REALISTIC ROAD-TRACK SYSTEMS SIMULATION USING DIGITAL COMPUTERS

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ABSTRACT

In order to realistically characterize guideway inputs to nonlinear vehicle models, it is necessary to generate a random process of specified spectral content using the digital computer.

The detailed procedure of constructing a realistic track system using a digital Random Number Generator (RNG) is presented.

The procedure begins with interpreting the real track profile Power Spectra Density (PSD) as a straight line in log-log representation. The RNG is then used to simulate the road roughness with desired white velocity characteristics. In addition the results of the Central Limit Theorem are used to construct a Gaussian white random sequence. The necessity of filtering the data is discussed and actual digital filtering is presented.

The entire procedure is paralleled by an example where a guideway with roughness of a conventional runway is simulated as the input to a high speed ground transportation vehicle.

NOMENCLATURE

$a, \Delta T$	time interval associated with RNG output
A	constant associated with two sided road roughness PSD
A'	constant associated with one sided road roughness PSD
DFT	Discrete Fourier Transform
E[]	expected value
f	frequency (Hz)
f_B	filter break frequency
f_C	filter cutoff frequency
FFT	Fast Fourier Transform
G	one sided PSD
H_{BP}	band pass filter transfer function
L	length of primary suspension
M_1	unsprung mass
M_2	sprung mass
PSD	Power Spectral Density
R	autocorrelation function
RMS	Root Mean Square value
S	two sided PSD
SAD	Semi-active damping
S_o	two sided "white" input velocity PSD
t	time
T	time increment
TACV	Tracked Air Cushion Vehicle (later renamed Tracked Levitated Research Vehicle)
V	vehicle forward velocity
V_1	unsprung mass vertical velocity
V_2	sprung mass vertical velocity
x	horizontal displacement along the roadway
X	total length of the roadway sample
y_o	roadway roughness
ξ	space lag
λ	wavelength
τ	time lag
ω	time frequency
Ω	space frequency
"'"	designates space derivative
"."	designates time derivative
< >	designates mean value

INTRODUCTION

The interest in vibration analysis using random process theory arose first in the aerospace field. With time, the statistical approach to the description of road surfaces gained wide acceptance in various areas such as automotive engineering [1], agricultural machinery [3], railway engineering [4], and finally high-speed ground transportation [6].

While linear systems allow relatively simple statistical analyses in the frequency domain, nonlinear system analysis tends to be restricted to the time domain. This necessitates the construction of realistic random time functions for use as input to the system. Whenever it is justified, linearization should be performed; but there are definitely systems where linearization is not possible. One striking example is semi-active control which will be discussed later. In these cases, in order to simulate actual roadway inputs to any dynamic system, it is necessary to characterize the road shape by a stationary random spatial function, $y_o(x)$. A number of statistical properties of y_o can then be defined.

The autocorrelation function $R(\xi)$ is given by

$$R(\xi) = E\,[y_o(x)\; y_o(x + \xi)] \tag{1}$$

where E designates the ensemble average. If ergodicity is also assumed the following relation holds for spatial autocorrelation

$$R(\xi) = \lim_{X\to\infty} \frac{1}{2X} \int_{-X}^{X} y_o(x)\ y_o(x + \xi)\ dx \qquad (2)$$

Defining "space frequency", Ω, by means of wavelength, λ, as

$$\Omega = 2\pi/\lambda,$$

the Power Spectral Density (PSD) becomes the Fourier transform of $R(\xi)$.

Thus $$S_{y_o}(\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\xi) \exp(-i\Omega\xi)\ d\xi \quad . \quad (3)$$

As a vehicle traverses this roadway with horizontal velocity, V, the time dependence of the spatial PSD is related through $\omega = V\Omega$

Thus $$S_{y_o}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{y_o}(\tau) \exp(-i\omega\tau)\ d\tau$$

$$R_{y_o}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} y_o(t)\ y_o(t + \tau)\ dt \qquad (4)$$

where

$$y_o = f(x) = f(Vt) = y_o(t)$$

and τ is now the time lag, $\tau = \xi/V$. Using equations (1-4), the following spectrum relations hold for the displacement and velocity PSD:

$$S_{y_o}(\omega) = \frac{1}{V} S_{y_o}(\Omega)$$

$$S_{\dot{y}_o}(\omega) = V S_{y'_o}(\Omega) \qquad (5)$$

where $y'_o(x)$ is the local slope of the road surface.

Finally, before starting with the road and model description, it is worth emphasizing the advantages offered by Gaussian or normal random sequences. For Gaussian processes, specification of the PSD function (or its inverse Fourier transform) determines the probability density function. Thus the availability of experimentally obtained PSD data is sufficient for complete probabilistic description of the input sequences [9]. Aside from its analytical advantages the assumption of the Gaussian random process is physically justified when the roadway random function is considered as the superposition of many random samples generated along adjacent lines parallel to the vehicle travelling direction. For example, in the case of an automotive vehicle, the superposition might be correlated with the width of the tire.

In the following sections, an approximately "white" random velocity function is "constructed" on the digital computer. This random sequence is subsequently used as input to a fully nonlinear heave mode model of a tracked air cushion vehicle (TACV).

THE TACV MODEL

The TACV is a proposed high speed (up to 300 m/h) ground transportation vehicle which is being considered as one solution for future mass transportation. The vehicle, shown schematically in Figure 1, consists basically of a plenum air cushion (primary suspension) and an inflated airbag secondary suspension. The static pressure in the plenum chamber provides the lift force which literally floats the vehicle. Although vehicle-roadway friction is negligible, the air cushion itself is too stiff to provide passenger confort and safety. Thus a secondary suspension is necessary.

In order to develop "semi-active" secondary suspensions for this vehicle, a fully nonlinear heave mode model was developed with provision for velocity guideway inputs at the base of the plenum chamber. The development of this model and associated semi-active secondary suspension characterization are fully documented in References [5], [6], and [11] and will not be pursued here. Bond graph techniques [12] were followed extensively with the result being a set of first order nonlinear state space equations which fully characterize the nonlinear heave mode dynamics of the TACV subject to random velocity inputs. The construction of this random sequence is the primary emphasis of this paper.

REAL ROAD-TRACK SYSTEMS CHARACTERIZATION

From experimental roughness measurements of different road-track systems, spatial PSD's of various road surfaces have been obtained. Figure 2, taken from Reference [3], is characteristic of these types of measurements. Data are available for a large number of roadway configurations. For the majority of the curves it can be seen that the spectra appear to be of the form

$$S_{y_{o(x)}}(\Omega) = A\Omega^n \qquad (6)$$

where

$$n \simeq -2.$$

This implies that most of the spectra are fit reasonably well by a straight line on a log-log plot (this line having 2:1 slope as is shown in Figure 2). The desirable consequence of roughness PSD being of the form $S_{y_{o(x)}}(\Omega) = A/\Omega^2$ is that slope PSD, $S_{y'_o}(\Omega)$, and input vertical velocity PSD, $S_{\dot{y}_o}(\omega)$, are now white, i.e., using Equation (5)

$$S_{y'_o}(\Omega) = A \qquad (7)$$

and

$$S_{\dot{y}_o}(\omega) = AV. \qquad (8)$$

White noise vertical velocity input has several advantages for testing system response. It allows relatively simple analytical treatment, provides uniform system excitation over a broad spectrum of frequencies, and thus allows the determination of each mode contribution to the overall system dynamics. Finally, and of particular interest for the purpose of this investigation, white noise is easily simulated using digital computer hardware.

IMPLEMENTATION OF FILTERED WHITE VELOCITY SEQUENCE USING THE DIGITAL COMPUTER

The presence of a Random Number Generator or RNG is commonplace on most large computers. Even some pocket calculators have built in RNG. The question arises, how is a RNG sequence processed in order that a meaningful representation of a particular random process can be obtained? Namely, how does one obtain "white" ground velocity input with a Gaussian distribution?

Since the computer representation of a stochastic process is necessarily discrete, this representation may be may be interpreted as a sample and hold operation on the continuous process (see Figure 3). This representation is justified since the operations of filtering and Fourier transformation bear the same results for both digital and continuous signals at the sampling instant. Next, due to the uniform probability density distribution of the RNG sequence, it will be assumed that all random numbers are uncorrelated. Thus, for a discrete time sequence composed of these random numbers the following autocorrelation function applies

$$R(\tau) = \langle \dot{y}_o^2 \rangle [1 - \frac{|\tau|}{a}] \text{ for } |\tau| \le a \qquad (9)$$

and $$R(\tau) = 0 \text{ for } |\tau| > a$$

where $a = \Delta T$ is the time increment associated with data output (Figure 4). The Fourier transform of this autocorrelation function divided by 2π yields the two sided PSD (Figure 4) as

$$S(\omega) = \frac{a\langle \dot{y}_o^2 \rangle}{2\pi} \frac{\sin^2(\pi f a)}{(\pi f a)^2} = \frac{a\langle \dot{y}_o^2 \rangle}{2\pi} \text{sinc}^2 \text{ (af)} \tag{10}$$

where $f = \omega/2\pi$.

For a finite record length, the given formulas apply approximately which means that there will be some correlation even for $|\tau| > a$. Moreover the PSD is then given by the convolution of the Discrete Fourier Transform (DFT) of the autocorrelation function and a rectangular window. As can be seen from the actual computation based on 10,000 data points (Figure 5) it is reasonable to neglect these effects.

From Equations (9) and (10) it is evident that sufficient control is obtained upon the "whiteness" of the velocity PSD function and its mean square value

$$\langle \dot{y}_o^2 \rangle = \frac{2\pi S_o}{a} \tag{11}$$

Here S_o is the desired ideal white noise velocity PSD. According to Equation (11) for the case of TACV, S_o was determined by choosing the vehicle traveling speed and road roughness PSD.

Thus $S_o(\omega) = 1.1 \cdot 10^{-3}$ ft^2/sec

for $V = 440$ ft/sec

and $A' \simeq 5. \cdot 10^{-6}$ ft (see Equation 8)

A' was chosen from Figure 2 for a good runway. This value for A' was then halved since data from Figure 2 are experimentally obtained and therefore valid for the one sided PSD. Hence $A = A'/2 = 2.5 \cdot 10^{-6}$ ft.

Once S_o has been fixed, the sampling interval is then merely chosen so as to insure sufficient "whiteness" of $S(\omega)$. Setting $a = 0.001$ sec in Equation (11) corresponds to the sequence being 87.5% "white" at the frequency of 200 Hz; namely

$$S(f = 200 \text{ Hz}) = 0.875\ S(f = 0) \ .$$

The mean square value is then

$$\langle \dot{y}_o^2 \rangle = 6.9 \text{ ft}^2/\text{sec}^2 \ .$$

At this point, the random sequence uniformly distributed between 0.0 and 1.0 as generated by the RNG could be rescaled to the desired $\langle \dot{y}_o^2 \rangle$. However, as mentioned previously, a Gaussian distribution was desired. Using the results of the Central Limit Theorem [10] a Gaussian probability distribution was constructed by summing twelve successive random numbers and adjusting the result to a zero mean value with the desired RMS value. Physically this corresponds to an averaging process correlated with the width of the primary suspension (in the case of the TACV, averaging is produced by the air cushion in the direction lateral to vehicle motion).

Before applying the velocity sequence obtained thus far it is necessary to low pass filter the "high" frequency content of the signal. This operation is both physically justified and numerically essential. Physically, it is not necessary to include input frequencies significantly higher than the characteristic frequencies of the dynamic system being studied. For nonlinear systems the high frequency cutoff should be extended somewhat beyond what would normally be dictated by linear considerations. Exactly how far beyond depends on the nature of the nonlinearities being investigated. In addition, for the air cushion vehicle traversing a guideway at velocity, V, input wavelengths shorter than the vehicle longitudinal length, L, would not be appropriate for a lumped heave mode model. This would dictate a high frequency cutoff, f_c, on the order of V/L.

Numerical justification for filtering high frequencies is due to sampling data at only discrete finite intervals of time, T, which is greater than the sampling time for RNG output. Thus frequencies higher than 2/T would be misinterpreted as lower frequencies thus producing erroneous low frequency results. This pheonomenon is known as "aliasing".

At the low frequency end of the spectrum, in order to avoid the inconvenience of non zero mean input displacements, a high pass filter is introduced with filter break frequency, f_B, chosen so as not to affect the system dynamics. In the particular example of TACV, f_B was chosen as 0.2 Hz and f_c as 35 Hz. A fourth order filter, which resulted from cascading a low and high pass filter was designed using phase space representation [14]. The state equations were then discretized and integrated using matrix exponentials [8]. The final form of the vehicle input velocity PSD is qualitatively presented in Figure 6.

The actual PSD was calculated using the Fast Fourier Transform technique [2,7] and the result is presented in Figure 7. To avoid excessive random error due to finite averaging intervals, frequency smoothing was performed according to [7]. The smoothed spectrum (small figure in the higher right corner of Figure 7) reflects the desired "whiteness" in the frequency range from 0 to 30 Hz which is adequate for the Tracked Air Cushion Vehicle.

The filtered "white" velocity record of data was separated by a sampling interval of a = 1.0 ms, and is now ready to be used as the vehicle input. If necessary, any inputs within the sampling interval can be approximated using linear interpolation.

The nonlinear set of differential equations describing the TACV was solved using the HPCG Subroutine from the IBM/SSP Package [15]. As an illustration of the results, time histories of input and output sequences are presented in Figure 8 and sprung mass velocity PSD's for two different damping schemes are given in Figure 9. The simulation was carried out for the TACV traversing the guideway described previously with forward velocity of 300 m/h. In order to avoid transient effects it was necessary to perform the simulation for a period of 1 sec prior to computing any output statistics.

CONCLUSIONS

A method of "constructing" a realistic road-track stochastic process for use as input to any dynamic system was developed using a Random Number Generator. The procedure was demonstrated for the particular case of a nonlinear TACV. For this case a random sequence of 10,000 processed numbers was used and required about 40 sec of Burroughs 6700 computer time. At current commercial computer rates, this costs less than $3.00.

The method yields the velocity-time histogram of an assumed stationary process. Virtually any kind of roadway can be accurately represented by using proper filtering. This type of modeling has far reaching implications with respect to specifying "ride quality" for ground transportation systems.

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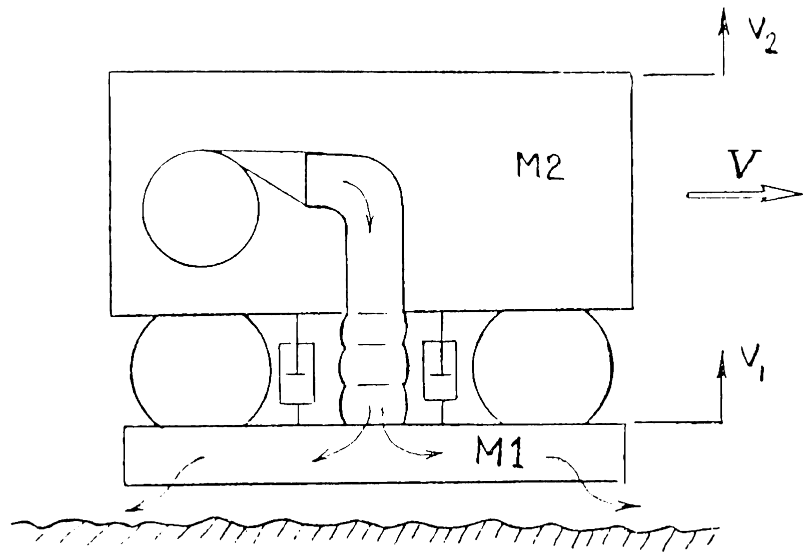


Figure 1 Tracked Air Cushion Vehicle Schematic

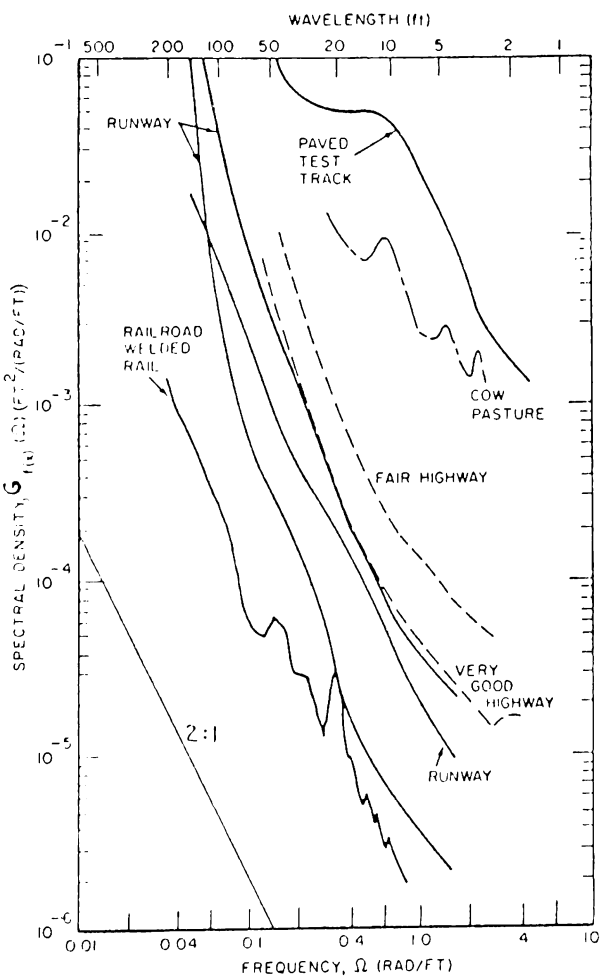


Figure 2 Power Spectral Densities of Various Terrains

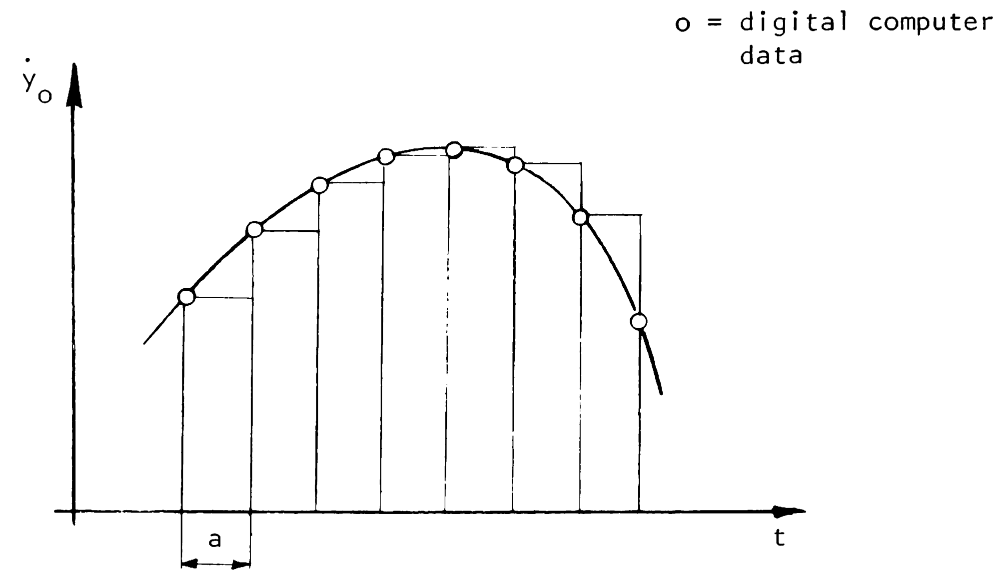


Figure 3 Continuous Time Representation of Digital Data

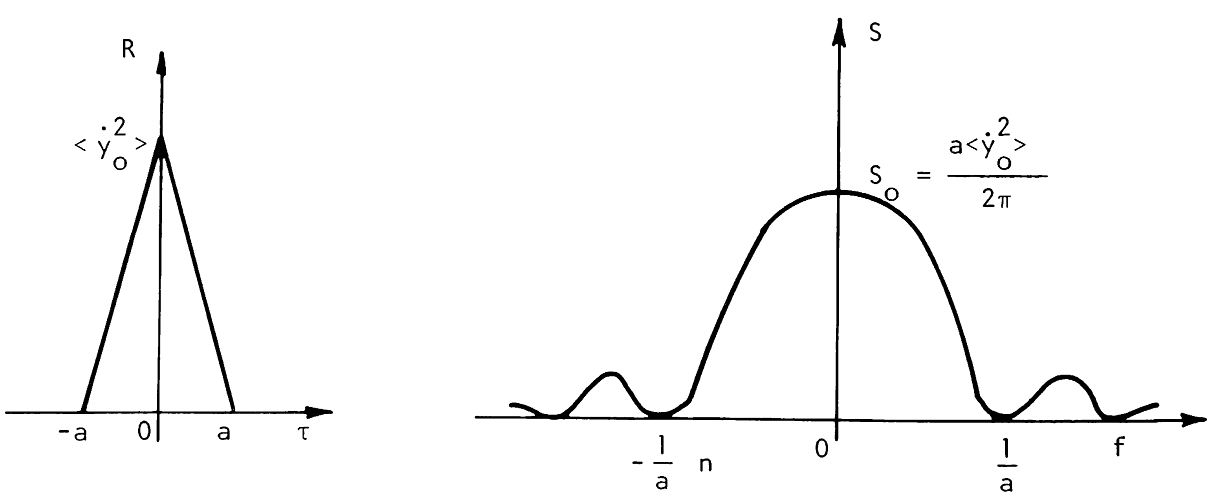


Figure 4 Autocorrelation and PSD of Infinite, Continuous time sequence which Corresponds to RNG Output

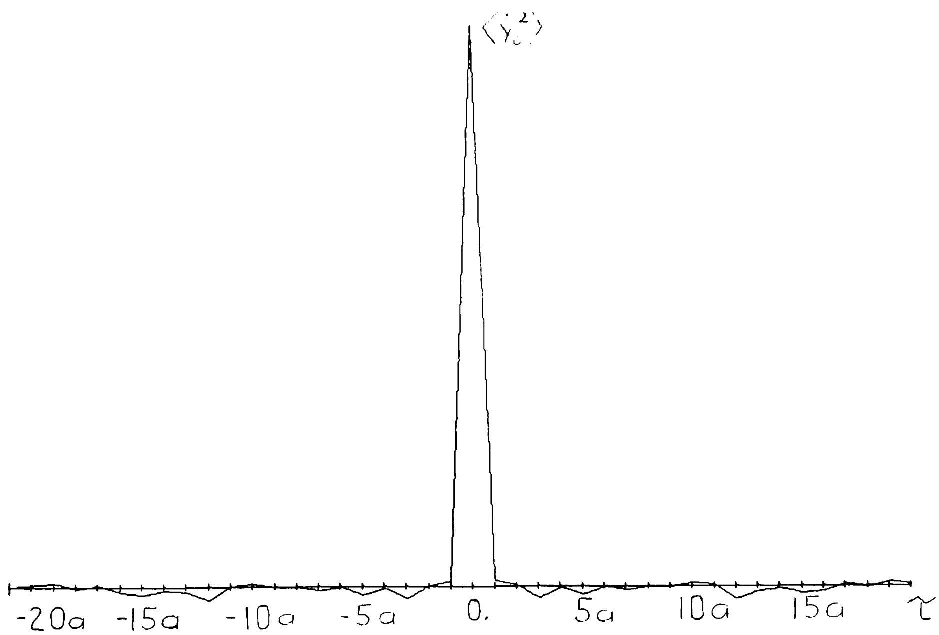
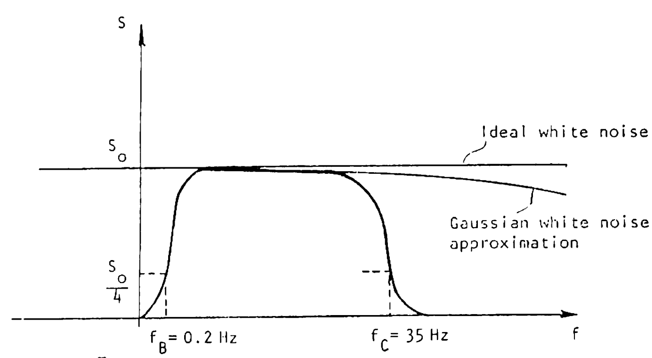


Figure 5 Actual Autocorrelation of RNG Sequence



$$\langle \dot{y}_o^2 \rangle = S_o \int_{-\infty}^{\infty} |H_{BP}(\omega)|^2 d\omega$$

Figure 6 Spectral Density for Band-Limited "White" Noise

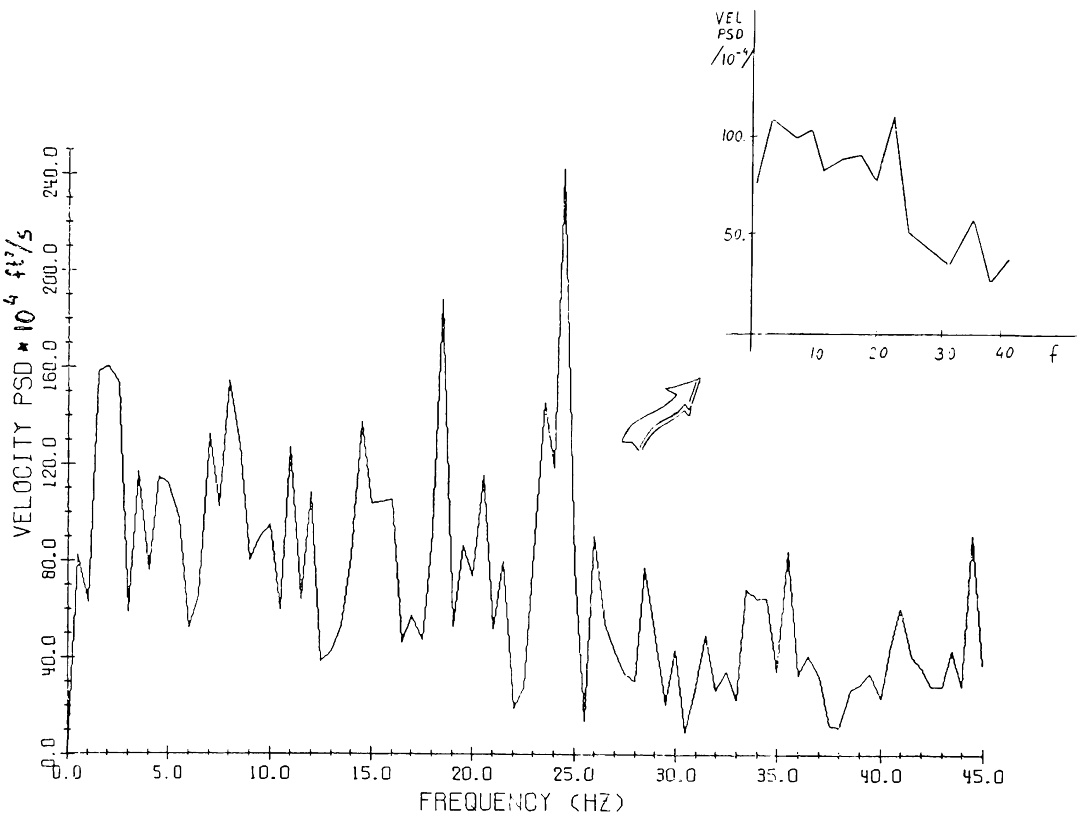


Figure 7 Actual PSD's Obtained from Filter Output

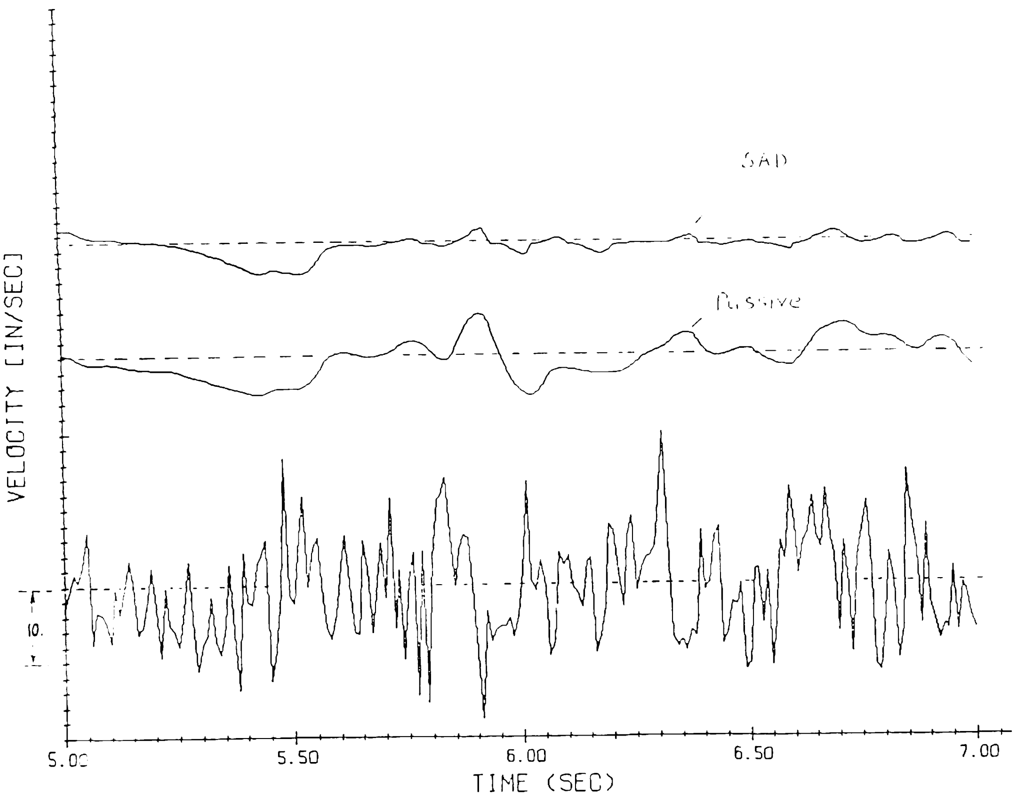


Figure 8 Time Histories of TACV Input and Output Sequences

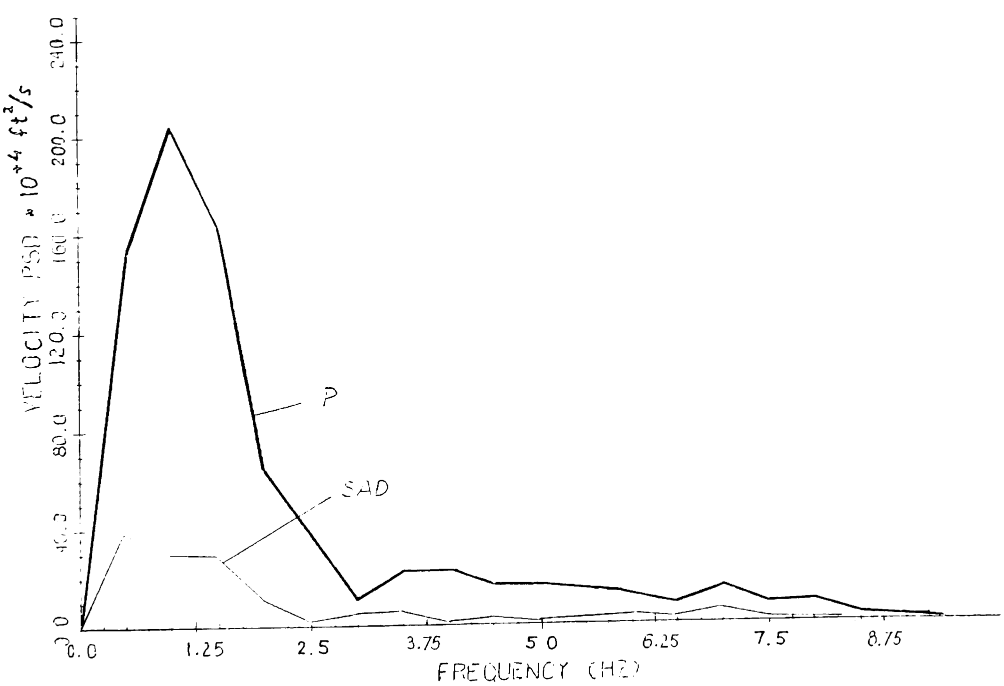


Figure 9 Sprung Mass Velocity PSD for Passive and Semi-Active Secondary Suspensions