

THE GENERATION AND USE OF PARAMETERIZED TERRAIN IN LAND COMBAT SIMULATION

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ABSTRACT

The complexity of land combat analysis, especially in high resolution simulation models, is enhanced by elaborate terrain models depicting a limited geographical area. Terrain is, therefore, treated as a given factor in the analysis and all results are conditioned on that particular location. This study presents and evaluates a methodology for parameterizing terrain for use in land combat analysis. The ultimate goals of these models are to provide methodology for treating terrain parametrically in high resolution land combat simulation models and to provide the capability of investigating the intervisibility segment length problem over terrain with characteristics indicative of typical terrain in any desired geographical location. The current procedure is to use digitized data which is compiled from actual terrain by engineer surveys and photo-interpretation. If a simulation is to be used for determining who will win a battle over a specific piece of terrain, such as the Fulda Gap, then this approach is necessary. However, if the model is to be used to determine weapons effectiveness, then representation of a specific terrain is not necessary, and need only be representative of the area to be modeled. Consequently, a less costly and time consuming approach is desirable. In particular, terrain can be created mathematically by using a modified bivariate normal probability density function. The individual bivariate normal hills can be used to create ridgelines. An additional advantage of this approach is that the macro-terrain features can be created at random, thereby providing multiple, unique realizations of a type of terrain. This capability overcomes the sensitivity of Army study results to a single sample of terrain. When used for line-of-sight calculations, the parameterized, continuous representation eliminates the need for interpolations required for digitized terrain. Additionally, the mathematically represented hills allow slope to be determined at any point on the map, thereby providing information for mobility determinations and route selection. The methodology and simulation can be employed independently or used as a pre-processor for other combat models.

I. INTRODUCTION

Many current army combat simulation models require digitized terrain inputs for their execution. This process is very costly and requires a significant

amount of computer storage. In addition, computation of line-of-sight and mobility require a great deal of computer running time.

The current methodology utilized to represent terrain is also questionable from a statistical point of view. A Vector Research Incorporated (VRI) study on terrain line of sight, concluded that "... present and past army study results, based on the analysis of combat results on a very limited sample of terrains, may have been determined by the terrain selection process and not by the actual weapon system or force design differences." More specifically it stated the following:

A. "There is extreme sensitivity in combat model results as the scenarios (terrain and movement assumption) are varied, even when variation is within a class of scenarios chosen for their a priori equivalence."

B. "This sensitivity can be slightly reduced, but remains extreme (with probabilities of win estimable only within plus or minus 25% even when battle results are used to redesign scenarios." (2)

These results imply that sufficient replications of each type of terrain should be run in order to reach a satisfactory statistical level of significance. For VRI's analysis at least fifty replications of each type of terrain were used.

In view of the problems stated above, research was done by Major Christopher Needels to develop a new methodology for the representation of terrain. (5) In particular, a modified bivariate normal distribution function was utilized to generate hill masses in a wide variety of configurations. Line of sight and movement routines were also developed to evaluate various intervisibility parameters over many terrain configurations for very low cost.

This paper represents an extension of the models developed by Needels to generalize the terrain generation routines. In particular, methodology was developed to generalize the hill mass generation from symmetric hill shapes to non-symmetric configurations. The methodology developed for this generalization is described in Section III.

In order to evaluate the effects of the number of hills, hill height, and slope on various intervisibility parameters, an experiment was designed to measure these effects. A description of the

PARAMETERIZED TERRAIN

design methodology and results of the analysis are given in Section III. Finally, the conclusions drawn from the current analysis and recommendations for future research are presented in Section IV.

II. REPRESENTATION OF TERRAIN

HIGH RESOLUTION COMBAT MODEL REPRESENTATION

The classic approach to representing terrain in land combat models is to utilize digitized terrain evaluation data from Waterway Experiment Station (WES) for the particular area of interest.

The Dynamic Tactical Simulator (DYNTACS) model serves as a good example of the state of the art in terrain representation and use. DYNTACS is a two-sided dynamic model of battalion level combat representing the detailed interaction of elements in a combined arms environment.

DYNTACS utilizes 100 meter grid squares for which macro-terrain elevations are determined from WES tapes. The simulation divides each grid square diagonally, thus providing a series of adjoining triangular terrains. The entire battlefield, therefore, is represented as a surface of equal size triangles which vary in slope depending on the elevation at their corners.

Needels provides a concise description of the DYNTACS methodology for line of sight calculation as follows.

"From the macro-terrain data, line of sight between any two opposing elements is computed. This is accomplished by first computing the angle between the horizontal and a straight line drawn between an observer and target (O-T Line). The program then conducts a search of the terrain along the path of the O-T line to see if any macro-terrain is higher than the O-T line itself. This search is accomplished by comparing the angle of the O-T line with the angle above horizontal of the Observer-Terrain line. If the latter angle is larger, there is no intervisibility. Over the duration of a battle with numerous elements this calculation may be made thousands of times. Consequently, not only is the time to prepare the tapes high, but also the time to compute lines of sight once the terrain is input to the model. Considerable effort by developers and users of this model has been expanded to streamline this subroutine." (5).

SYMMETRIC REPRESENTATION

The basic models developed by Needels (hereafter

referred to as SIMTER) which provide the basic foundation for the methodology developed in this paper are described in this section. (5) The basic motivation for SIMTER was to develop a random terrain representation which could be replicated quickly but at the same time be representative of a particular "type" of terrain. The approach adopted for SIMTER generated a variable number of hill masses. The shape and height of the hills could be varied, but each hill was symmetric, since it was generated directly from the modified bivariate normal (MBVN) density function.

SIMTER has two parts. The first part is the main program which creates terrain and, at the discretion of the user, plots both a three-dimensional drawing and a contour map. Used strictly for terrain generation, it can be used as a preprocessor for other combat models by producing grid points and elevations similar to those provided on computer tapes by WES. The second part of SIMTER is a movement/line of sight routine which moves a target along specified routes across the generated terrain.

A NEW REPRESENTATION METHODOLOGY

The methodology developed in this paper provides for the generation of hill masses utilizing MBVN density function to represent masses that are not symmetric. This development provides a significant increase in flexibility to represent hill masses more realistically. The mathematical development of this methodology is presented in the next section, after which actual results from executing the model are given.

III. THE EXTENDED MODEL

UNSYMMETRIC TERRAIN - SINGLE HILL

The unsymmetric slope of terrain can be generated by a cutting method using the MBVN as follows:

$$HI = MH \cdot \text{EXP} \left\{ - \frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{x-\mu_x}{\sigma_x} \right) \left(\frac{y-\mu_y}{\sigma_y} \right) + \left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\} \quad (1)$$

where MH: maximum height of hill

HI: height of hill

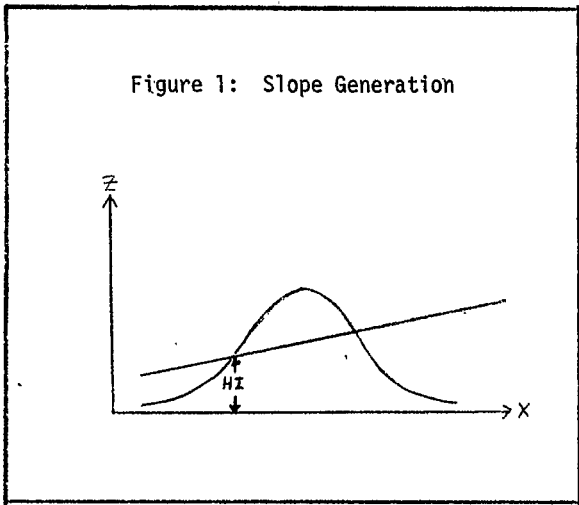
μ_x, μ_y : location of hill center

σ_x, σ_y : spread of hill mass in x,y plane

ρ : ellipse factor of hill

The unsymmetric slope of the hill can be represented by the vertical value of the MBVN subtracted from the vertical value of a linear function as shown in Figure 1 by the value HI.

Figure 1: Slope Generation

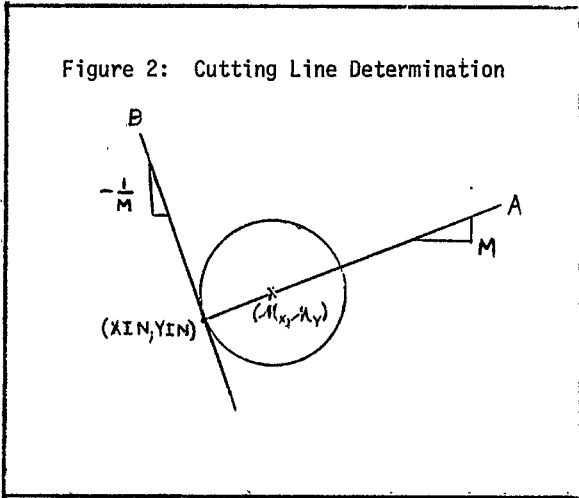


Two basic questions must be addressed:

1. Where to initiate the cutting of the MBVN function?
2. How to compute the initial cutting position?

If M is the direction of the cutting line (Line A), then the cutting plane is through the location of the center of the hill (μ_x, μ_y) as shown in Figure 2.

Figure 2: Cutting Line Determination



If the initial point of cutting is (XIN, YIN) and HI is height at the initial point, this point is on the cutting plane and also on the MBVN distribution function. Thus, if we consider the graph in two dimensions as shown in Figure 2, Line B is tangent to the circle at the point that the vertical value on the MBVN function is HI and the slope of Line B is $-1/M$. If the initial point of the

cutting is (XIN, YIN) this point is on Line B, Line A and the circle at a height HI on the MBVN function. We can compute the initial point (XIN, YIN) as follows.

From (1), letting $x = XIN$ and $y = YIN$, we obtain

$$-2(1 - \rho^2) \ln(HI/MH) = \left(\frac{XIN - \mu_x}{\sigma_x}\right)^2 - 2\rho \left(\frac{XIN - \mu_x}{\sigma_x}\right) \left(\frac{YIN - \mu_y}{\sigma_y}\right) + \left(\frac{YIN - \mu_y}{\sigma_y}\right)^2 \quad (2)$$

From (2) and the relationship shown in Figure 2, we obtain

$$-\frac{1}{\sigma_x} \left[\left(\frac{XIN - \mu_x}{\sigma_x}\right) + \rho \left(\frac{YIN - \mu_y}{\sigma_y}\right) \right] = -\frac{1}{M} \quad (3)$$

$$-\frac{1}{\sigma_y} \left[-\rho \left(\frac{XIN - \mu_x}{\sigma_x}\right) + \left(\frac{YIN - \mu_y}{\sigma_y}\right) \right]$$

From (3), it follows that

$$\left(\frac{XIN - \mu_x}{\sigma_x}\right) = \frac{\sigma_x + M\rho\sigma_y}{M\sigma_y + \rho\sigma_x} \left(\frac{YIN - \mu_y}{\sigma_y}\right) \quad (4)$$

$$\text{Let } C = \frac{\sigma_x + M\rho\sigma_y}{M\sigma_y + \rho\sigma_x} \quad (5)$$

From (2), (4) and (5),

$$-2(1 - \rho^2) \ln(HI/MH) = \left(1 - \frac{2\rho}{C} + \frac{1}{C^2}\right) \left(\frac{XIN - \mu_x}{\sigma_x}\right)^2 \quad (6)$$

$$\text{Let } D = 1 - \frac{2\rho}{C} + \frac{1}{C^2} \quad (7)$$

$$\text{Then } XIN = \mu_x \pm \sigma_x \sqrt{\frac{-2(1 - \rho^2) \ln(HI/MH)}{D}} \quad (8)$$

Also, let

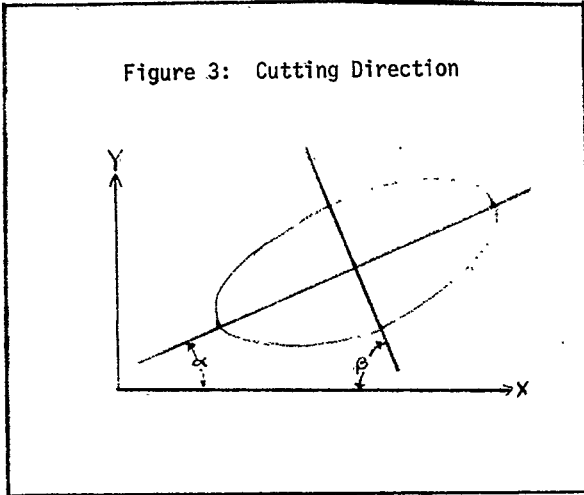
$$E = C^2 + 2\rho C + 1 \quad (9)$$

Then,

$$YIN = \mu_y \pm \sigma_y \sqrt{\frac{-2(1 - \rho^2) \ln(HI/MH)}{E}} \quad (10)$$

PARAMETERIZED TERRAIN

Figure 3: Cutting Direction



If the cutting direction goes through the longer axis as shown in Figure 3, then

$$\cot 2\alpha = \frac{\sigma_x^2 - \sigma_y^2}{-2\rho\sigma_x\sigma_y} \quad (11)$$

The cutting direction, α , is determined by

$$\alpha = \frac{1}{2} \operatorname{arccot} \frac{\sigma_y^2 - \sigma_x^2}{-2\rho\sigma_x\sigma_y} \quad (12)$$

If the cutting direction goes through the shorter axis as shown in Figure 3, then

$$\beta = 90^\circ - \alpha \quad (13)$$

MACRO-TERRAIN MODEL

So far in the discussion of the new approach to modelling terrain, only a single hill has been discussed. This has been important as a mathematical foundation, but lacking in practical value. To be of use in a combat simulation, the hills must be created collectively in a size, quantity and configuration so as to represent a desired type of terrain. For example, a user may want terrain characterized by a few, low rolling hills; or perhaps rugged, mountains with peaks of widely varying elevations. Both can be modelled by adjusting parameters.

The Extended Simulated Terrain Model (SIMTER-X) has two parts. The first is the main program which creates terrain and, at the discretion of the user, plots both a three-dimensional drawing and a contour map. Used strictly for terrain generation, it can be used as a preprocessor for other combat models by producing grid points and elevations similar to those provided on computer tapes by WES.

The second part of SIMTER-X is a movement/line-of-sight subroutine (MOVLOS) which moves a target along specified routes across the generated terrain. This part will be discussed in further detail later in the paper.

In addition to those input parameters used to describe a single hill, others are used to aggregate the hills into a map. These include the following:

1. The dimensions of the battlefield in meters.
2. The grid interval (e.g., 10 meters or 100 meters).
3. The number of hills to be created.
4. How much the peaks of the hills are to vary.
5. How much the spread of the hills are to vary.
6. How many ridge lines, if any, are to be created.

Given the input parameters, the model proceeds as follows.

1. The first step of the program is to randomly select the centers of mass for the desired number of hills. This operation is performed by drawing Uniform (0, 1) random numbers and multiplying them by the size of the battlefield in the X and Y directions, thus producing an array of paired grid points. If ridge lines are desired, then the random points in either the X or Y direction, but not both, can be biased by drawing random normal deviates about a preselected ridgeline center. Although not a requirement, this particular program makes a hasty plot of the centers of mass points for visual reference of the random process.
2. The second step is to create the dimensions of each MBVN hill. If the user desires that all hills be of the same size and shape, then this step is complete. However, for most cases the height, spread and slope of each hill will be varied according to input parameter values which are actually standard deviations for the variations. For example, if a user desires hills which average 200 meters high, but vary about this average value by approximately 40 meters, then 200 becomes the mean value for the peaks of the hills which vary in individual elevations according to normally distributed random numbers whose standard deviation is 30. The same is done for the spread and slope of the hills.

If only movement and line-of-sight calculations are to be made, then MOVLOS subroutine is called and the program is terminated. A major advantage of this approach to terrain modelling is that no 10 meter, 100 meter, or other grid system is necessary. The terrain representation is continuous; therefore, the elevation at any point on the map can be found without storing any digitized (discrete) map information. Only the hill parameters need be stored.

3. If a matrix of grid values is desirable, as would be the case if the program were used in lieu of the WES computer tapes, then the third part of

the SIMTER-X main program is the creation of an evenly spaced grid system, to include the elevation at each grid line intersection. For all the maps and drawings in this study, a 100 meter grid interval was used.

Since the MBVN distribution has some finite value (elevation) in all directions, regardless of how far removed from its center, the elevation of each hill must be computed for each point in the grid system. As each hill is checked for its elevation at any selected point, it is compared with its predecessor. If it is lower, it is discarded; otherwise it is saved for comparison with the next hill's elevation at that same point. After each hill has been searched, only the highest value is stored. It is this value, along with the other maximum values at each grid interval, which make up the terrain surface. If there is a large number of hills, the search process can be streamlined by truncating the MBVN if the height of the density falls below some specified value.

For example, consider two intersecting hill masses as shown in Figure 4.

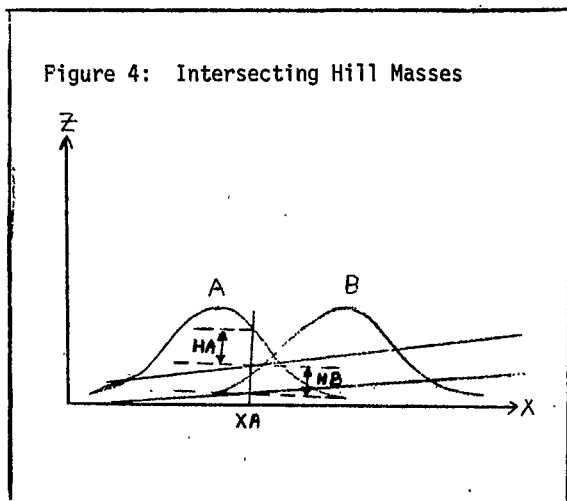


Figure 4: Intersecting Hill Masses

HB defines height of hill "B" after cutting.

HA defines height of hill "A" after cutting.

The height of hill "A" is greater than that of hill "B" at the point XA but the height of hill "B" is greater than that of hill "A" after cutting. If several hills are in close proximity to each other, those hills must be adjusted by comparing the height after cutting each one.

MOVEMENT/LINE-OF-SIGHT MODEL

The movement/line-of-sight subroutine (MOVLOS) performs three main calculations: movement, line-of-sight, and percent of target area visible. To perform these functions additional input parameters are required. The first of these is the

location of a stationary observer or defender, the second is an array of coordinates which depicts the route of a moving target, and the final input group includes the dimensions of the target. For this program, the target is presumed to be rectangular so that height, width, and length are required.

The first step is to determine the elevation of the observer by searching for the highest hill at the observer coordinates. This is accomplished by substituting the observer XY coordinates into the equation of each MBVN hill. The highest resulting functional value is the elevation of the observer. This procedure is likewise carried out for the elevation of the target. Using the three-dimensional coordinates of both the observer and the target, the subroutine forms the equations for the Observer-Target (O-T) line in three space. The projection of the O-T line intersects at right angles with lines drawn from the hill mass centers.

The elevation of the O-T line is also computed at each intersection. This is done using the ratio of sides of similar triangles. If any hill is higher than the O-T line along the line's path, then intervisibility does not exist.

To be of use in a high resolution model, the line-of-sight subroutine must answer more than "yes" or "no" to the question of intervisibility. Since the probability of a hit (P_h) is partly a function of target size, the program must determine what effective area of the target is visible to the firer. This, in turn, is a function of the target's dimensions, the percent of area exposed above the terrain, and the angle it is facing with respect to the observer. Using basic trigonometry, MOVLOS computes the area of the target projected in the direction of the O-T line.

The dynamics of the program are provided by the movement portion of MOVLOS. At discrete time intervals, the target moves along the preselected routes toward its objective. For all runs of SIMTER a one-second time interval was used. At each second an instantaneous velocity and a line-of-sight are computed. If the velocity is not held constant, it is then solely a function of the slope of the terrain. The slope is found by taking the directional derivative (DEL) of the MBVN in the direction of the target.

Target movement along the surface slope of the hill is computed as follows:

$$ZH = MH * \text{EXP} \left\{ - \frac{1}{2(1-\rho^2)} \left[\left(\frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left(\frac{x-\mu_x}{\sigma_x} \right) \left(\frac{y-\mu_y}{\sigma_y} \right) + \left(\frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\} - (X-XIN) \tan \theta - (Y-YIN) \tan \phi \quad (14)$$

where:

MH = height of the dominant hill at (x,y).

θ, ϕ = angles determined by the slope of the cutting plane.

PARAMETERIZED TERRAIN

The directional derivative, DEL, of the MBVN in (14) in the direction of target movement, γ , is given by

$$\text{DEL} = \frac{\partial ZH}{\partial x} \cos \gamma + \frac{\partial ZH}{\partial y} \sin \gamma \quad (15)$$

The target speed, VEL, dependent on the slope of the terrain, is computed by

$$\text{VEL} = V + W (\text{DEL}) \quad (16)$$

where

V = target speed on flat terrain

W = speed factor of slope.

The output of the Movement/Line-of-sight sub-routine includes the elevation, coordinates, speed, per cent exposed, and area exposed at each one-second time increment. Additionally, a summary is provided at the end of the simulation. The information printed is listed below:

1. The number and length of intervisibility segments.
2. Total distance traveled with intervisibility.
3. Total distance traveled without intervisibility.
4. Total distance traveled.
5. Average distance traveled with intervisibility.
6. Per cent of time in which intervisibility existed.

A sample two-dimensional terrain, indicating one route from the target initial position (T) to the observer position (O) is given in Figure 5. The three dimensional view of the terrain and route, along with the resulting intervisibility segment length data, is given in Figure 6.

III. TEST AND EVALUATION OF SIMTER-X MODEL RESULTS

To ensure that the model was responding to all changes in parameters, a series of verification runs were conducted. It was not the intent of these replications to establish the sensitivity of line-of-sight to variations in terrain. The latter was the subject of the previously mentioned VRI study (2).

The methodology for parametrically describing terrain as a function of the number of hills, the center and spread of each hill, and the slope was developed in the previous sections. In this section the effect of the number of hills, height of hills, and the slope cutting method on various intervisibility measures is investigated. Three random terrains were generated (with each terrain having six, twelve, and eighteen hills) resulting in nine basic terrain configurations. For each

configuration three average hill heights (100, 200, and 300 meters) were utilized.

Finally, for each of the 27 resulting configurations, four slope cutting methods were used as follows:

1. Down slope-cut from observer side to target side.
2. Side slope-cut from right to left as viewed from the target.
3. Up slope - cut from target side to observer side.
4. Symmetric slope as viewed from any position.

A total of 108 unique terrain configurations were generated for the investigation. For each of the configurations, the mean and standard deviation describing the center and spread of each hill is specified. In addition, the cutting angle for determining slope must also be specified.

If the cutting angle is constant, then the hill can be described as follows:

$$\text{HILL} = F(\text{Peak, Position, Standard Deviation, Slope})$$

In the constant cutting angle case, only the slope is variable, with peak, standard deviation and position constant. In other words, if the slope varies, the standard deviation of the resulting distribution also varies.

The objective of the methodology developed was to maintain a constant standard deviation of the distribution function resulting from each cutting angle, so that only the slope of the hill varies. Otherwise the determination of how slope affects the intervisibility parameters would not be possible, since it would be confounded with the standard deviation (spread) of the hills.

For each terrain an observer position and initial target position were selected. Two routes from the target to the observer position were selected in accordance with accepted tactics of advance routes.

Two cases of target movement were considered in the analysis.

1. Constant Speed

The target moves along the route at a constant speed of six meters per second, independent of hill slopes. The constant speed runs serve as the base case for measurement of the percent of time intervisible along the route.

2. Variable Speed

The speed of the target is a function of the terrain slope being traversed. The speed of the vehicle is computed by the relationship as follows:

$$\text{velocity} = 6.0 + 4.0 (\text{slope})$$

This slope is the directional derivative value at the instantaneous target position on the hill.

Figure 5: Sample Terrain With Routes

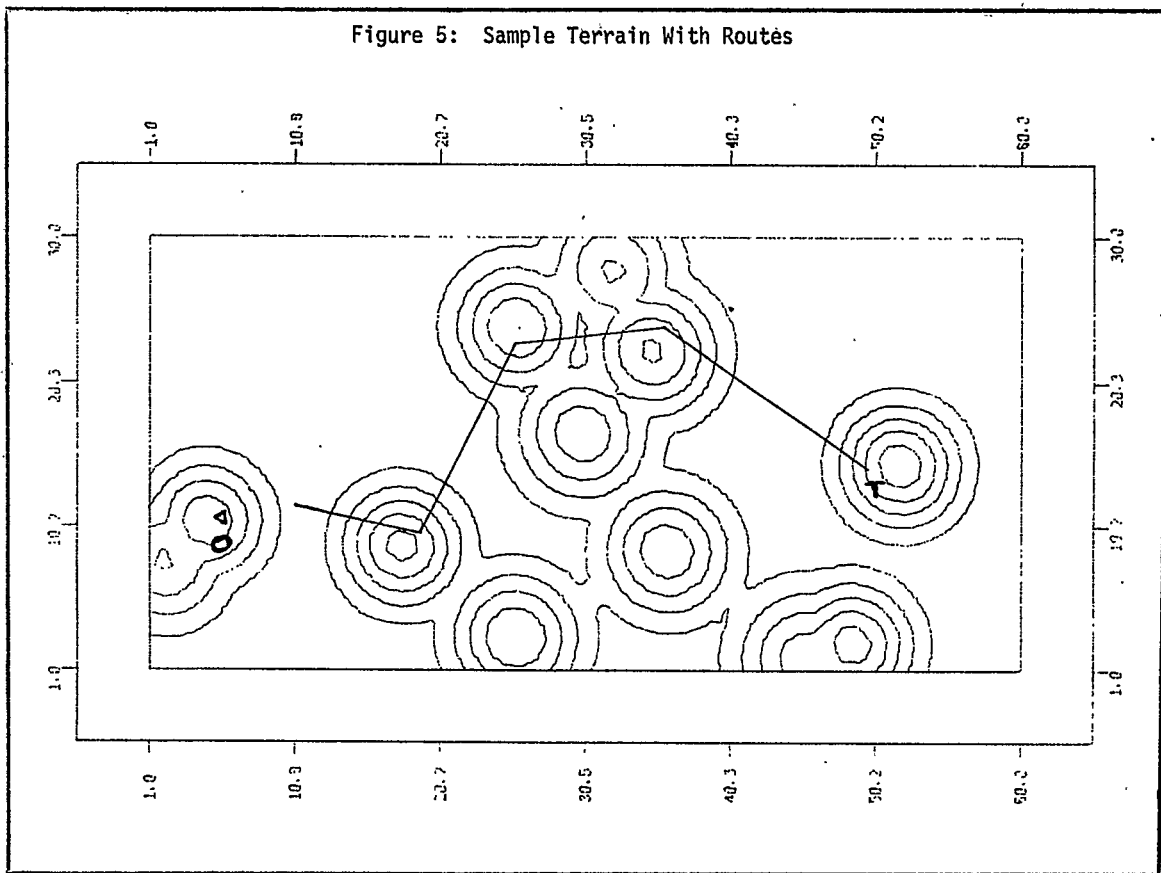
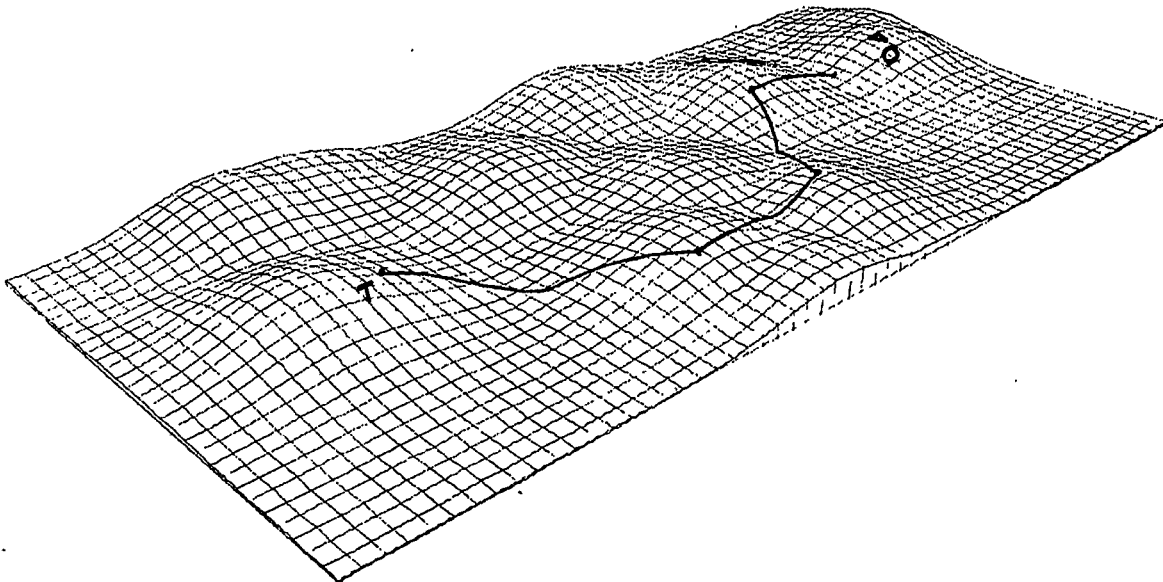


Figure 6: Three Dimensional View of Routes

INTERVISIBILITY SEGMENT LENGTHS				
	1	1119.704		
	2	442.767		
DISTANCE UNCOVERED	DISTANCE COVERED	TOTAL DISTANCE	AVG DISTANCE INTERVISIBLE	PER CENT TIME INTERVISIBLE
1562.471	3388.935	4951.436	781.236	0.277



PARAMETERIZED TERRAIN

AVERAGE ELEVATION OF TERRAIN PEAKS (METERS)

	Number of Hills	100		200		
		Percentage of time Intervisible	Average Distance Intervisible Per Segment	Percentage of time Intervisible	Average Distance Intervisible Per Segment	
D I R E C T I O N O F	UP	6	88.4	2819	84.6	2215
		12	52.8	1618	51.9	1203
		18	21.1	410	17.3	352
C U T T I N G	DOWN	6	80.2	2520	77.6	1930
		12	49.6	1401	43.3	919
		18	16.4	263	10.6	191
A N G L E	SIDE	6	71.3	2104	61.3	1617
		12	36.6	835	28.5	516
		18	8.1	105	5.1	84
	SYMMETRIC (No Cutting)	6	75.7	2210	69.2	1810
		12	39.1	717	35.6	693
		18	12.2	151	9.4	106

Table 1: Results of SIMTER-X Execution

For the three cases of up, down and side slope, a cutting angle of fifteen degrees was used. The symmetric case was generated with a cutting angle of zero degrees using the methodology previously described.

A portion of the results derived from execution of the SIMTER-X model is given in Table 1. These results are for the variable speed case previously described. Space does not permit the presentation of the full range of results obtained by varying such factors as the spread of the hills, vehicle speed capabilities, maximum hill height, etc.

The results presented in Table 1 are presented to exhibit the capabilities of the model and to provide for initial model validation. Note that the number of hills has the greatest influence on the percent of time intervisible and the average distance intervisible per segment. The direction of the cutting plane slope, which represents various orientations of steep and gentle slopes, also influences the dependent variables. The average height of the hills exhibits the least influence on the intervisibility parameters.

IV. CONCLUSIONS

If exact terrain modelling is not required in a combat simulation, then representative terrain

can be created using a modified bivariate normal distribution and cutting plane methodology. Since there is no requirement for survey or photographic interpretation in order to mathematically model terrain, this approach is significantly less costly and time consuming than digitizing terrain. An additional advantage of mathematical representation is that replications of a "type" of terrain can be randomized, thereby improving the statistical level of confidence in a combat model output.

The parameterized terrain is continuous; therefore, the elevation and location is exact and not a linear interpolation between discrete points. This makes it possible for line-of-sight calculations to be more accurate. On the other hand if the program is to be used as a terrain preprocessor for a high resolution combat model, then SIMTER-X can produce a digitized output from the parameterized representation.

The results of the tests indicate that the parametric representation of terrain is both useful and realistic. In view of these conclusions the following actions are recommended:

1. Parameterized terrain should be run against digitized actual terrain in a high resolution model such as DYN TACS or Army Mobility Model. Sufficient replications of the randomized parametric terrain should be conducted in order to establish steady state results.

2. Distributions other than the bivariate normal should be examined, e.g., the beta.

3. The feasibility of representing actual terrain with the MBVN should be examined by pre-selecting centers of hills as they appear on a map.

4. The SIMTER-X simulation should be evaluated for use as a mobility model. Routes can be selected or readily changed commensurate with vehicle performance.

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