$) for lag $0\leq l\leq 64$) is shown in Exhibit 6. It should be noted that the autocovariance of Model I declines to zero and then shows a tendency to cycle around this

EXHIBIT 6
Autoregressive Structure



value while the autocovariance of Model II declines to zero much more quickly and tends to dampen down almost immediately. A plot of the autocorrelation structure of the series is not necessary since it is the ratio of the autocovariance at lag l to lag 0 and would display in this case the same graphic representation. The rapid decline in the autocovariance is consistent with the bursts of transactions followed by random movements to the next price as mentioned earlier. These graphs suggest that the bursts of transactions are longer, in general, for Model I than for Model II since the specialist will intervene in the market within his quote range whenever allowed. Thus the pattern of serial dependence as displayed by the autocovariance is the same between Model I and Model II but the specialist has reduced slightly its degree and duration.

The power spectral estimates ad discussed in (5 and 7, 879-889) are shown in Exhibit 6. The same difference appears to exist between Model I and Model II in the frequency domain as in the time domain. Again the pattern of the spectral estimates appears to be similar between the models but the power over the frequency range is reduced. It should be noted that both series present non-zero spectral estimates only in the low-order frequency range. Both series can then be characterized by a sinusoidal wave of long duration with only a few low-order harmonics contributing to the variance of the series. This description of the price series is especially apt for Model II and is true in general for Model I. Model I, however, does display a secondary peak but not a spike around a frequency of 7 which has disappeared in the estimates of Model II. The remaining frequencies appear as low-order white noise.

The shape of the spectral estimates in both models suggest that (1) the series are a Dirac delta function (5, 29-30) or constant, (2) the series can be characterized completely as a linear trend, or (3) the series can be characterized as a low-order autoregressive process. The first explanation can be disposed of since a Dirac function has infinite variance which contradicts all the preceding tests. To determine if a linear trend exists in the series the sequence of clipped prices were detrended and spectral estimates were again computed. This resulted in a pattern of spectral estimates with the same shape as that displayed in Exhibit 6 but at considerably reduced power. The secondary peak in Model I was retained.

Cross spectral analysis demonstrated that the correlation (squared coherence) between the two series was low but that they are in phase (phase=0.0) at the fundamental frequency. Although these results are consistent with the previous analysis, reasonable measurements of coherence and phase angle are difficult to achieve (5, 406-407) and space considerations require that these results be only suggestive support. Additional analysis was performed on the spectral estimates using a test suggested by Nalor (9, 259-263) to determine if the ratio of spectral estimates for the two models was the same. This test confirmed their differences in amplitude. The preceding analysis was also performed on the first differences and log-first differences of both data but these transformations failed to reveal additional information.

In summary, it is concluded that the time series of prices in Model I and Model II can be characterized by a low-order autoregressive process with a linear trend. The activity of the specialist caused the removal of some information from the "pure" series, namely a secondary peak in the low-order harmonics and a lowering of the power of the low-order estimates. However, it is noted that the specialist did not either induce periodicities that did not exist before nor did he substantially change the general characteristics of the time series.

CONCLUSIONS

From the preceding analysis, the price stability attributes of order service, minimum price variance and market efficiency were reasonably demonstrated by the existence of a specialist acting under institutional constraints. It is quite apparent that the specialist will greatly improve the ability of the market to clear its potential volume and achieve a significantly lower price variance. However, improved order efficiency and lower price variance were achieved at some cost due to the loss of information content in the price series. The authors do not believe that the information loss was great since both series could be similarly characterized as low-order autoregressive processes, and hence should offer about the same ability to forecast. However, this loss will be the subject of future research.

Finally, the role of the institutional constraints needs to be emphasized. In a previous paper (13), the authors demonstrated that an unconstrained specialist in a completely random environment can remove all information content from the sequence of transaction prices. That any information exists in the prices Model II must therefore be considered largely the result of the institutional constraints imposed upon the specialist. It is therefore concluded that a market characterized by price stability as defined by this paper must contain a specialist and enforceable rules of operation which limit or prevent the specialist from moving the price toward his preferred position.

BIBLIOGRAPHY

1. Baumol, W. J., The Stock Market and Economic Efficiency, The Millar Lectures, No. 6, 1965, (New York, Fordham University Press, 1965.

2. Fama, E. F., "The Behavior of Stock-Market Prices," The Journal of Business, Vol. 38, No. 1 (January, 1965).

3. Francis, J. C., Investments: Analysis and Management, (New York, McGraw-Hill, 1972).

4. Granger, C.W.J., and Orr, D., "'Infinite Variance' and Research Strategy in Time Series Analysis," Journal of American Statistical Association, Vol. 67, No. 338 (June, 1972).

5. Jenkins, G. M., and Watts, D. G., Spectral Analysis and Its Application, (San Francisco, Holden-Day, 1968).

6. Leffler, G. L., and Farwell, L. C., The Stock Market, 3rd Ed., (New York, The Ronald Press, 1963).

7. Lenthold, R. M., "Random Walk and Price Trends: The Live Cattle Futures Market," Journal of Finance, Vol. 27, No. 4 (September, 1972).

8. Lintner, J., "Dividends, Earnings, Leverage, Stock Prices, and The Supply of Capital to Corporation," The Review of Economics and Statistics, Vol. 44, No. 3 (August, 1962).

9. Naylor, T. H., Computer Simulation Experiments with Models of Economic Systems, (New York, John Wiley & Sons, 1971).

10. New York Stock Exchange, Inc., Constitution and Rules, May 1, 1972, (Chicago, Commerce Clearing House, 1972).

11. Phone interview with Mr. Norman Miller, Assistant Director, Research Department, New York Stock Exchange, February 15, 1973.

12. Siegel, S., Nonparametric Statistics for the Behavioral Sciences, (New York, McGraw-Hill Book Company, 1956).

13. Swanson, P. J., and Graham, R., "The Concept of Market Efficiency under a Constrained Specialist: A Simulation Study," paper presented at the MIDWEST FINANCE ASSOCIATION CONFERENCE, Chicago, Illinois, April 16, 1973 (unpublished).

14. West, R. R., "Simulating Securities Market Operations: Some Examples, Observations and Comments," Journal of Finance and Quantitative Analysis, Vol. 5, No. 1 (March, 1970).

15. Work of the Stock Exchange and Brokerage Office Procedure, (New York, New York Institute of Finance, 1969).

ACKNOWLEDGEMENTS

The authors are indebted to Mr. Arnold Kotler, Executive Assistant to the Vice President, Floor Department, New York Stock Exchange for his helpful comments concerning the floor operations and specialist trading rules. Of course, the responsibility for the interpretation of these operations and rules as discussed in this paper is solely that of the authors and not intended to reflect that of any member of the Exchange.