ESTIMATING THE TOTAL REAL PROPERTY VALUE OF GEOGRAPHIC AREAS FOR EQUALIZATION--A COMPUTER SIMULATION

James M. Kraushaar

Steven Hershey

ABSTRACT

A computer model of the equalization rate determination procedure is presented. Several changes in sampling and estimation parameters are examined using a full factorial design.

INTRODUCTION

Property valuation is a major function of governments at all levels [1]. Local governments make assessments of each property under their jurisdiction mainly to collect property taxes. Larger governmental units, like state and county governments, use the total property value of each town and village under their jurisdiction to perform a variety of governmental functions. For example, total property value is used to collect taxes from each town and village to allocate funds among the towns and villages and to control the taxing and borrowing powers of the towns and villages [2].

STATEMENT OF THE PROBLEM

It would seem a simple task for state and county governments to obtain the total property value of localities since each town or village determines the value of all the properties under its jurisdiction. However, for many reasons, the individual assessments made by each town or village often lead to total property values that are not comparable among districts if the state or county governments simply aggregate the local assessed property values. For example, each district may use a different method for determining property assessments [3]. Also, local assessments may have accurately reflected property market value at one time, but may not have been updated in subsequent periods to reflect inflation [4]. For these and other reasons similar properties in different towns or counties could have widely varying assessed valuations [5].

State and county decisions based on assessed property values are not equitable without current and comparable total property values for each town and county [6]. Therefore, state and county governments have had to develop their own techniques to provide consistent, rapid, and accurate total property market value estimates for the towns and counties under their jurisdiction.

State and county governments cannot determine the property value for each property in all towns and counties under their jurisdiction because of the large number of properties involved. Instead, for each town and county only a sample of properties is appraised at market value by a qualified appraiser. These appraisals along with recent property sales information are used to estimate the total property value [7]. The number of properties sampled and appraised is necessarily small because the accurate appraisal of properties is time consuming and costly [8]. Therefore, the efficiency of the procedures used to estimate total property market value can greatly affect the quality of the outcome. It is, therefore, necessary to review and modify these procedures in order to ensure that the most up-to-date efficient procedures are used.

JUSTIFICATION FOR STUDY

The state or county is then left with only one choice when current equalization techniques are not satisfactory [9], namely, experimentation. Real world experimentation is difficult in controlled industrial environments; but in bureaucratic government organizations, the problems are greatly multiplied. Therefore, the development of a simulation model of the equalization process provides a very useful mangement device.

The computer simulation model of real property values in a geographic area allows various statistical, political, and economic scenarios to be tested without the problems associated with real world experimentation. For example, the effect of a change in sampling strategy can be measured before the actual new strategy is implemented. Or, a forecasted large increase in real property value can be evaluated in terms of the ability of current sampling and estimation procedures to accurately reflect the economic change.

OBJECTIVES

The major objective of this study is to examine several administratively controlled factors that affect the determination of total real property value for government units such as counties, cities, and towns. Briefly, these factors are:

1. The sampling method used to collect information about appraisal market value and recent comparable sales.

2. The estimation method used to determine the total property market value for a specific year from the sample of information collected.

3. The trending method used to incorporate previous years' assessed and market value changes into the current estimate of total market value.

4. The various political and economic forces affecting the assessed and market value of real property.

METHODOLOGY

This study is a significant extension of previous work done by the author for residential properties only [10]. The current model, although similar in some respects, is oriented towards the whole equalization process and attempts to help solve broad managerial problems rather than to help answer more narrow statistical questions.

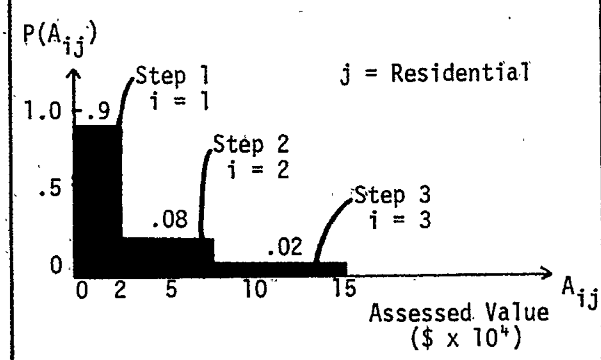
MODEL OF ASSESSED AND MARKET VALUES

Assessed property values are assumed to follow a step function. The assessed value of the k-th property is:

$$A_{ijk} = \text{STEP}(XLO,XHI,P,i,j) \qquad (1)$$

where XLO and XHI are the smallest and largest assessed value for a step. P is the probability of being in the i-th step of the j-th property type. An example of the assessed value step function used for residential properties is presented in Figure 1.

FIGURE 1



The market value of the k-th property [11], i-th step, and j-th type is assumed to be linearly related to assessed value as follows [12]:

$$M_{ijk} = B0_{ij} + B1_{ij}A_{ijk} + E(0,\sigma_{ij}) \qquad (2)$$

where $B0_{ij}$ and $B1_{ij}$ are the y-intercept and slope, respectively, for the i-th step and j-th property type. $E(0,\sigma_{ij})$ is a normally distributed error term mean zero and standard deviation of σ_{ij}.

MODEL FOR CONDUCTING MARKET VALUE SURVEYS

The equalization rate determination procedure used in this study has three steps [13]. Step one stratifies the assessment role by assessed value or property type [14]. Step two selects a random sample of properties from the stratified assessed role for appraisal [15]. Step three estimates the total market value necessary for a determination of the equalization rate.

After stratification appraisals are allocated among the strata proportional to [16]:

$$\frac{N_h(S_{mh}^2 + R^2S_{ah}^2 - 2RP_hS_{mh}S_{ah})}{\sqrt{C_h}} \qquad (3)$$

where for the h-th stratum,

N_h = Total number of properties

S_{mh} = Standard deviation of property market values

S_{ah} = Standard deviation of assessed values

P_h = Coefficient of correlation between market and assessed value

C_h = Relative cost of making a property appraisal

R = Ratio of market to assessed value for all properties

Three estimates of total market value are calculated as follows:

$$\hat{M}_{sme} = N \sum_{h=1}^{L} f_h\bar{m}_h \qquad (4)$$

$$\hat{M}_{cr} = \frac{\sum^{L} N_h\bar{a}_h}{\sum^{L} N_h\bar{m}_h} A \qquad (5)$$

$$\hat{M}_{sr} = \sum^{L} \frac{\bar{m}_h}{\bar{a}_h} A_h \qquad (6)$$

where:

N = Total number of properties in all strata

f_h = Fraction of properties for stratum h

$\overline{m}_h$ = Sample mean appraised value

$\overline{a}_h$ = Sample mean assessed value

A_h = Total assessed value

A = Total assessed value for all properties

L = Total number of strata

A MONTE CARLO SAMPLING METHOD FOR MEASURING THE EFFICIENCY OF SAMPLING AND ESTIMATION PROCEDURES [17]

The procedure used in this study to estimate the true sampling error of the ratio estimator is dissimilar to the highest order product moments method. This method employs a simulated sampling distribution for the ratio estimator by empirical sampling [18]. Random samples are repeatedly selected from the population of properties according to the procedures discussed in the last section. For each sample, an estimate of total residential market value is calculated. The variability in these estimates provides a simple estimate of true sampling error without resorting to the complex estimators which have product moments beyond second order.

The mean square error for an estimator of total market value is estimated by randomly selecting n_r samples of size n from all possible samples and using the formula:

$$\hat{MSE}_{\hat{T}g} = \sum_{l=1}^{n_r} (\hat{T}_{lg} - T)^2/n_r \qquad (7)$$

where T_{lg} is the estimate of total market value from the l-th sample and for the g-th estimator type (g = 1, 2, or 3; unbiased mean expansion, combined ratio, separate ratio). T is the known total market value for the simulated population:

$$T = \sum_{k=1}^{N} m_k \qquad (8)$$

where m_k is the market value for the k-th residential property and N is the total number of simulated properties.

The estimated variance, $\hat{V}(\hat{T}_g)$, for an estimator of total market value based on n_r estimates of total value is:

$$\hat{V}(\hat{T}_g) = \sum_{l=1}^{n_r} (\hat{T}_{lg} - \hat{\overline{T}}_g)^2/(n_r - 1) \qquad (9)$$

where the mean of the n_r estimates of total value for the g-th estimator type is:

$$\hat{\overline{T}}_g = \sum_{l=1}^{n_r} \hat{T}_{lg}/n_r \qquad (10)$$

The bias exhibited by the two ratio estimators of total value can be estimated by:

$$\hat{BIAS}_{\hat{T}_g} = \hat{\overline{T}}_g - T \qquad (11)$$

where g = 1, 2, or 3 as defined above, $\hat{\overline{T}}_g$ is the mean of the n_r estimates of total market value, and T is the known total market value.

COMPUTER SIMULATION MODEL

A simulation model composed of two separate modules was developed to measure the effect of administrative changes in procedures used to conduct market value surveys and establish equalization rates.

The first module creates a hypothetical population of properties with assessed and market value using step functions discussed earlier. Input parameters determine the property type mix and the property value range for the hypothetical population. The output is a file with type, assessed value, and market value of each property.

The second module performs the sampling and estimating tasks listed below:

1. Stratification
2. Iterative optimal allocation
3. Stratified random sampling
4. Estimation of total property market value
5. Estimation of estimator efficiency

The file of properties is stratified by either assessed value or property type. The sample allocation is calculated by (3). If $n_h > N_h$, the calculation is reiterated. A simple random sample without replacement is selected from each stratum. Three estimates of total market value are calculated using (4), (5), (6). After repeating the sampling process, estimates of the bias, standard error, and mean square error are calculated using (7), (9), (11).

Input parameters determine the sample size, number of strata, strata boundaries, relative stratum costs, and number of times the sampling is to be repeated.

Output includes strata parameters (e.g., mean, coefficient of variation) as well as estimator efficiency measures (e.g., bias).

The computer programs were written in FORTRAN IV using the ANSI compiler at California State University, Fresno and the WATFIV compiler at Syracuse University. The WATFIV compiler proved indispensable in debugging the program quickly. The actual experimental runs were made on an IBM 370/155 with the WATFIV compiler. The program is also running on a CDC 3150 machine with an ANSI FORTRAN compiler. Hand calculations to verify program results were made using APL and an IBM 370/155.

Module one consists of 665 statements which compiled and executed in approximately 3.2 and 19 seconds, respectively. The object code and array area required 30K and 37K bytes of storage, respectively [19].

Module two consisted of approximately 680 statements which compiled and executed in approximately 3.45 and 90-110 seconds, respectively. The object code and array area required approximately 37K and 40K bytes of storage, respectively.

EXPERIMENTAL DESIGN

A complete factorial design for three factors and three replications was performed [20]. The factors and levels are as follows:

FACTOR	LEVELS OF FACTOR
A: Estimator Type	1. SUME 2. RATIO 3. RATIO
B: Population Type	1. Largely Residential 2. Largely Commercial
C: Stratification Method	1. Assessed Value 2. Property Type

RESULTS

Tables 1 and 2 present the experimental results. Examination of these tables indicates a very small sampling error. Table 3 confirms this observation as all main and interaction effects for $\sqrt{MSE}$ are highly significant. Similar results were obtained for |Bias| but have not been presented.

The use of a multiple random number generator and 500 repeated samples per replication unquestionably contribute to the smallness of the error term. It appears that less than 500 repeated samples per run would suffice for this experiment.

TABLE 1

$\sqrt{MSE} \times 10^6$

	SME	CR	SR
Largely Residential Assessed Value Stratification	6.7	4.2	4.4
	6.4	4.3	4.4
	6.6	4.4	4.6
Property Type Stratification	157.2	36.7	37.7
	162.5	37.4	37.2
	169.2	38.8	39.0
Largely Commercial Assessed Value Stratification	204.8	12.3	17.9
	192.7	11.9	18.2
	200.8	12.6	18.4
Property Type Stratification	1268.1	86.0	92.5
	1234.6	83.8	92.7
	1258.9	84.9	91.4

TABLE 2

$|BIAS| \times 10^4$

	SME	CR	SR
Largely Residential Assessed Value Stratification	6.5	12.9	39.8
	15.0	9.0	21.3
	10.5	9.6	22.3
Property Type Stratification	58.7	205.1	529.8
	494.6	301.7	46.1
	2168.0	163.1	501.1
Largely Commercial Assessed Value Stratification	968.0	6.4	744.6
	390.5	26.3	812.4
	2621.2	30.5	709.5
Property Type Stratification	331.5	1063.4	746.6
	2444.0	872.3	720.8
	676.9	1671.8	1226.9

TABLE 3

COMPLETE FACTORIAL ANOVA FOR $\sqrt{MSE}$

Source	df	ss	ms
A	2	1090662	545331
B	1	494865	494865
AB	2	746926	373463
AC	2	609500	304750
BC	1	239055	239055
ABC	2	368182	184091
ERROR	24	755	31.5
TOTAL	35	4057130	

CONCLUSIONS

Based on the $\sqrt{MSE}$ criteria, assessed value stratification is clearly superior to property type for both the largely residential and the largely commercial populations. An interesting question requiring more experimentation is whether assessed value and property type stratification should both be used.

It appears more difficult (i.e., the $\sqrt{MSE}$ is larger) to estimate total market value for the commercially dominated property mix. However, at the lowest $\sqrt{MSE}$ level, the coefficients of variation are approximately .4% and .6% for the commercial and residential mixes, respectively. Also, there is little difference between residential and commercial mixes with assessed value stratification.

In general, the ratio estimators are to be preferred to the simple unbiased mean expansion estimator since they have lower $\sqrt{MSE}$. The combined ratio is to be preferred to the separate ratio; although with a residential mix stratified by assessed value, there is very little difference.

These conclusions are based solely on the $\sqrt{MSE}$ as a measure of precision. If costs are not comparable, then a more appropriate measure of precision is $\sqrt{MSE}$/Cost.

If costs are roughly equal for different estimation and stratification methods, the combined ratio estimator should be used with assessed value stratification for both largely residential and largely commercial populations.

FUTURE RESEARCH

For this simulation model to be useful, two additions must be made. First, the model should have the capability to include recent sales data. Second, the model should be dynamic by taking previous equalization rates into account and by allowing changes in assessed and market values over time.

REFERENCES

[1] Bergren, A. L., "Equalization in New York," National Tax Association Proceedings for the 49th Annual Conference (1956), pp. 225-226.

[2] Beeman, W. L., The Property Tax and the Spatial Pattern of Growth Within Urban Areas, Ph.D. Dissertation, Syracuse University, 1968.

[3] Bancroft, D. A., Variations of Property Tax Assessments in Alberta, Ph.D. Dissertation, Syracuse University, 1971.

[4] Cheng, P. L., "The Common Level of Assessment in Property Taxation," National Tax Journal 23 (1970), pp. 50-65.

[5] Sazama, G. W., "Equalization of Property Taxes for the Nation's Largest Central Cities," National Tax Journal 18 (1965), pp. 151-161.

[6] Rosett, R. N., "Inequality in Real Property Tax of New York State and the Aggravating Effects of Litigation," National Tax Journal 24 (1971), pp. 37-43.

[7] Some states, for example California, do not currently use recent property sales information in the equalization rate determination except as they affect appraised value.

[8] For example, New York State samples often represent less than 1% of the properties.

[9] Ross, M. H., "The Property Tax Assessment Review Process: A Case for Regressive Property Taxation?" National Tax Journal 24 (1971), pp. 37-43.

[10] Kraushaar, James M., "A Simulation Model for the Comparison of Sampling Strategies Used in Estimating Total Residential Market Value for a Geographic Area," Proceedings of the Winter Simulation Conference, 1976, pp. 211-220.

[11] Market value has many definitions. This study uses a consistent definition of market value but is not constrained to a single definition.

[12] Empirical analysis of several areas in California and New York indicate this assumption is reasonable.

[13] Often a fourth step of either forecasting equalization rates (e.g., California) for future years or combining past years' equalization rates (e.g., New York) is performed.

[14] California stratifies by assessed value but has considered property type stratification, while New York uses both assessed value and property type stratifications.

[15] Appraisal in this study is the determination of market value.

[16] California uses this method of allocating sample appraisals.

[17] This method was originally described in an earlier article by one of the authors. See reference 10.

[18] This technique has been widely used by other researchers. See, for example, Hutchinson, "A Monte Carlo Comparison of Some Ratio Estimators," Biometrika 58 (1971), pp. 313-321.

[19] The times and storage requirements presented are part of the WATFIV compiler output for an IBM 370/155.

[20] The APL function ANOVA in library 4 workspace STP1 at Syracuse University was used to do this analysis.