# A SIMULATION OF U. S. COAST GUARD RESPONSE TO DEMANDS FOR SERVICE

Joseph A. Smith
U. S. Coast Guard R&D Center
Groton, Connecticut

ABSTRACT

SARSIM is a simulation which mimics the actual operation of the Coast Guard search and rescue system and records its aggregate behavior. The most promising (near-optimal) configuration can be identified by repeating the experiment for various resource and crew allocations and comparing the performances.

Historical data was used to form the basis for occurrence distributions. The data base provides invaluable input to a model of this type in which many dependent occurrences affect the outcome of the activities monitored by the simulation.

Statistics such as utilization factors, service times, waiting times, response times and various cost figures are tallied during each exercise to provide a measure of system performance under alternative mixtures of resources and crews.

A measure of effectiveness (MOE) was derived to enable the experimenter to make multiple runs in an exploratory mode. The accumulated average of the resource cost per distance multiplied by the number of incidents in queue awaiting service was calculated for each station. Overall mission performance, consisting of a sequence of basic activities, was synthesized using a time profile for each candidate resource.

## I. INTRODUCTION

One of the U.S. Coast Guard's prime responsibilities is to provide assistance to those in distress within the coastal regions of the United States. This duty entails an enormous expenditure of manpower and equipment to meet growing demands for service. In addition to the burden of providing resources to meet the needs of a rapidly expanding marine law endorcement role, the Coast Guard must dispatch search and rescue forces when distress situations occur. Coast Guard upper management is faced with extreme complexity, considering the stochastic nature of these operations. In order to gain a deeper grasp of the varying quantities, a search and rescue simulation model, SARSIM, was developed by the National Bureau of Standards and representatives of the U.S. Coast Guard in 1971. The model was successful in providing insight into the resource allocation process and produced many worthwhile suggestions regarding the effects resulting from alterations in allocation of search and rescue (SAR) resources. The simulation model described in this paper is an outgrowth of the above product and many of the features of the original prototype have been incorporated into the new version.

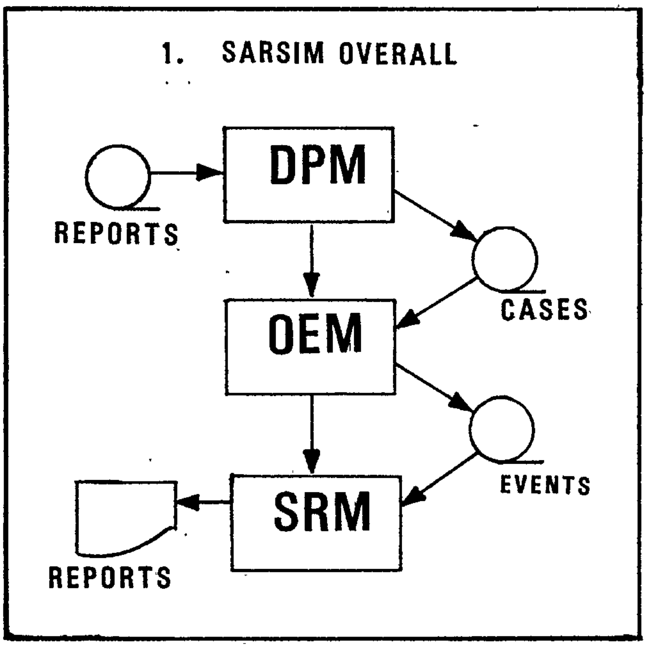
The real-world system can most simply be described as one in which a set of stations are assigned to strategic locations along the coastline in order to provide assistance to distressed individuals and property. When a call for assistance arrives at a station, the type of distress may not be known, especially if a search activity is required, so stations must be prepared to send additional resources to the scene and possibly interrupt resources already engaged in lower priority cases.

The distribution of incidents is functionally related to the time of occurrence, location, type of distress and the environment. The dissimilarity of customer demand from incident to incident and the diversity of resource capability in conjunction with the attempt to achieve parity between numerous necessities with limited means, creates a formidable decision-making task. Resource availability, parallel servicing, state-dependent service rates and numerous additional uncertainties also contribute to the intricacy of the stochastic processes that govern events which affect Coast Guard response to public demands for service.

Simply stated, the goal of the SAR system is to respond to demands in the most economical way without sacrificing the quality of services rendered and to minimize to the greatest extent possible the danger to personnel involved in the mishap. The simulation was designed with this goal in mind.

The computer program was coded on a Univac 1108, Exec VIII operating system using a Simscript II.5 compiler. The program consists of three major sub-divisions: The Data Preparation Member (DPM), the Originate Events Member (OEM), and the Simualte Response Member (SRM). See Figure 1.

The Simscript timing mechanisms are not used in the DPM and OEM as these two members are 'static' programs functioning as processors of the historical data base. The processed, refined data is input to the SRM in the form of external event




files where the resource responses are simulated in time and recorded for analysis.

Historical data was used to form the basis for the incident distributions. The data base provides invaluable input to a model of this type where so many dependent occurrences affect the outcome of the activities monitored by the simulation. This data-base is transformed into Simscript notation; Entities, Attributes and Sets, by the DPM which is described in detail in Section III.

The OEM accepts a number of direct access files containing the exogenous events produced by the DPM and generates a time-ordered set of incidents that ultimately drive the SRM. Descriptions of the OEM and SRM will be found in Sections IV and V respectively.

Output statistics from the SRM, include a tally of the number of 'assists' a given station makes, the priority of the cases not serviced within assigned time limits, utilization figures, cost figures per resource type, queue length and virtual waiting times. Provision was also made for time series output to files to be later subjected to spectral analysis during validation phase of the project.

A measure of effectiveness (MOE) has been developed which measures, in effect, the efficiency of service by calculating at each station the total expected variable cost per unit time and attempting to minimize this parameter with alternative mixtures of resource types. A measure of productivity was derived to compare various mission profiles by considering the times spent at the station and enroute to scene as non-productive, dividing the sum of these times by the total activity time for each resource and subtracting this value from one. Increased values of this measure yield higher levels of productivity and less standby time and time enroute which is non-productive.

Several methods were combined for presenting evidence of the validity of SARSIM, which included graphical presentation and statistical testing techniques. Section VII deals with these matters in detail. One method was developed which verified the queueing behavior at each station by comparing the average queue size derived from an approximation given by Rider (16) with the simulation output for the station.

In the next section we give an overview of the physical model being simulated. The utilization is described in Section VI and ongoing work on verification and calibration is described in Section VIII.

## II. CONCEPTUAL MODEL

The model of the Search and Rescue (SAR) activities is characterized by heterogeneous categories of customers serviced by a wide variety of Coast Guard resources, depending on the urgency and the nature of the distress. The resource's ability to satisfy the customer's demands is a function of each resource's physical attributes and the environmental conditions existing during the time of service. For example, a rescue vehicle would more likely arrive on the scene earlier in calm seas than it would in rough seas. Transit speed would be reduced by the increased wave height. An aircraft or helicopter might not be able to respond to a call if the visibility fell below minimums. Also, towing requirements depend on weather conditons, the displacement of the vessel being towed and the resource's towing capability.

Calls for service are distributed over time and geography according to distributions that are known only empirically and the occurrences of incidents tend to cluster at certain locations along the shore. In addition, SAR activity tends to be concentrated about certain particular monthly, daily and hourly time periods. For instance, July and August have the most activity, with Fall and Spring falling off in a fairly symmetrical pattern. Analysis of historical records show that Saturdays and Sundays are about twice as busy as the average weekday, while the period from noon to six PM is usually the busiest period of the day.

Customer demands vary from case to case. Some of the services rendered by Coast Guard (CG) vehicles in the past are; towing vessels in to shore, dewatering and refloating flooded vessels, escorting, extinguishing fires, salvaging, refueling, delivering equipment, making emergency repairs at sea, rescuing and providing medical assistance.

In many instances the demand(s) are not known at the time a call for service is received by the CG. As a result a CG vehicle incapable of supplying the required services might be the responding resource. For example, when a vessel is reported missing or overdue, the responding CG resource may have no information about the nature of the distress or the type of assistance that might be needed. This would necessitate additional calls for assistance once the demands were known.

The CG resources are situated along the shore where the locations of the distress incidents most often occur historically. The stations are grouped into twelve districts, each normally containing from thirty to fifty stations(facilities) and about eighty to one hundred and fifty resources. The resources are categorized into some fifteen to twenty different types. Early in 1974, CG active strength included two hundred and sixty-six Cutters, two thousand smaller boats, one hundred and eight helicopters and fifty fixed wing aircraft.

Small boats handle the bulk of the demands, supported by fixed wing aircraft. Roving Patrol Cutters occasionally assist while operating independently at sea. Units other than CG resources, such as Civil Air Patrols and the CG auxiliary will often respond to a call for assistance.

Resources with appropriate attributes assigned to a station might not respond to a call for assistance if, for example, they were all busy servicing other customers of higher priority, down for repairs or inadequately manned. Consequently, availability, maintainability and reliability must be taken into account in addition to the resource's attributes.

The time-dependent queueing behavior of the SAR system is a function of the priority of the case, the compound interaction between customers entering the system, the inventory of resources at each station, readiness status at the time of the incident, resource assignment policy and weather. The complexities involved in modeling this type of system create a difficult analytical problem. All of this makes simulation a very attractive alternative due to the more elusive analytical solutions.

In terms of Queueing theory, the SAR system appears to be Priority-discipline, multiple server type model where service is based on a priority system. Only limited results have been derived from mathematical analysis of priority discipline models and most of these for the single-server case. The SAR model is a multi-server, priority discipline with an unknown arrival time distribution. Although usable results have been derived from some multi-server models of this type, the service is usually assumed to be non-preemptive i.e., customers being served cannot be returned to the queue if a higher priority customer enters the queue. Simulation can deal with the above conditions as well as enabling the experimenter to observe the processes at certain points of interest in order to gain insight at the quantum level of activity.

## III. DATA PREPARATION - DPM

An Assistance Report is prepared by the CG after each SAR incident is serviced and the reported information is divided into four main categories: (1) identifying information such as the Date Time Group (DTG) of notification, the location of the distress, if known, weather conditions, station receiving the call, etc., (2) data concerning the distressed unit such as; tonnage of the vessel, nature of the distress, value of the vessel, severity codes etc., (3) reporting command data, and (4) assisting USCG resource data such as the type of vehicle or resource used to service the distressed unit, DTG underway and the DTG on the scene, type of assistance rendered, total time on the sortie, etc.

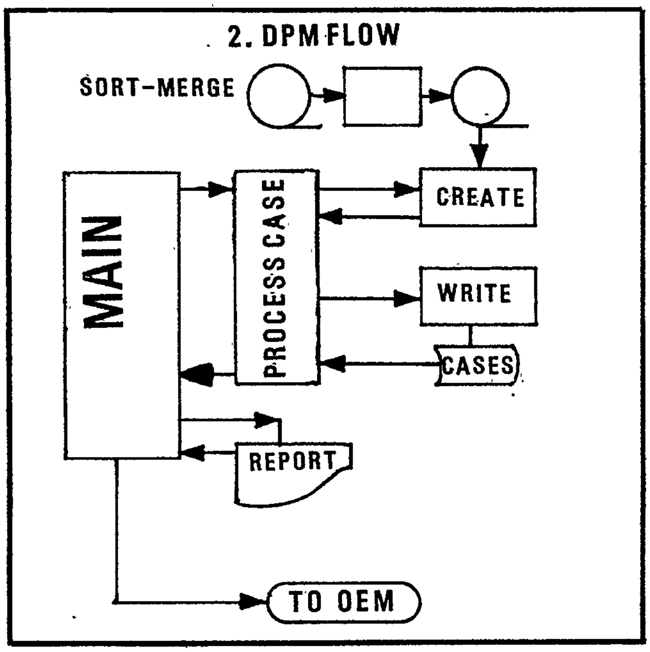
The reports are coded and punched on IBM cards. Utility programs 'clean' the data and sort merge the records onto magnetic tape. Analysis of the data reflects the fact that incidents do not occur uniformly in time. There is a tendency for the incidents to occur in the afternoon of summer weekends. This produces a situation where CG vehicles have little to do during some periods of time but are overloaded during busy intervals.

Historical records produced by the reports have been analyzed thoroughly by CG personnel and others in the past. These studies suggest that relatively large numbers of cases occur in fairly warm weather, most incidents occur on clear days (high visibility), many recreational boats are involved in calls for assistance and a large percentage of cases require towing capability.

One study in particular (8) conducted extensive tests on the frequency distributions derived from the data obtained from one facility on the west coast. A Hypothesis test was used to see if the yearly means were the same from year to year. For every factor except visibility, the test indicated that the means were different. Another test demonstrated that the means of the values of the nature of distress, the severity to persons and property, and weather conditions on scene were significantly different at the station of interest than at other stations within the Districts. A very interesting result was that severity to personnel and property was found to be essentially random and not dependent on weather conditions, distance offshore or nature of distress. All other variables when tested however, did show large correlation coefficients.

The computer model combines historical data and even-paced simulation for the purpose of evaluating and predicting the performance of the SAR system's various components i.e., CG vehicles and service facilities.

In the data preparation phase, a minimum amount of case information is processed by the DPM, which formulates the dependent elements of the incident as closely to the real world as possible. See figure 2.
The raw data from the reports must be 'massaged' to produce values for the performance metrics that will be used to verify and calibrate the




model. The data must also be refined until it is suitable as input to the simulation model. The refinement process involves the deletion of extraneous data as well as transformation of certain variables.

A Fortran program was developed to decode and sort the records generated by the reports into specified geographical boundaries. This program can selevtively create frequency distributions of certain elements of the record. This feature was used to verify the input to the DPM.

The DPM reads the sorted records from the tape produced by the Fortran program and merges the records into a consistent case. It checks and validates the input data and makes the attributes of events and entities compatible with the next members; the OEM and SRM. All dates are converted, via the Simscript II.5 ORIGIN.R routine, to a length of time relative to the origin date, January 2, 1972. This conversion simplified calculations, improved program readability and correctness.

The DPM distinguishes two different kinds of incidents; the search incident and the non-search incident. In the case of a search incident, the centroid of coverage and the type and total times underway of resources engaged in the search activity are filed as attributes of the search event. The day of the month and the hour in which the search event occurred are also recorded with the event. For the non- search incident, each demand and service time are recorded along with the location, weather factors, severity codes and DTG of notification.

Each time an incident is processed in the above manner it is output to direct access files which contain the events that ultimately will drive the SRM.

The events are seperated into two major time periods: Peak and Non-peak. The peak period is usually the summer months in which it is known that the bulk of incidents occur. The period is then further broken down into two types of days; weekend and weekday. The weekend days include holidays. The DPM breaks these down even further considering sunrise and sunset to seperate day and night.

The output files are segmented for selected random access through the use of two temporary entities; "In.keycode" and "Srch.keycode". The keycode attributes include the record number of the case and its corresponding timeslot. Also, the time of incident occurrence is included for later ranking of the set of keycodes in the OEM.

Since it is desirable to include in the model the capability of creating a non-historical caseload at locations where no incidents occurred historically we must also file the distressed units attributes of displacement and type in the keycode entity.

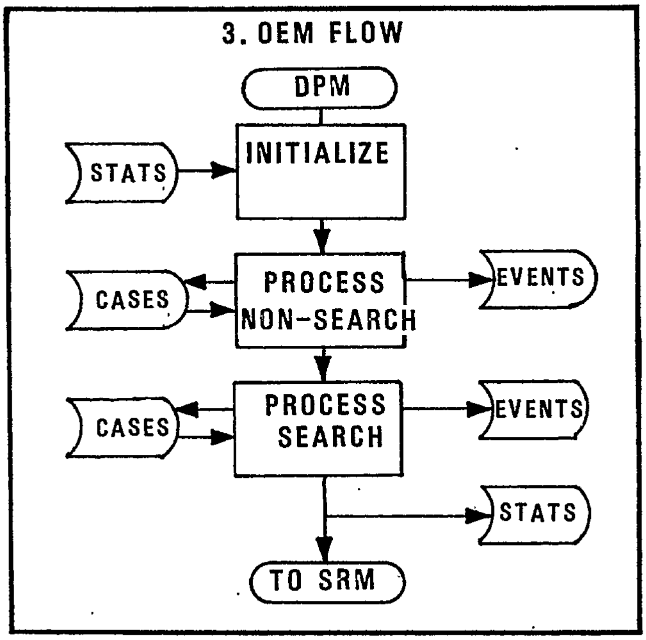
An arrival rate histogram is developed for the number of search incidents per month. This histogram is input to the OEM and is used to generate the number of search events via the Poisson distribution. An empirical distribution of the number of days per occurrence is also tallied for non-search incidents for each of the four time periods: 1. Weekday Peak Period, 2. Weekend Peak Period, 3. Weekday Non-peak period and 4. Weekend Non-peak Period. The OEM uses a uniform random varitate to draw the number of non-search incidents that will occur each day from this histogram.

Finally, a large amount of descriptive statistics are output from the DPM in the form of a lengthy report. This report is used by the analyst to check the OEM's output and to verify the simulation's results. Two way histograms are produced which yield the geographical locations by demand, event type, distressed unit type, period, resource type, etc. For example, one may wish to ascertain the geographical locations of recreational vehicles in the peak period during weekdays. The distribution of cases generated by the OEM (scenario) should closely resemble the histogram produced by the DPM. Appendix C.

## IV. ORIGINATING THE EVENTS-OEM

The OEM produces a time-ordered set of events for each run to be applied to the SRM. The events are generated external to the SRM in order to assure that alternative runs use the same historical case load to compare output statistics. The OEM can produce a historical case load, or a non-historical case load based on user input parameters.

When creating a historical scenario, the OEM first determines how many incidents are to occur in the area of interest by sampling from the occurrence frequency distribution histogram which was tallied in the DPM for non search incidents. One sample will yield the number of non-search incidents that will occur for one day of the simulation. Once this quantity has been established, the program randomly selects the index number of and incident for each incident that is to occur on that day. After these indices are all selected, the entity keycodes are withdrawn from the set of keycodes which are conveniently ranked by lowest time of notification. See Figure 3.




The keycode entities then point to the record location for the non-search event and its attributes within the DPM files. Each event for the day is read from the files, properly formated for input to the SRM external event files, tallied for OEM output statistics and written sequentially out to the SRM External Event files. The process is repeated for each day of the time frame to be simulated. An analogous method is employed to generate the search events, however, these events are generated monthly via the Poisson distribution with parameters determined by the arrival rate histogram developed within the DPM. The appropriate number of events are then sent to external Events files. Statistics similar to those obtained in the DPM are tallied to compare the generated case load with the historical data base.

The analyst may wish to increase or 'grow' cases selectively when planning for future resource requirements. The OEM allows for this option by multiplying the historical frequencies by values supplied by the user when generating scenarios but cautions the user not to use this feature temerariously due to the impossibility of validating a future case load, i.e., validation can only be attempted on historical case loads. The user may also select certain types of incidents which he wishes to sample, but again warnings are issued against placing too much confidence in the results. Nevertheless, since simulation provides insight into the processes investigated, enlightenment can be derived from these exercises if supported by analysis such as statistical forecasts.

A further enhancement of the OEM is its ability to 'strip' the location from the historical cases and then move these cases to sites randomly generated around an origin specified by the user. Operating in this manner is very useful when expecting a non-historical case load due to a projected change in the shore environment, e.g., a new Marina or Naval Base under construction. The events can be selected by type of distressed unit, since one might not want 50 ton vessels operating 1 mile off shore. The arrival rates must be specified by the analyst, possibly from past experience. The non-historical mode could be used perhaps to help justify the addition of new facilities in a locality of interest.

An algorithm for generating the coordinates of the non-historical cases was developed that assumes that the coordinates sampled are pair-wise independent and are drawn from a bi-variate normal population. See Appendix A for derivation of the algorithm. The longitude and latitude of the sampled points are generated within sectors of a circle centered at a desired location. The routine accepts the mean radius emanating from this specified origin and replaces the historical coordinate attributes of the selected incident with the generated values. The OEM merges the non-historical events with the historical and creates the SRM External Event files in the same manner as described previously.

During Validation and Calibration, the SRM requires the actual cases at the actual times that they occurred historically for selected geographical areas. The OEM has the ability, through the use of the keycode entity and ranked sets, to easily provide the SRM with the proper events ordered temporally as well as geographically. It also tallies resourse statistics such as service times and waiting times to be compared and calibrated with the SRM output statistics.

The Monte-Carlo method of utilizing antithetic realizations, which the SRM will average to obtain the final estimates of queue length and virtual waiting time estimates, has been incorporated into the OEM. The OEM obtains the realizations by simply assigning the pseudorandom numbers alternately while selecting events from the appropriate Time-slot files thereby generating two scenarios instead of one per execution. This produces an ensemble of scenarios within the SRM External Event files. This will reduce the variance of the estimates output from the SRM because of the negative correlation artifically introduced by the OEM.

## V. SIMULATING RESOURCE RESPONSE-SRM

The SRM in effect simulates the SAR system by first setting up an appropriate mathematical model that relates the component quantities and then experiments with the mathematical model. The experiment proceeds by drawing samples from the proper External Event file, created in the

OEM, and then calculating costs, resource utilizations, queue lengths and waiting times, given the use of specified resource assignment rules. Having observed the manner in which a system behaves over a time period under such simulated conditions, the system may be altered in an attempt at improvement. Artificial system histories are constructed and averaged in order to obtain estimates of system performances and suggestions for improvement.

The model begins by locating stations at given fixed locations within the area being simulated which is usually a CG District. Each of the stations are assigned a number of resources which can vary by type thereby yielding different capabilities. The resources are central to the method as they provide system dynamics through the response given to calls for service, i.e., without resources, no activity would transpire in the SRM .

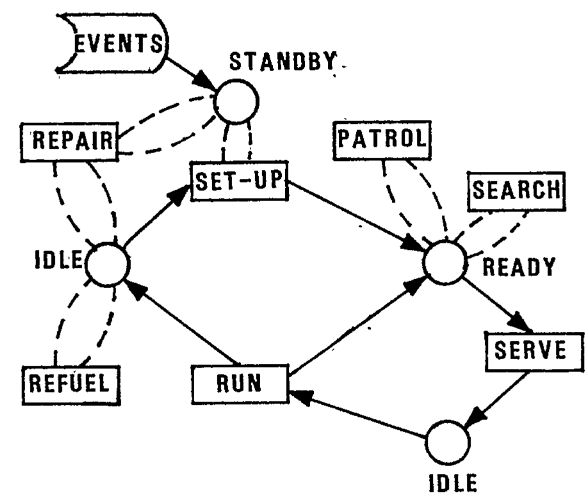
The heart of the model is the Resource Assignment routine. When a case is presented to the system, the program must decide which resource (vehicle) to send out on the case. It makes the decision based on the following factors:
Availability: To be a candidate a craft must be available: it cannot be out on another case (unless the severity of the case at hand demands an "interrupt" because of higher priority), refueling, or undergoing maintenance. The downtime distribution parameters must be supplied for each resource and these times are determined from the Log normal distribution.
Capability: The model does not allow a resource to be used that will not satisfy the needs of the distressed unit. For example, it does not send a helicopter to do a towing job or a boat to provide air escort. Environmental factors are considered part of capability. Each resource has restrictions concerning sea states, wind, visibility, etc., which limit its capability.
Tolerance times: For all available and capable resources, the SRM determines which vehicles can arrive on-scene within certain tolerance times. If no vehicle can arrive within the tolerance time indicated for that case, the model sends the (available, capable) resource which can arrive first, regardless of cost.
Cost: If more than one resource type can arrive at the scene within the tolerance time, the model takes the least expensive. While it is true that the Duty officer who makes the decision may not think explicitly of cost, it is usually the case that he implicitly does so by not wanting to send a bigger craft than is required.

Policy selections at the station level are also input to the intialization which determine whether or not a station receiving the call for services uses only its own resources, or whether resources from adjacent stations may be utilitzed.

A special type of resource called a patrol is allowed to randomly roam about an area specified initially. This entity may be allowed to participate in servicing a case or engage in search exercises. The Resource Assignment Policy Routine, (RSAP), must take this entity into consideration as well as all the other factors mentioned above.

As each external event occurs, stimulus is provided which causes the internal events to react to the demands for service. See Figure 4.

4. SRM BEHAVIOR




The needs are examined to determine which set of resources are to service the needs. If resources are available to service a given need of the incident, a particular station is assigned on the basis of the RSAP. If no suitable resources are idle when the external event occurs ongoing service to a case of lower priority may be interrupted. If no suitable resources can be assigned, the case is placed into a queue with subsequent periodic review of status. When resources complete services, they become available for reassignment to other incidents or return to the station.

Endurance is determined by the resources fuel function attribute which calculates the amount of fuel remaining at simulation time given the consumption rate and fuel capacity. This places yet another constraint on the selection algorithm as the allowable range of the resource must be taken into consideration. The resource also must be provided with a speed, a function of sea swell. An average transit speed for given discrete levels of sea state is used for calculating the time enroute the scene and can be a limiting factor when using RSAP to allocate resources depending on the environmental conditions at the time of the call for service.

Another important constraint on the RSAP as yet has not been mentioned and that is the times of sunrise and sunset. E.g., an aircraft would not be sent to a case if the time of the call was so close to sunset as to make the assignment infeasible due to lack of operating time necessary to perform the mission. A function which uses precaculated coefficients for each area simulated yields the times of sunrise and sunset for each day (5). The SRM calls upon this function each day and uses the values of sunrise and sunset when assigning resources.

After the SRM has processed all the events within the desired time frame, a report is generated with a variety of statistics such as: total number of different types of incidents processed, number of these completed and in queue, number of times specific kinds of service could not be rendered and the reasons, utilization statistics by shift and resource type, breakdowns by each station, the number of sorties of each individual craft, average transit times, average waiting times, and frequency distributions of variables of interest. In addition, optional weekly, monthly and special interim reports can be generated if required by the analyst. Furthermore, huge amounts of data concerning the disposition of each event are optionally output to a tape which is acessible through the use of a data management system. In this way, the analyst can obtain snapshots of certain behavior after the simulation has run its course to gain further insight into the operations studied.

## VI. MANAGEMENT CONCERNS

The SARSIM model involves the conduction of experiments and studies of the SAR system which is characterized by a rather complex mathematical queueing model. As in most simulations, experience and extensive technical knowledge of the particular problem area must guide the analyst in his efforts to apply the model. The managerial problems are many and varied and therefore program flexibility is a requirement. Some managerial problems encountered are: establishment, relocation or disestablishment of shore stations or air stations within a Coast Guard District, changes in manning levels at individual stations or throughout a district, relocation of resources from one station to another or other changes in the relative mix or availability of different kinds of resources, introduction of new types of resources, either as replacements for or in addition to existing resources, and deciding whether action should be taken in anticipation of radical changes in demands for service.

Selection of a course of action can, of course, be made in the traditional fashion of utilizing individual judgement based on experience and expert opinion. In some situations it may be possible to experiment with changes in policy or equipment, but empirical trials in actual situations are often time-consuming and costly, and often entail risk. SARSIM offers an alternative of modeling and analyzing the operation and estimating the likely outcome of the proposed courses of action. The results of experiment are presented to the decision-maker for consideration along with other analyses.

The SAR system can be defined as that interaction of resources and processes which results in some measure of system performance as function of time. It is desirable to describe this system so that analysis may be performed, such as studying trade-offs among the various input variables, e.g., determing the specific combination of input variables which will yield an optimum (maximum or minimum) for a specific system output performance measure. Other analytical applications include alternating SAR resource locations, minimizing the expected customer (distressed unit) waiting time across a range of operational situations and over possible variations in system input parameters, subjecting to cost restrictions, and concurrent minimizing of collateral measure, such as percentage of adjacent personnel casualties. There exists a family of input variables, each of which assumes a range of values,of which the decision-maker ulimately seeks the specific set that will provide the optimum values for the system's output measures.

The model attempts to provide relative measures of performance so that the analyst can possibly glean some indication as to what alternative locations of resources approach the optimum measure.

Decisions regarding the amount of service capacity to provide are usually based on two considerations. One is the cost incurred by providing the service and the other is amount of time waiting for that service. It is readily apparent that these two considerations create conflicting pressures on the decision-maker. The objective of reducing service costs recommends a minimum level of service while on the other hand long waiting times are undesirable, which recommends a high level of service. Therefore it is necessary to strive for some type of compromise. The objective is to determine the level of service which minimizes the total expected cost of service and the expected cost of waiting for that service.

The simulation reduces these costs to relative costs for comparisons. The relative cost per operating hour, Cj is given for each resource type. The cost of waiting would be arbitrary and is omitted, i.e., we are only looking for a relative measure here.

A measure which gives the relative efficiency of service given the mean service rate per busy resource, the number of resources, nj, and the mean incident arrival rate is the expected total relative variable cost/unit operating hour:

$$E(C) = \Sigma n_j c_j + L$$

where $L = A * W$ Expected Line Length

and $A$ = mean incident arrival rate

$W$ = expected waiting time in system

The analyst would consider the run with the lowest E(C) as the 'best' alternative exercise as regards the mix of resources.

Another measure was developed to create a time profile for each candidate resource to be used to measure overall mission performance. The total mission time is found by summing the elapsed time duration of each activity in the activity sequence of the simulation. This time, T is determined by the equation:

$$T_m = \Sigma T_{sb} + \Sigma T_{enr} + \Sigma T_{srch} + \Sigma T_{serv}$$

where $T_{sb}$ = Time spent idle at station

$T_{enr}$ = Time enroute to scene

$T_{srch}$ = Time spent searching

$T_{serv}$ = Time spent servicing a case

If $T_{sb} + T_{enr}$ is considered to be time wasted, a measure of productivity, P, can be assessed by the following equation:

$$P = (1.0 - \frac{\Sigma T_{sb} + \Sigma T_{enr}}{Tm}) \; 100\%$$

Increasing values of P imply higher level of productivity and less time spent in transit which is non-productive.

It must be kept in mind that these measures only relate to the simulation and are used only to pin point possible 'best' allocations. The simulation must always be accompanied by common sense and further analysis from other sources in order to make judgements concerning SAR problems.

VII. CURRENT SIMULATION EFFORT

The model is currently undergoing enhancement and fine-tuning to improve the validity. Program results have been compared with hand calculations, in sets of a few to several simple test cases. The programs have been run through a set of analysis runs designed to detect errors and inconsistencies. Input parameters are being adjusted and reasonableness of output levels when new parameter values are used are being checked and corrections being made where necessary. Tests are being conducted to determine the model's ability to produce simulation results which replicate what occurs historically using the SAR data base. The tests will follow closely those which were applied to the earlier version of SARSIM. (14)

In addition, the program members were designed with the ability to record the state of the various queues at unit intervals and store the resulting time series' in order to apply spectral analysis to the data generated by the simulation. The differences in the statistical properties of queue length are easily identified using spectral analysis. More generally, this estimation procedure provides a tool for comparing simulated time series with real world data and for understanding the implications that various alternative assumptions have on the output of SARSIM. (3) Another unique method which checks complex internal behavoir patterns of the model is exemplified through the application of an approximation to average queue size. See Appendix B. The arrival and service rates as a function of simulated time are input to the approximation during a run and the SRM-produced mean queue size is compared with the solution to the analytical model.

VIII. REMARKS

The model described herein is an outgrowth of a previous simulation jointly developed by the National Bureau of Standards and the Coast Guard in 1971. The program was placed on operational status to ferret out shortcomings, to define them precisely, and to determine what enhancement to the program would prove valuable in making it a more effective management tool. The present project was designed to enhance SARSIM's validity and utility on the basis of the experience and knowledge accumulated during its history of operation. Some of the enhancements introduced into the model were: Multiple run capability, empirical arrival rate distributions , ability to create non-historical case loads, a wider selection of RSAP's, and various methods for increasing the precision of the model outputs. In addition, the excellent algorithmic and "state-of-the-art" programming features of Simscript II.5 in conjunction with an improved Univac 1108 Exec VIII Operating system has greatly increased program effectiveness.

BIBLIOGRAPHY

1. Box, George P. and Jenkins, Gwilyn M. Time Series Analysis Forecasting and Control. Holden Day, San Francisco, 1970

2. Feller, William. An Introduction to Probability Theory and its Applications Volume II. John Wiley and Sons, Inc., New York, 1966

3. Fishman, George S. and Kiviat, Philip J. "The Analysis of Simulation - Generated Time Series," in Management Science, Vol. 13, No. 7, p. 525

4. Gaver, Donald P. and Thompson, Gerald L. Programming and Probability Models in Operations Research. Brooks/Cole, Monterey, Ca., 1973

5. Goltz, Gene and Weiner, Howard. "Computer Program for Design and Performance Analysis of Navigation and Power Systems." Report, U.S. Coast Guard R&D Center, Groton, CT

6. Hamilton, F.M., Pritchett, C.W., Hudgins, H.H. "Technical and Operational Characteristics of High Performance Watercraft." Report, U.S. Coast Guard R&D Center, Groton, CT

7. Hillier, Frederick S. and Lieberman. Introduction to Operation Research. Holden Day, Inc., San Francisco, 1967

8. Holleman, Kenneth C. "An Analysis of the Need for Coast Guard Search and Rescue Facility." Thesis, Naval Postgraduate School, Monterey, CA., 1969

9. Jenkins, Gwilym M. and Watts, Donald G. Spectral Analysis and Its Applications.

Holden Day Inc., San Francisco, 1969

10. Kiviat, P.J., Villanneva, R. and Markowity, H.M. Simscript II.5 Programming Language. Consolidated Analysis Centers Inc., Los Angeles, 1975

11. Lass, Harry and Gotlieb, Peter. Probability and Statistics. Addison Wesley, Mento Park, CA., 1971

12. Mitchell, T.R., Cohan, L.S. and Heider, C.H. "The Utility of High Performance Watercraft for Selected Missions of the U.S. Coast Guard." Report, Center for Naval Analysis, Univ. of Rochester, Arlington, VA., 1972

13. "A Search and Rescue Model for the U.S. Coast Guard." Report, National Bureau of Standards, 1971

14. "Search and Rescue Simulation Model. Phase III. Calibration/Validation/Investigation." Report, National Bureau of Standards, 1971

15. Papoulis, Athanasios. Probability, Random Variables and Stochastic Processes. McGraw-Hill, New York, 1965

16. Rider, Kenneth. "A Simple Approximation to the Average Queue Size in the Time-Dependent M/M/I Queue." in Journal of the Association for Computing Machinery, Vol. 23, No. 2, 1976

ACKNOWLEDGEMENT

The author wishes to thank LT Tim Jamison, USCGR, for his work in helping to prepare this paper and would also like to express his appreciaiton to CDR Don Naples and Mr. Bob Reust for their support, advice and encouragement throughout its development. The author would also like to acknowledge the excellent programming assistance given by Mr. Gary Walters and Mr. Henry Hood.

## APPENDIX A

The material contained in this appendix consists of theoretical background information that forms the basis for implementation of the routine used in the OEM to generate the coordinates for a non-historical case load.

First, the formulas which were used to generate the two coordinates must be derived from Probability Theory. The assumption is made that the coordinates to be sampled are pairwise independent and sampled from a bivariate normal population. The density function for this distribution is:

$$f(x,y) = \frac{1}{2\pi VAR} \text{EXP}(-(x^2 + y^2)/2\ VAR) \quad \text{EQ1}$$

with zero mean and equal variance, VAR.

The random vector Z as seen in figure 5 below can be written as a function of the two random coordinates X & Y as follows:

$$Z = \pm\sqrt{X^2 + Y^2} \quad \text{EQ2a}$$

The random angle, W, can also be expressed as:

$$W = \text{ARCTAN}\ (X / Y) \quad \text{EQ2b}$$

The region $D_{zw}$ consists of two sectors as seen shaded in figure 5. If we are given 2 functions, g(x,y) and h(x,y) of real variables x and y and two random variables (r.v.) X and Y, we can form the r.v.

$$Z = g(X,Y) \quad \text{and } W = h(X,Y)$$

These r.v. have a joint distribution and density F(z,w) and f(z,w) respectively.

These functions must be determined in terms of g(x,y), h(x,y) and the joint density f(x,y) of X and Y.

With z and w two real numbers, denote $D_{zw}$ the region of the x,y plane such that:

$$g(x,y) \leq z \quad \text{and } h(x,y) \leq w$$

Since $\{Z \leq z,\ W \leq w\} = \{(X,Y) \in D_{zw}\}$ we conclude that the probability of the event $\{Z < z,\ W < w\}$ consisting of all outcomes, o, such that $Z(o) \leq z$ and $W(o) \leq w$, equals the probability mass in region $D_{zw}$. Hence

$$F(z,w) = \iint_{D_{zw}} f(x,y)\, dx\, dy$$

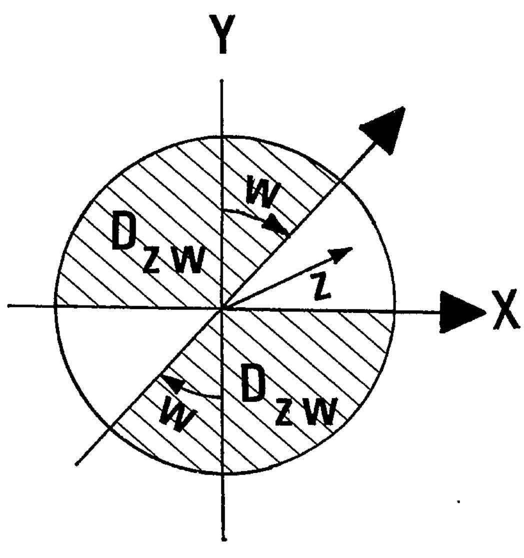
The total mass in the circle $\sqrt{x^2 + y^2} \leq z$ equals:

$$1 - \text{EXP}(-z^2 / 2\ VAR)$$

but the density has circular symmetry; therefore the mass in the shaded sectors equals:

$$(1 - \text{EXP}(-z^2 / 2VAR))\ ((\pi + 2w)/ 2\pi$$

5. DIFFERENTIAL SURFACE AREA



This equals F(z,w) for z>0 and $|w| < \pi/2$
If z >0 and $w > \pi/2$, the $\sqrt{x^2 + y^2} < z$
and ARCTAN (x/y)< w holds for every point of the circle $\sqrt{x^2 + y^2} < z$. Hence

$$F(z,w) = 1 - EXP(-z^2 / 2\ VAR)$$

Finally if z< 0 or $w < -\pi/2$, then F(z,w) = 0
In all cases F(z,w) equals a product of a function of z times a function of w. Therefore r.v. Z and W are independent. Thus Z has the Rayleigh distribution and W is uniform in the interval $(-\pi/2, \pi/2)$

Setting the cumulative densities to uniform random number RU and suitably transforming the equations we get:

Sample Bearing Angle = $\pi RU - \pi/2$ EQ 4
and
sample Distance = E(Z) $\sqrt{\pi/2\ Ln\ RU^2}$ EQ 5

where E(Z) is Rayleigh mean which equals

$$\sqrt{VAR\ \pi/2}$$

The Algorithm then is as follows:

Step 1: Input the mean, E(Z), and origin, LAT, LONG in degrees.

Step 2: Using EQ 5 generate a sample yielding the distance from the present origin.

Step 3: Using EQ 4 determine the bearing B.

Step 4: Determine Latitude of non-historical case relative to zero degrees latitude:

Y = ( d sin(B) ) (1/60) degrees

Step 5: Determine the Longitude of the case in degrees relative to zero degrees longtitude

X = C d cos(B)

where C is longitude correction factor given by C = 111.415 cos(Y) - 94.55 cos (3Y) + .012 cos (5Y)

Step 6: Find the location, (XX,YY), with respect to the preset origin

YY = Y + LAT

and

XX = X + LONG IF LONG LT X

= X - LONG IF LONG GT X

APPENDIX B

This will be a brief description of the technique used to check the internal queueing behavior of the SARSIM model.

The queues formed by the distressed units waiting for service can be estimated by numerically solving the time-dependent M/M/1 queueing equations. An approximate time-varying description of the queueing behavior of this type of situation is given by Rider (16), when the expected queue size changes slowly compared to the service rate(the queue includes any case being serviced). Because only a single first order equation is involved in this approximation, it is solved by rather simple integration techniques.

The basis of the approximation for the differential equation, DE (queue size), is an exact equation derived from the family of time-dependent equations for the M/M/1 queue.

Let

Pij(t) = Probability that there are j cases in the SAR system at time t given there were i cases in the system at time 0.

A(t) = Arrival rate of cases at time t

S(t) = Service rate of cases at time t

e(t) = A(t)/S(t)

Then the equation for expected queue size is:

$$Q(t) = \Sigma j P_j(t)$$

Using the queueing equations, we can write a DE for expected queue size:

$$Q'(t) = A(t) - S(t)(1.0 - P_o(t)) \qquad \text{EQ 1}$$

This equation means that the queue will increase with the arrival rate and decrease with the service rate multiplied by an "efficiency factor", the probability that a resource is busy at time t.

EQ 1 yields the steady state value of P when Q'(t) = 0 and $A'(t), S'(t) \rightarrow 0$.

The equation can be used formally to solve for Q(t) as:

$$Q(t) = Q(0) + \int_o^t A(t)dt - \int_o^t S(t)dt + \int_o^t S(t)P_o(t)dt$$

It is usually not possible to obtain an analytic solution to this equation. In order to write EQ 1 in closed form we need an approximation relating $P_o(t)$ to Q(t). Rider derives such an approximation given as:

$$P_o(t) \approx (1.0 - a(t,T))(1.0 - e(t)) + a(t,T)/(1.0 + Q(t)) \qquad \text{EQ 2}$$

where a(t,T) = EXP(-S(t)*T)
and T is the time period, usually not crucial to accuracy.

If EQ 1 and EQ 2 are combined the resulting equation is simplified to the form

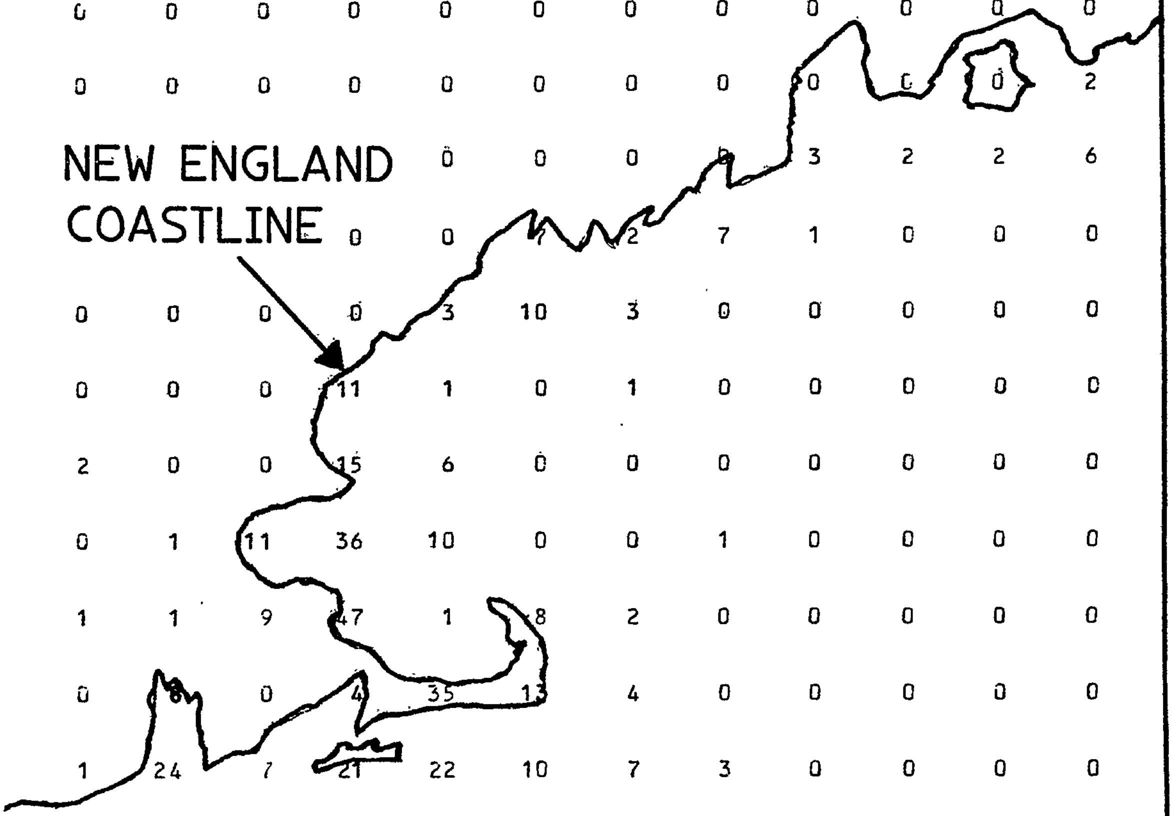
$$Q'(t) = S(t)EXP(-S(t)*T)(e(t)-q(t)) \qquad \text{EQ 3}$$

where $q(t) = Q(T)/(1.0 + Q(t))$

The predicted values of $P_0(t)$ and $Q(t)$ are calculated by numerical integration for each time period, 1 through 24, using the arrival and service rates supplied by the SRM. The SRM accumulates the simulated values for $P_0(t)$ and $Q(t)$ for each time period and the resulting curves are plotted for comparison.

APPENDIX C

## 6. GEOGRAPHICAL HISTOGRAM




LONGITUDE 71 40 71 20 71 0 70 40 70 20 70 0 69 40 69 20 69 0 68 40 68 20 68 0 67

| LATITUDE | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 44 40 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 44 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| 44 0 | | | | | 0 | 0 | 0 | [illegible] | 3 | 2 | 2 | 6 |
| 43 40 | | | | 0 | 0 | 7 | 2 | 7 | 1 | 0 | 0 | 0 |
| 43 20 | 0 | 0 | 0 | 0 | 3 | 10 | 3 | 0 | 0 | 0 | 0 | 0 |
| 43 0 | 0 | 0 | 0 | 11 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 42 40 | 2 | 0 | 0 | 15 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 42 20 | 0 | 1 | 11 | 36 | 10 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 42 0 | 1 | 1 | 9 | 47 | 1 | 8 | 2 | 0 | 0 | 0 | 0 | 0 |
| 41 40 | 0 | 8 | 0 | 4 | 35 | 13 | 4 | 0 | 0 | 0 | 0 | 0 |
| 41 20 | 1 | 24 | 7 | 21 | 22 | 10 | 7 | 3 | 0 | 0 | 0 | 0 |
| 41 0 | 5 | 14 | 3 | 5 | 1 | 7 | 1 | 0 | 0 | 0 | 0 | 0 |
| 40 40 | 1 | 0 | 0 | 0 | 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |