THE BUNCHING OF BUSES ON A ROUTE

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INTRODUCTION

Intracity bus riders have at one time or another experienced the frustration of having several buses arrive together at a designated stop. The entire situation is made more frustrating by the fact that the buses come after the prospective riders have waited a considerable amount of time. The New York Times (1) has editorialized on the situation directing their hostility toward the bus drivers. We believe they are misdirected. It is our belief that the bunching of buses on an intracity route is a natural consequence of the characteristics of this system. Furthermore, once buses amass, they tend to remain together. This paper shall explore these two working hypothesis.

In order to do so we must study an artificial situation. Real life is too overwhelmingly complex to see underlying principles. Passenger loads vary temporally and by location. Times between stops vary with the route, time of day, etc. To understand the underlying causes, a simplified model of reality must be used. By putting into a model as much realistic detail as is needed to study a particular system (as can always be done using simulation), generalizations are difficult to educe. The details distract from the broad view, the specificity negates the transportability of the results. By concentrating on an abstraction of reality that hopefully still contains the pertinent parameters, gross generalizations can be perceived.

Simply, the model that was kept in mind was that of an equally loaded route. By this is meant passengers arrive at stops according to identical distributions. Further clarification and specification of the model shall be given later.

The organization of the rest of this paper is as follows. Intuitive arguments shall be used to defend the two hypotheses. We realize that intuition alone is never a firm basis. One person's intuitive foundation may be another's quagmire. So that a simple model that can be solved analytically shall be presented. This model is used to shed further light on the question of how long a bus will take to catch up to an earlier dispatched bus. This model, however, proves to be too simple, not representing reality enough. Although progress could be made in devising and solving more realistic models analytically, this was not done here. Rather simulation was used, as it provides a flexible tool enabling less stringent restrictions than an analytic model could provide. Simulation was the main tool of our analysis.

INTUITIVE EXPLANATION

Look at the situation intuitively. A force of attraction operates between buses in the following sense. Say a bus ahead of another encounters a stop where more people than usual are waiting to board. Its delay at this stop will be greater than average. Meanwhile the bus immediately following it will (sometimes) reduce the headway between them. So that when the second bus arrives at the particular stop at which the first bus was held up, fewer people than average will likely be there. In fact, each stop it encounters from then on will have fewer people than average waiting because of the reduced head-time. Of course it could be true just as well that the second bus was the one to encounter a heavily loaded stop. By the same reasoning the second and third buses would become closer to each other. The instability of the situation is apparent. Buses closer than average will continue to approach each other till they come together, at least on average.

The second hypothesis is intuitively clearer. Once together, buses will not separate substantially since the first bus to a stop will be slowed picking up the waiting passengers. (This is actually an extreme case of the earlier intuitive arguments.) The buses may even leapfrog one another. We return our attention to the first hypothesis, the less intuitive of the two, and use a simple model of the distance between two buses. The main virtue of this model is that it can be solved analytically.

A SIMPLE MODEL

The first bus leaves the terminal at time zero; the second bus leaves c time units later. Let the time between stops including the boarding and exiting of passengers be exponentially distributed, each time independently drawn. The distribution parameter for Bus One is b_1 and Bus Two b_2.

The situation can be thought of as in terms of a single server queue. The "stops" would include the intertravel time between actual stops. Then Bus One advancing a "stop" can be thought of as an arrival of a customer requiring service to a single serve queue. Bus Two advancing a "stop" would be the departure of a served customer. With the customer being served being counted as on the queue, the number of customers on the queue is analogous to the stop head-way between buses. Also the length of a busy period for the queue server is exactly analagous to the length of the time until Bus Two catches up to Bus One for the first time.

If there are j customers in the queue initially, it can be shown (see e.g. Prahbu (2)) that the mean busy period length is $j/(b_2-b_1)$ for $b_2>b_1$ and infinity otherwise. The probability that Bus One is j stop ahead of Bus Two by the time Bus Two leaves the terminal c time units later is given by a Poisson distribution with parameter b_1c. It is easily shown that the mean time till Bus Two catches up with Bus One is then $b_1c\exp((b_2-b_1)c)/(b_2-b_1)$ for $b_2>b_1$ and infinity otherwise. Since each service requires on average $1/b_2$, Bus Two will on average travel a number of stops of b_2 times this mean time.

According to this model, if the buses operate at the same rate, they will on average never come together! Not unless the rate of Bus Two is greater than Bus One will (on average) the buses meet. Actually the simplifications used in formulating this model overstated the mean time to meet. Of prime importance, no account has been made in this model of the interdependence of the interarrival times of buses and their delay at stops loading and discharging passengers. Our intuitive explanation was based on this positively correlating factor. This suggests that if the boarding and discharging time is negligible, the buses may not aggregate.

To further explore these questions a simulation model was built. The correlating factors are implicitly contained in this model.

SIMULATION MODELS

The simulation model was designed to represent a homogenous bus system. In the simulation the buses go from stop to stop first letting people off, then letting waiting people on. The buses follow a circular route and wait a fixed amount of time at a single terminal before restarting the circuit. The interarrival of people to the stops is stochastic. All stops use identical interarrival distributions and in that sense are equally loaded. Each person on the bus travels a number of stops determined by a single distribution. That is, all people are treated in an identical manner (e.g., independent of where they got on). Any people still on the bus at the last stop get off; the bus starts empty from the terminal.

Two simulations were written in GPSS. The first was a two-bus system in which each bus could take on an unlimited number of passengers. Investigations of the time till they met were made. A second model consisting of five buses, each with controlled capacity, was used to follow the bus interdistance changes over a time period.

An exponential distribution was used for the interarrival times of passengers at each stop. The justification for the use of this distribution has been given many times (see e.g. Saaty (3)). A property of the exponential distribution allowed us to use a single stream of arrivals that were assigned stop numbers randomly (actually a GPSS function). Each stop then saw exponentially interarriving passengers with equal rates, the original rate divided by the number of stops.

Common to both simulation models were the chosen parameter values: fifty stops; a ten minute wait at the terminal; a travel time of between two and six minutes uniformly distributed; five seconds to enter or exit the bus; an entering passenger exiting four stops later twenty-five percent of the time, five stops later fifty percent of the time, and six stops later twenty-five percent of the time. Of course these are arbitrary values. Other values were used to confirm qualitatively the results reported here.

The question to be answered with the two-bus simulation model was: How long it would take the second bus to catch up to the first under normal operating conditions? But what is normal operating conditions? Surely not steady state. Running a simulation to steady state, a standard solution, cannot be done here as steady state constitutes an absorbing state--the aggregation of the buses. What was done was to initially position the buses as far apart as possible (twenty-five stops) and under several initial waiting passenger loadings determine the length of time till the buses were together. Figure One shows the results under a variety of passenger input rates with initially no passengers waiting at stops and initially with one hundred people (stops randomly chosen). Notice that the ordinate is labeled "average number of stops for both buses." The buses start twenty-five stops apart so that at meeting one bus goes twenty-five stops fewer than the other. The longest time till the buses meet is at the higher interarrival rates. At a rate of fifty seconds between arrivals the system is underutilized though; only one passenger per fifty seconds arrives to the entire system of fifty stops. Yet the buses will still catch up relatively quickly, requiring less than twelve cycles.Somewhat surprising, the effect of initialization was small. This is attributed to the relatively short entering and exit times compared to the interstop time used.

Results from the five-bus model are shown in Figure Two. The plots are realizations of forty and eighty hours using eight hour snapshots of the distance in number of stops each bus was from Bus One. Realize that buses fifty stops away are effectively at the same stop. Even though the time frame is long, the general trend towards aggregation for all the rates is clearly evident. At an interarrival rate of ten seconds/customer, the buses pair together within the first eight hours and all five buses aggregate within about twenty-four hours. The capacity of the fifty passengers per bus was not reached for any of the rates and so did not affect these results.

SUMMARY

We have leaned towards conservatism whenever assigning values. The two-bus model nonetheless demonstrated the definite tendency of buses to aggregate. Although in the model the buses started twenty-five stops apart, they met fairly soon and what is more, the dispersion from the average value was narrow (small variance). The results of Figure One show a linear or close to linear relationship. The exponential growth predicted by the queuing model was not fulfilled. The queuing model's inadequacy is evident--the correlating factors not taken into account in the queuing model are of primary importance.

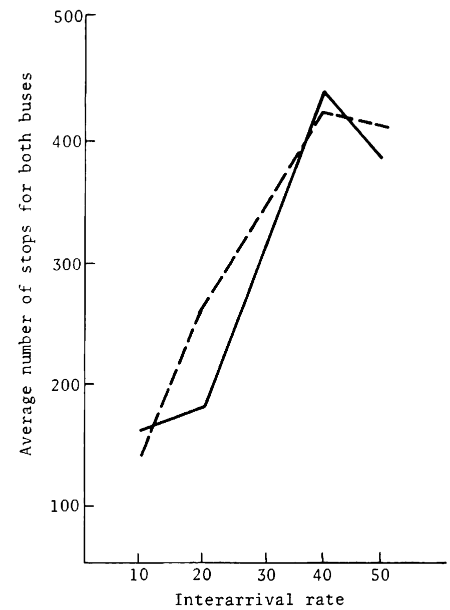

Figure 1. Average number of stops till two buses catch up to one another. Initially they are twenty-five stops apart on a fifty stop circuit. Further details in text. The two numbers in parentheses are the number of replications and the standard deviation. The dip proved not to be significant.

The results of the five-bus simulation back up the second claim: once the buses bunch they tend to remain so.

This paper accomplishes a limited goal: the hypotheses have been defended. Further work is required. A quantitative model relating the input variables to the buses meeting time needs to be done. Other questions arise as well. Should aggregation be prevented? If so, what control procedures would work. Much interesting work remains.

REFERENCES

1. New York Times January 7, 1975, page 32.
2. Prahbu, N.U., Queues and Inventories, John Wiley, 1965.
3. Saaty, T.L., Elements of Queueing Theory with Applications, McGraw-Hill, 1961.

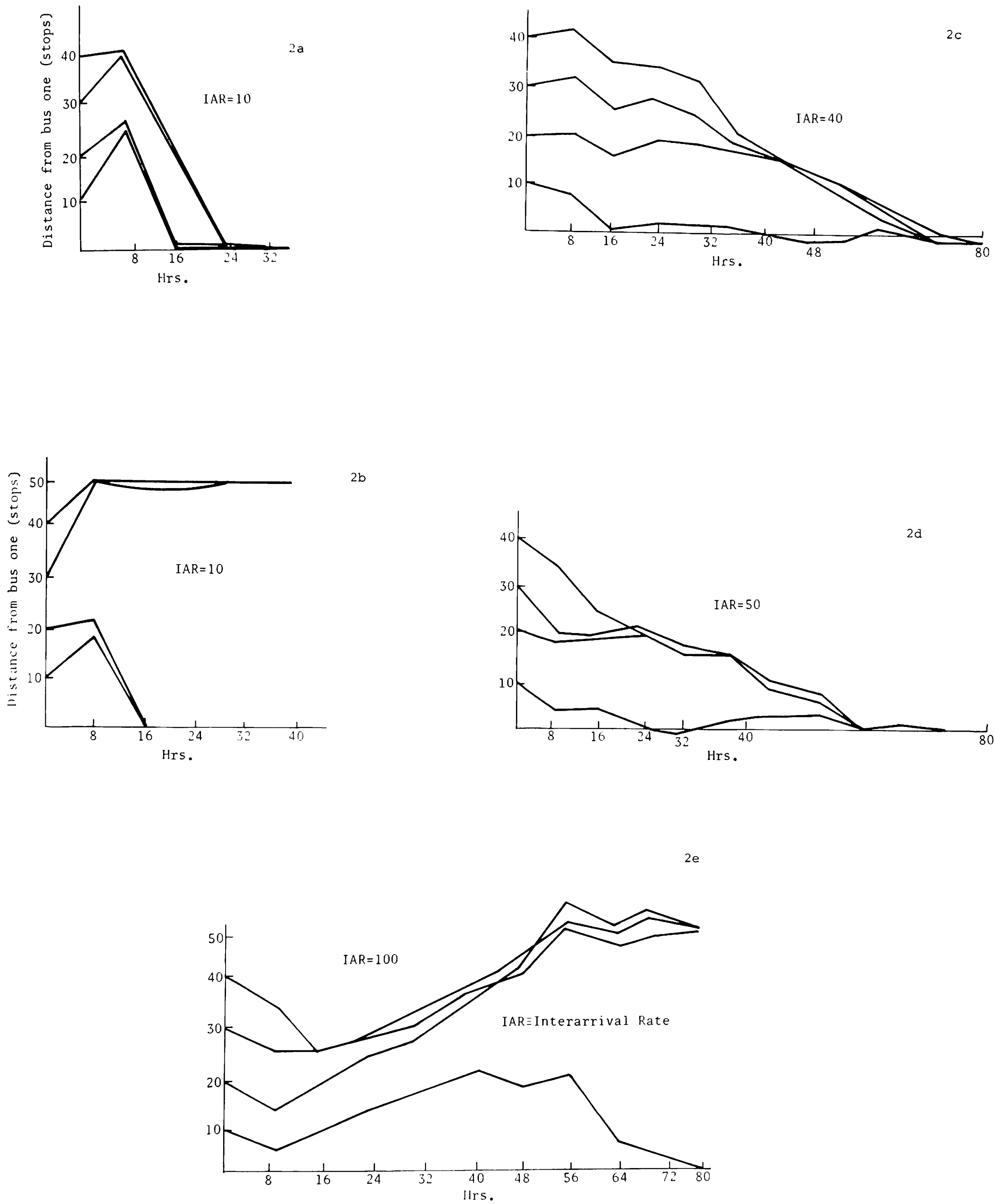

Figure 2 Realizations of the distance from Bus One of five buses in circuit for various customer interarrival rates. Snapshots taken every eight hours.