A Simulation Approach to Examining Traditional EOQ/EOP and Single Order Exponential Smoothing Efficiency Adopting a Small Business Perspective

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Abstract

This paper addresses the question of how well classical economic order quantity (EOQ) and economic order point (EOP) logic performs under conditions of varying demand over time. A second question addressed is how well does first-order exponential smoothing serve as a forecasting tool in the face of varying demand. A small business perspective is adopted.

The above questions are addressed for the following reasons:

1) EOQ and EOP logic are still widely taught in schools of business, often without consideration of the current academic and practical surge of interest in material requirements planning (MRP);
2) Small businesses will be the last to join the MRP "bandwagon" due in part to the current lack of software and cost effective computer resources -- and therefore will remain the most likely candidates for continued use of EOQ and EOP in the traditional fashion;
3) Small businesses most likely conduct less forecasting than larger organizations who provide staff to accomplish tasks such as forecasting -- and therefore could potentially benefit greatly from a simplistic tool such as first-order exponential smoothing.

A simulation approach is adopted for addressing the above defined research questions.

INTRODUCTION

Traditional economic order quantity (EOQ) logic states that, for a given inventory item, total variable cost is the sum of ordering costs and inventory carrying costs. This logic presupposes that ordering costs are balanced against (equal to) inventory carrying costs. Generally, safety stock carrying costs and out-of-stock costs are ignored, the latter often because of the inability of measure stockout costs in any acceptable fashion. Also ignored is the interaction between the optimal order period calculation and the safety stock calculation.

This paper presumes that many small businessmen are simply not aware of the logical problems associated with EOQ and economic order point (EOP) usage. Any lack of awareness concerning assumptions, independent-dependent demand principle, and the like, may account for a portion of the historical problems encountered by users of traditional EOQ/EOP. Many business users of order-quantity/order-point theory often adopt the posture that the traditional calculations are "close enough" and that additional refinement would be both cost and time inefficient. This paper will not argue with this logic as used by many small businessmen -- but will explore the question of how well traditional EOQ and EOP logic performs under conditions of varying demand.

A second presumption of this paper, as witnessed by this author in dealings with small businessmen, is that small businessmen often do not forecast in a formal fashion. The "dart board" or "what do you think" method tends to dominate many small enterprises. This paper will additionally explore how well a simplistic forecasting technique, specifically first-order exponential smoothing, performs employing data that possesses a potentially wide range of variability, such as could be expected to be experienced in a real-world situation.

To address both the EOQ/EOP portions of this study and the forecasting portion, a simulation approach was adopted.

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Simulation Approach

SIMULATOR DESIGN

The inventory simulator designed presumes normally distributed demand data with a weekly time frame (rectangular distribution of usage within any given week). The user specifies the historical mean usage per week and historical weekly standard deviation. No trend or seasonality is presumed. Treatment of linear trend, seasonality and additional potentially realistic demand patterns will be considered in a subsequent paper. EOQ is computed in a normal sense employing the traditional square root formula. Item unit cost, carrying cost as a percent of item unit cost, and setup costs are all specified by the user. Lead time in weeks is also user defined. For the results to be described below, expected lead time may be specified as from one to ten weeks. No variability in actual versus input lead time was permitted, an obviously unrealistic condition, but one that would only distract from the specific objectives of the simulation design. EOP is computed as the sum of mean demand through lead time plus safety stock. Mean demand through lead time is the product of mean weekly demand times the expected value of the lead time (L) in weeks. Safety stock is computed as the square root of the lead time in weeks multiplied by the weekly standard deviation, all times the standard normal deviate (Z) value corresponding to a desired stockout rate [3].

Notationally,

EOP = Mean demand through lead time + safety stock

or

$$EOP = \overline{X} \cdot L + \sqrt{L} \cdot S_x \cdot Z$$

For simulation purposes, a Z-score of 1.645 was used throughout which presumes a desired service rate of 95 percent (stockout rate equals 5 percent). Also, a constant random number seed and therefore constant pattern of demand was used to test and compare results obtained when varying different parameters. The Box-Muller method for generating normally distributed random numbers with a prescribed mean and standard deviation was employed [1].

Using selected data as follows, an interesting result was obtained

$\overline{X}$ = 100 units/week

S_x = 10 units/week

Item unit cost = $5.00

$$EOQ = \sqrt{\frac{2 \cdot 5000 \cdot 25}{5(.25)}}$$

$$= 447$$

Annual demand (D) = 5000 units/year based on 50 weeks/year

Carrying cost = 25%/year as a percentage of item unit cost

Setup cost = $25.00/setup

Simulation period = 1000 weeks or 20 years

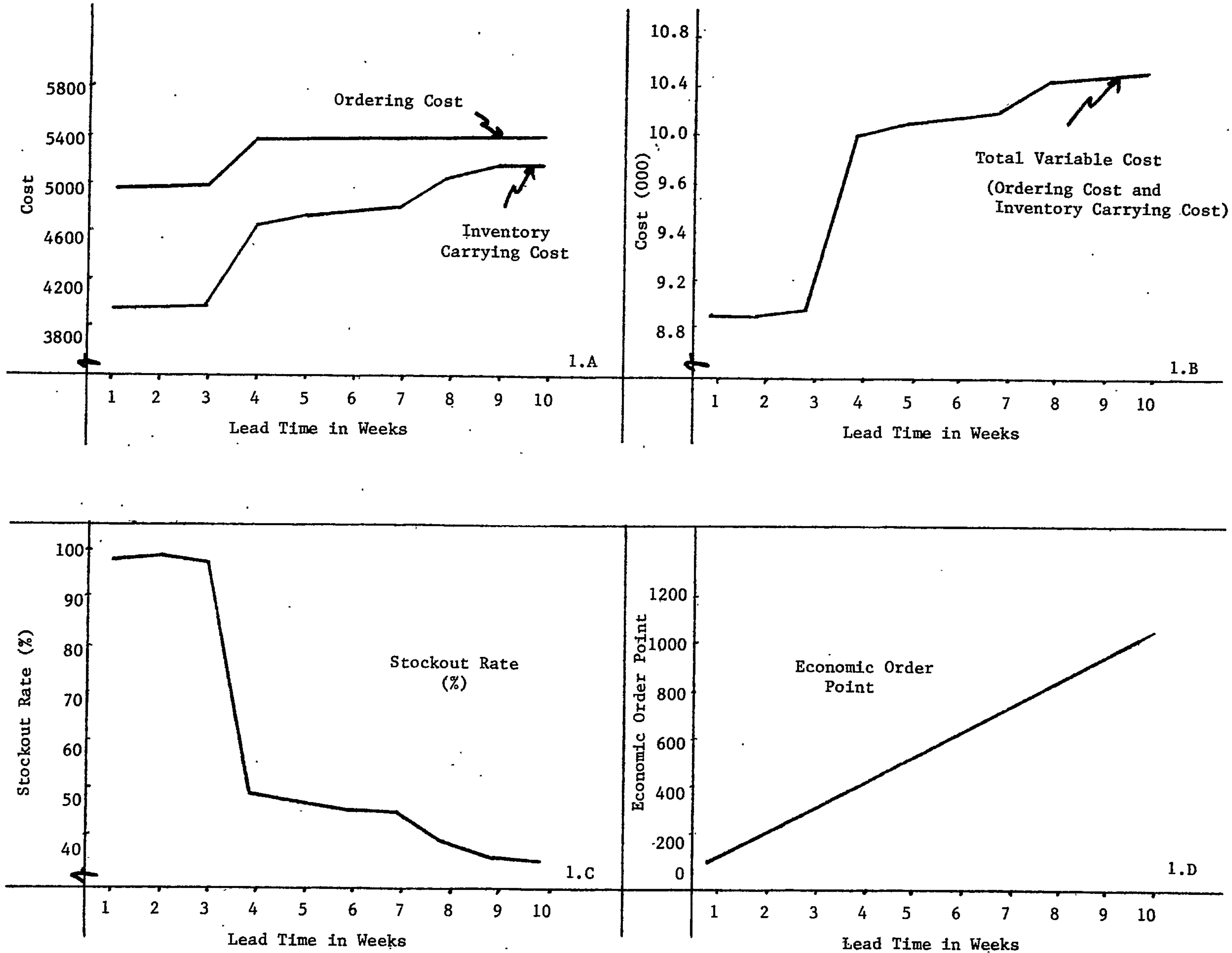
with regard to the simulated stockout rate. Figure 1-C indicates that the stockout rate varied from a maximum of 98.0 percent when lead time was one week to approximately 35 percent for a lead time of 10 weeks. EOP was computed in the traditional fashion as described above. Multiple orders up to five orders outstanding at one time were permitted. These stockout rate results were in obvious contrast to the theoretical stockout rate of 5 percent.

Some of the variability in stockout rate results obtained may be attributed to the fact that an EOQ of 447 (based on the above data) and an estimated annual demand of 5000, suggests approximately eleven "exposures to stockout" per year. The desired stockout rate of 5 percent (Z = 1.645) implies only one stockout approximately every two years. Again, the interaction between the optimal order period calculation and safety stock calculation is ignored -- a particulary common occurrence by small businessmen who may not be familiar with the basics of EOQ logic, let alone the finer points associated with the interaction between seemingly independent calculations.

The simulator was designed such that orders were placed at the completion of a week's business activity at which time a review of the inventory situation could be presumed. This assumption is inconsistent with the general theory of order placement at the instant a new order is suggested, but, and the opinion of this author, fairly consistent with the operating practice of many small businessmen. Therefore, a "timing" problem exists in the EOP calculations, adding further to any distortion caused by any inconsistency between the order period and safety stock calculations.

This timing problem results from the condition of when an order <u>should</u> be placed relative to when an order <u>is</u> placed.

FIGURE 1



No Adjustment to Order Point Calculation

Simulation Approach

Assuming a uniform distribution of demand within a week's time span, an order should be placed anywhere from very early in an ordering week to the actual end of the week. In other words, the theory of EOP assumes a continuous distribution for order placement but, and in reality, a discrete distribution for placing orders may be the case.

Since, and on the average, one-half week will have transpired from the time an order should have been placed to when an order was placed (end-of-week), adjusting the EOP calculation within the simulator by adding one-half week's demand to EOP, produced stockout results as depicted in Figure 2-C. As can be seen from this figure, the stockout rate varied from a maximum of 33.6 percent for a one-week lead time to 10.9 percent for a ten-week lead time. Randomization of demand data can now be attributed to producing stockout rates inconsistent with pure theory. When an adjustment to EOP of 0.75 of one week's demand was made (graphics not shown), the stockout rate varied from a maximum 10.3 percent for a lead time of one week to a minimum 3.1 percent for a lead time of nine weeks. Adjusting the EOP by a full week's demand produced a maximum stockout rate of 0.9 percent (lead time = 1 week) to zero percent in several lead time cases -- which is much more in line with the theory and acceptable levels in practice. Of course, the tradeoff for a reduction in stockout rate is the increase in inventory levels and therefore inventory carrying costs. Figures 1-B and 2-B will attest to the cost impact of the adjustment to EOP presumed in the Figure 2 results. Again, lead times from 1 to 10 weeks only were considered in this portion of the simulation. Also, lead time variability produced less than a 10 percent deviation in the number of orders placed over the simulation period when no adjustment to EOP was made and virtually no deviation in the numbers of orders for the cases of one-half, three quarters, and a full week's demand adjustment to EOP.

These results suggest that an addition of between three-quarters weeks' demand and a full week's demand be added to the EOP calculation to produce results consistent with a 95 percent service rate, when lead time can vary from 1-10 weeks.

A comparison of total variable cost curves (ordering costs plus carrying costs) is presented in Figure 3-A under conditions of no adjustment to EOP for the timing of the order placement (line B), an adjustment of one-half week (line A), three quarter's week (line D) and a full week (line C). Corresponding stockout rates are presented in Figure 3-B.

The majority of the increase in the slopes of any of these total variable costs (TVC) curves (lines A,B,C,D -- Fig. 3-A) may be attributed to the rate of increase in the carrying cost portion of the TVC curve. Ordering costs are relatively insensitive to lead time variations, at least over the range tested. A re-examination of Figures 1-A and 2-A will suggest that carrying costs increase at a much faster rate than do ordering costs as lead time increases. Also, it can be presumed that the trade off for an increase in carrying costs is the decrease in the stockout rate. This latter point is only common sense and well-known by all inventory managers.

First-order exponential smoothing was incorporated in the same inventory simulator as the forecasting tool for future sales, where

$$\text{Forecast}_{\text{new}} = \alpha \cdot \text{Current sales} + (1 - \alpha) \cdot \text{Forecast}_{\text{old}}$$

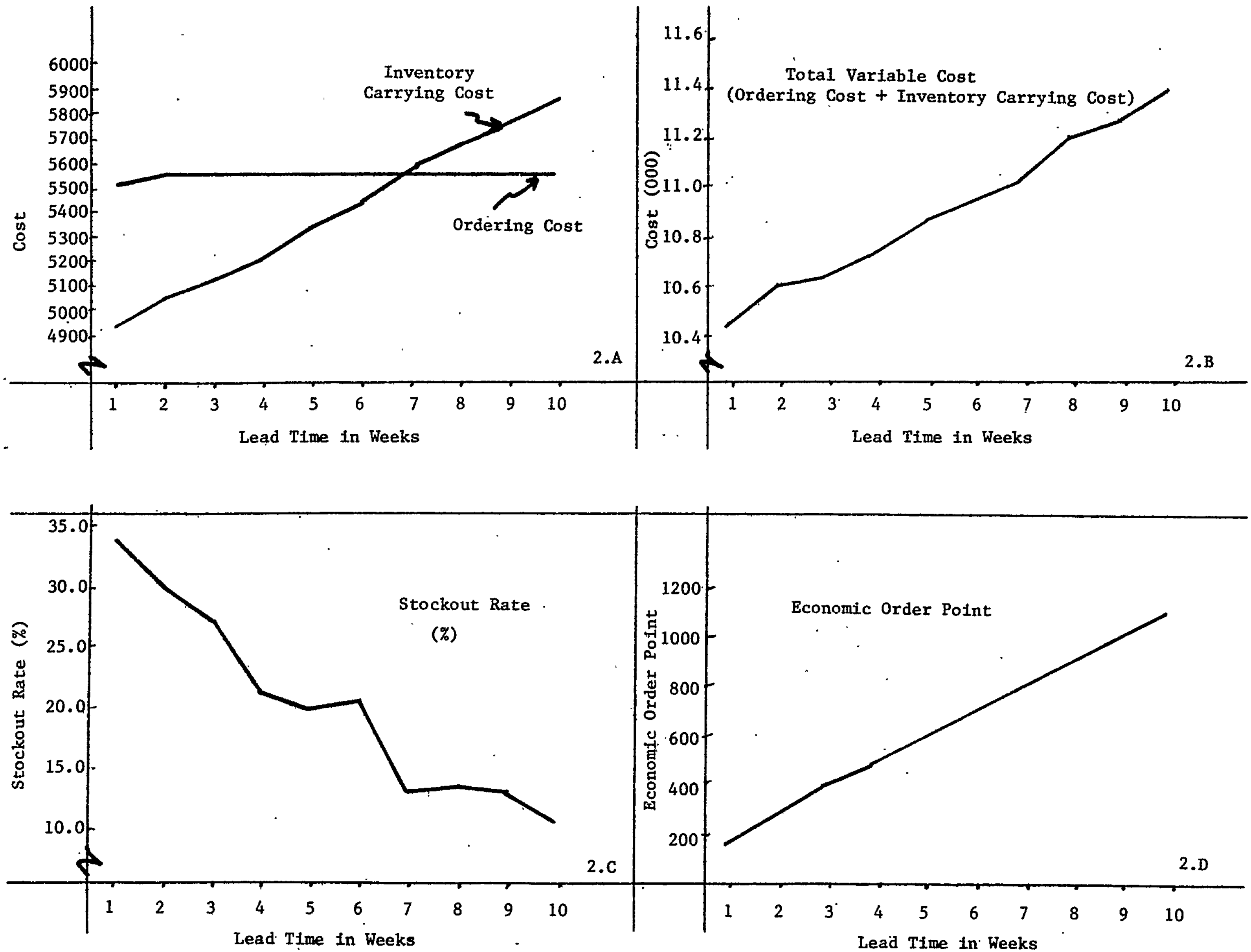
and α is the weight ($0 \le \alpha \le 1$) given to the current weekly sales. A constant α of 0.2 was used in all simulations throughout this study. For each week of the simulation period, the difference between the randomly generated weekly sales result and the forecast for that week was used to compute mean absolute deviation (MAD), the running sum of the forecast errors (RSFE) and a tracking signal (TS). Again, first-order smoothing was chosen since this is a simplistic tool that could be incorporated into a small business operation with a minimum of training or changes to existing operations and procedures. The question is how well does single-order smoothing work under conditions of varying demand.

TRACKING SIGNAL SIMULATION RESULTS

The same simulation design which produced results as discussed above and depicted in Figure 1 through 3, were also used to measure the change in tracking signal values, a relative measure of forecast error, from low to high standard deviation (and therefore coefficient of variation) values. Using a constant mean weekly demand of 100 units, a lead time of 10 weeks, and an EOQ of 447 units, and while allowing for multiple outstanding orders, a variable standard deviation and variable EOP, four levels of tracking signal results were monitored.

A tracking original (TS) value or result is defined on a weekly basis as RSFE divided by MAD. A TS value of 2-3(±) was counted as a level one warning of

FIGURE 2



One Half Week Adjustment to Order Point Calculation

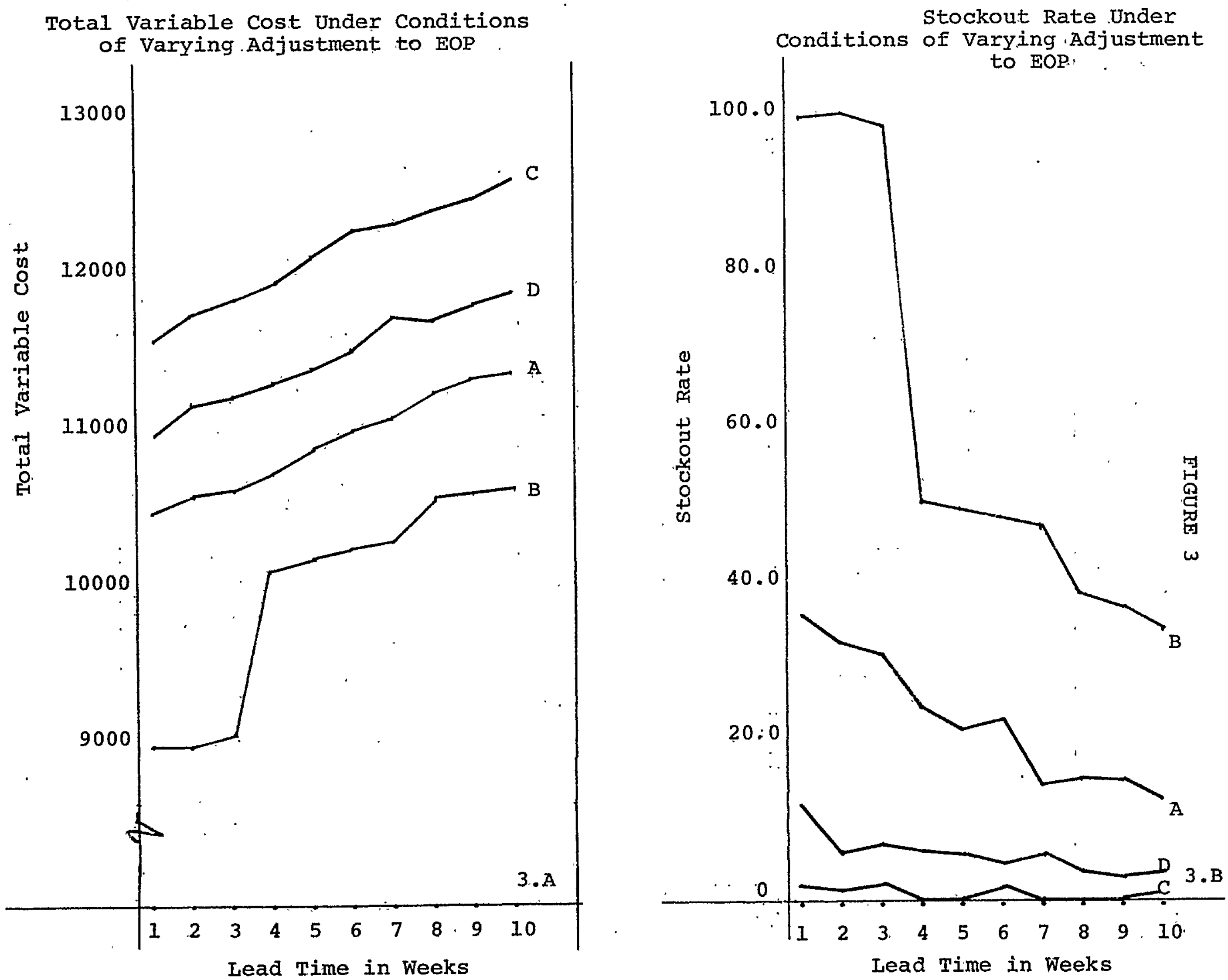

Total Variable Cost Under Conditions of Varying Adjustment to EOP
Total Variable Cost
13000
12000
11000
10000
9000
C
D
A
B
3.A
1 2 3 4 5 6 7 8 9 10
Lead Time in Weeks
Stockout Rate Under Conditions of Varying Adjustment to EOP
Stockout Rate
100.0
80.0
60.0
40.0
20.0
0
B
A
D
C
3.B
1 2 3 4 5 6 7 8 9 10
Lead Time in Weeks

FIGURE 3

significant forecast deviation; a TS value of 3-4 (±) was a second level warning, a TS of 4-5 (±) a third level warning and a TS of ≥ (±)5 was a fourth or final level warning that the forecast was not performing satisfactorily. While the TS values selected for measurement here were somewhat arbitrarily chosen, any TS values selected in a realistic range would produce similar results.

As can be seen from Figure 4-A, the range of first-level warnings varied from a low of 160 weeks out of a possible 1000 weeks or 16 percent for a coefficient of variation (CV) of 10 percent, to a high of 18 percent for a CV of 110 percent. For many small businessmen, this increase may not be significant considering the magnitude of change in CV considered. Again, Figure 4 presumes a lead time of ten weeks. Adjustments to EOP for the timing problem discussed in the previous section is not a factor in the results presented in this figure. From Figure 4-B second level warnings varied from a low of 85 out of 1000 or 8.5 percent (CV = 85%) to a high of 9.9 percent (CV = 105%). It is interesting here that the frequency of level two warnings as a function of standard deviation (or CV) exhibits a near sinusoidal pattern. An upward tendency in second level warnings is seen for high CV values beyond the low point of 8.5 percent (CV = 85%). Third and fourth level warnings also show an upward drift for high CV values with a maximum percentage change of approximately 52 percent (low = 2.9%; high = 4.4%) for level three warnings (Figure 4-C) and 118 percent (low = 1.1%; high = 2.4%) for level four warnings (Figure 4-D). The net result is quite clear - very high coefficient of variation (CV) values produces the greatest number of warnings at each of the four selected TS levels except level two where the final CV value tested produced the second highest frequency of TS flags. This is a strong suggestion that the single smoothing forecasting rule may deteriorate rapidly for high CV values. However, it must be noted that almost any forecasting rule will perform poorly given CV values in this range, and in fact, first-order smoothing may not be worse than other more sophisticated rules under such conditions of demand variability (test comparisons were not made). It is interesting to note that the total number of TS signals received, of all levels, varies only 18 percent from a CV of 5 percent to a CV of 110 percent. While the severity of the error increases as a function of CV, the tracking value of first-order smoothing may be sufficiently acceptable to the small businessman who may not be using any form of forecasting. The question for the small businessman is whether or not this tool will provide forecasting results superior to that currently being used.

COST AND STOCKOUT PERFORMANCE UNDER VARYING EOP CONDITIONS

The final portion of this simulation looks at cost and stockout performance as a function of both CV and adjustments to EOP for the timing problem discussed earlier. Figures 5 through 8 present simulated costs and stockout performances considering no adjustment to EOP for the timing of an order release (Figure 5), for a one-half week's demand adjustment to EOP (Figure 6), a three quarters' week's demand adjustment (Figure 7), a full week's demand adjustment (Figure 8). Each point on each of these figures represents a 1000-week simulation where CV again ranges from 5 percent to 110 percent in increments of 5 percent.

Figure 5 assumes that EOP is calculated in the traditional sense and as discussed earlier above (no adjustment to EOP) or,

$$EOP = \bar{X} \cdot L + \sqrt{L} \cdot S_x \cdot Z$$

where

L = lead time estimate (deterministic) in weeks and

Z = standard normal deviate value corresponding to the desired service level.

With L = 10 weeks, Figures 5-8 each depict inventory carrying cost, ordering cost and total variable cost as well as stockout rate performance and average inventory per week over the range of CV values considered (5%-110%) under the EOP assumptions stated immediately above for each of these figures. The results associated with each of these figures continues to assume that orders are placed at the end of each business week at the conclusion of that week's activity. The interesting result depicted in Figure 5 is the discrepancy, as a function of CV, between inventory carrying cost and ordering cost, which theoretically are equal. It may be generalized from these results that inventory carrying cost will tend to outstrip ordering cost as CV increases. As will be seen in Figures 6-8, this conclusion is independent of any adjustment to EOP. The stockout rate over the range of CV values considered (Figure 5-C) varies from approximately 55 percent for a CV of 5% to

FIGURE 4

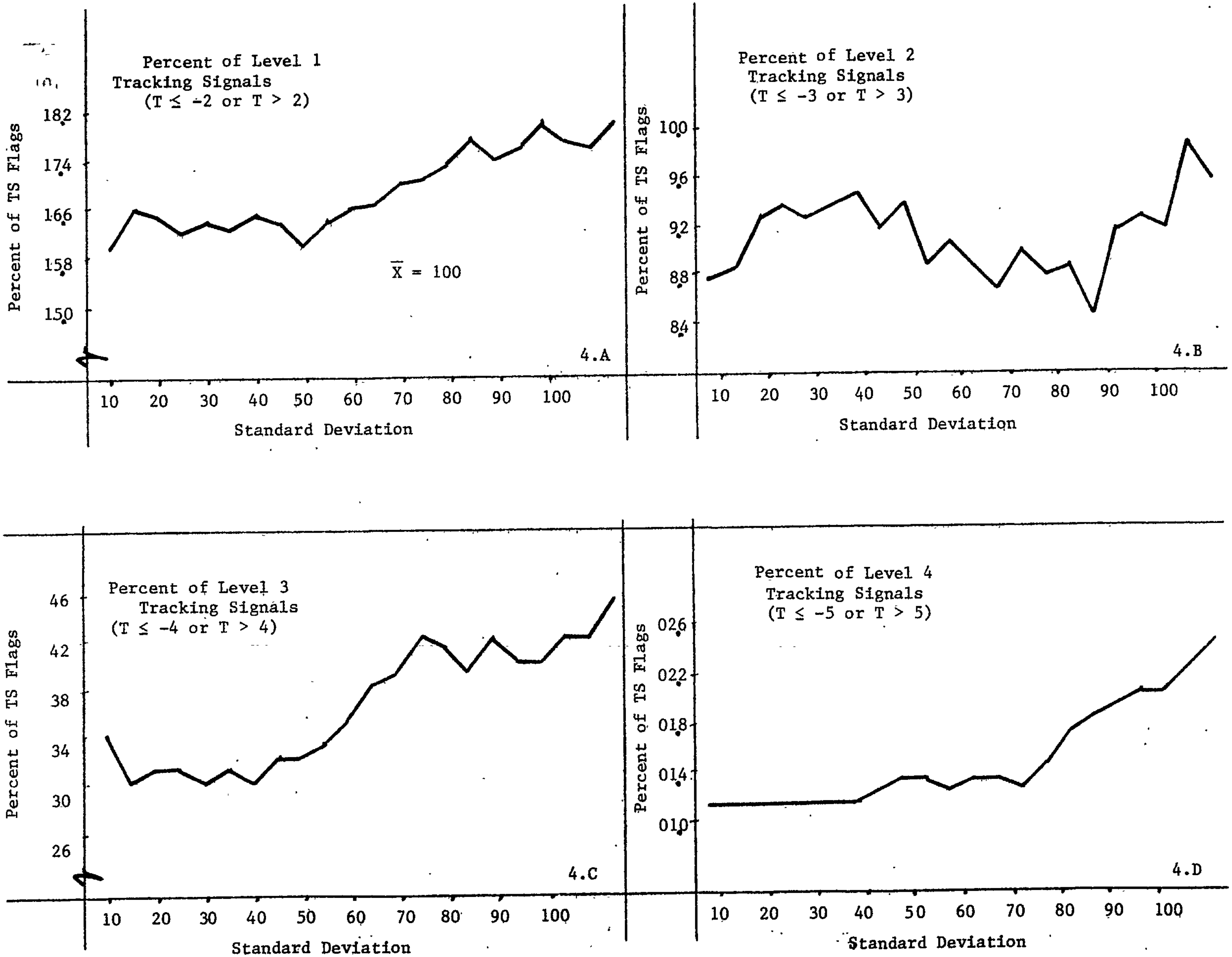
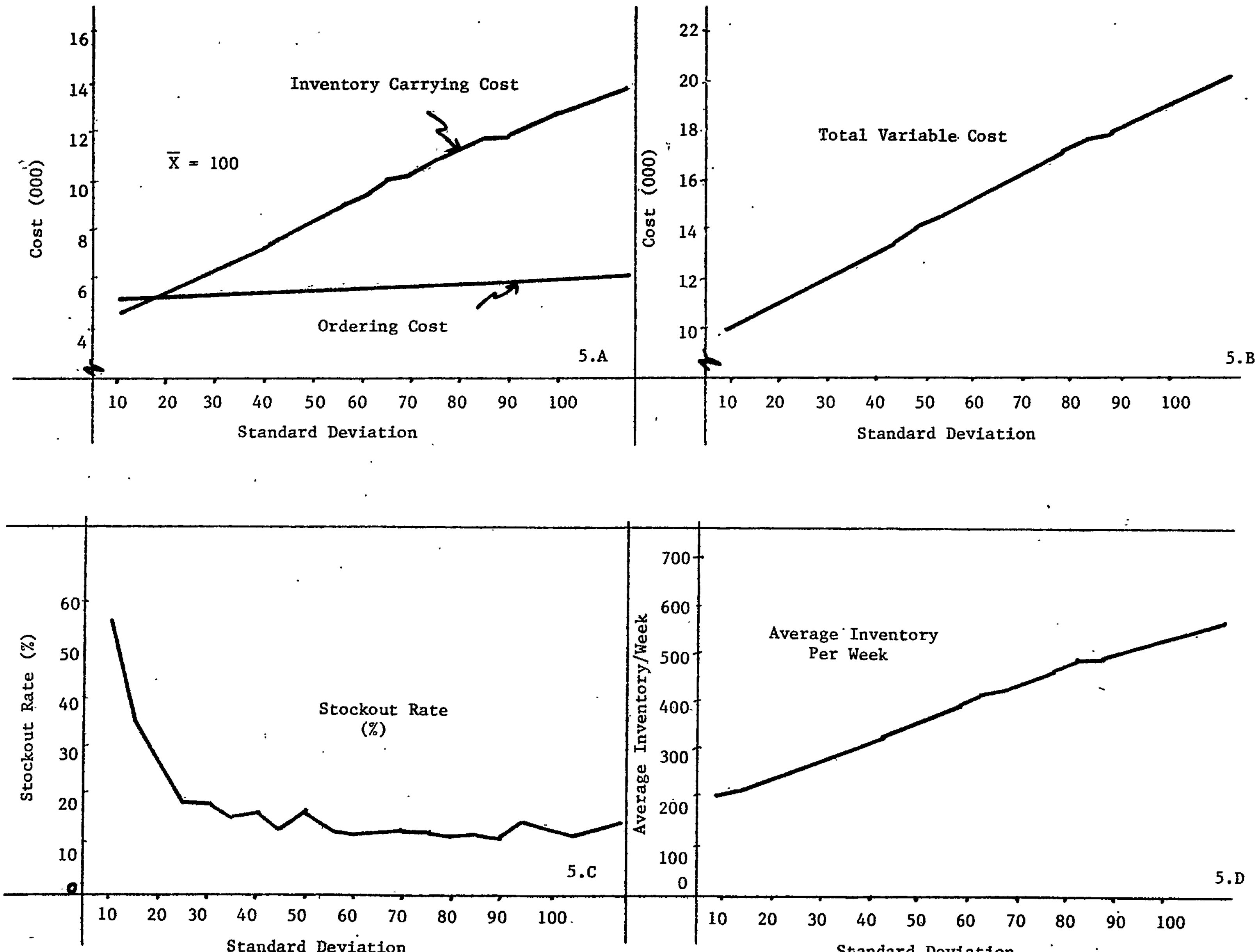
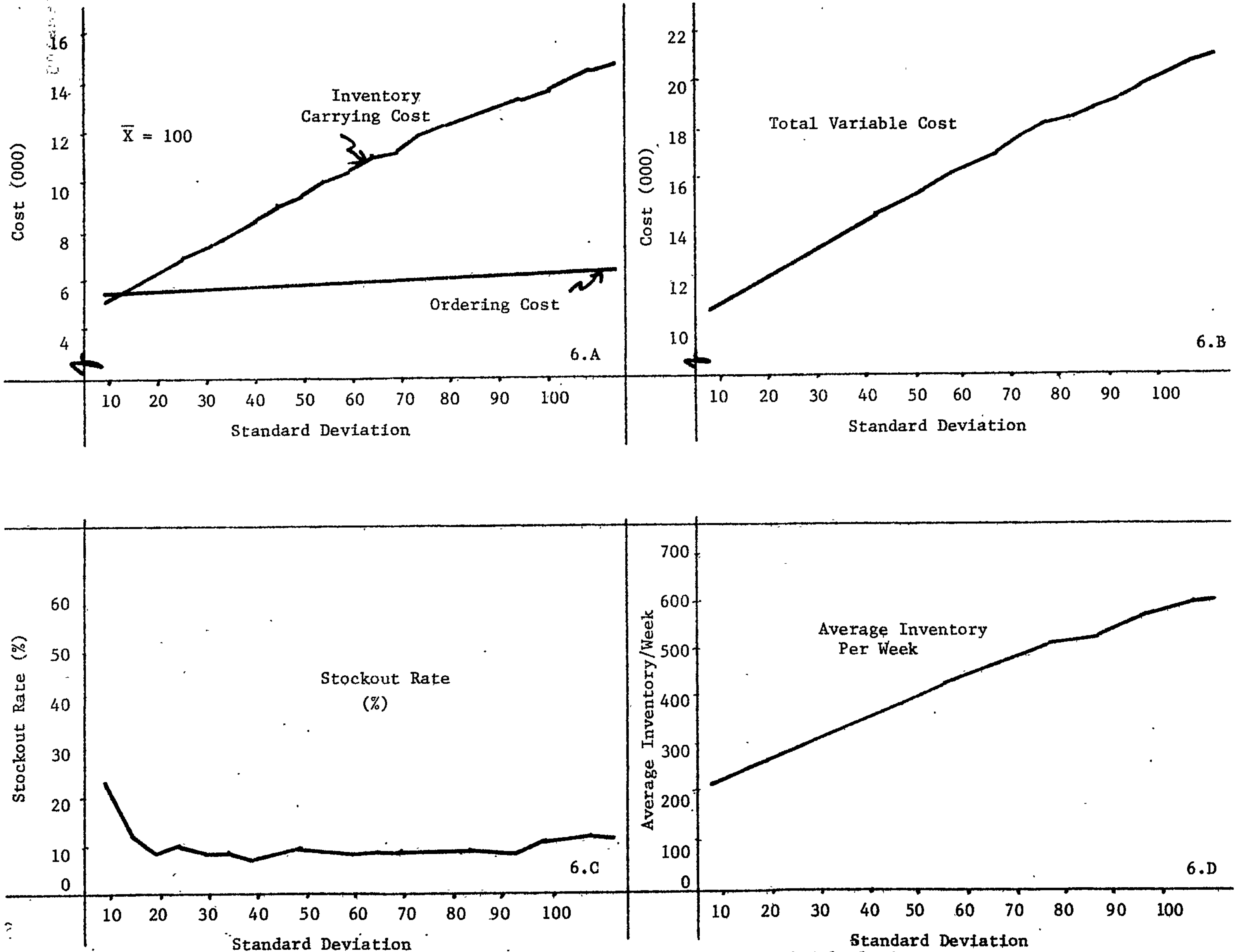

Percent of Level 1
Tracking Signals
(T ≤ -2 or T > 2)
182
174
166
158
150
Percent of TS Flags
X̄ = 100
4.A
10 20 30 40 50 60 70 80 90 100
Standard Deviation
Percent of Level 2
Tracking Signals
(T ≤ -3 or T > 3)
100
96
92
88
84
Percent of TS Flags
4.B
10 20 30 40 50 60 70 80 90 100
Standard Deviation
Percent of Level 3
Tracking Signals
(T ≤ -4 or T > 4)
46
42
38
34
30
26
Percent of TS Flags
4.C
10 20 30 40 50 60 70 80 90 100
Standard Deviation
Percent of Level 4
Tracking Signals
(T ≤ -5 or T > 5)
026
022
018
014
010
Percent of TS Flags
4.D
10 20 30 40 50 60 70 80 90 100
Standard Deviation

FIGURE 5



No Adjustment to Order Point Calculation

FIGURE 6



One Half Week Adjustment to Order Point Calculation

13.9 percent for a CV of 110 percent, with virtually no change in stockout rates when CV varies from 50 percent to 110 percent. Also, the number of orders will increase as CV increases, therefore increasing the potential exposures to stockout. The rate of increase in the TVC curve can almost totally be attributed to the rise in inventory carrying costs; similarly for the average inventory per week curve.

Figure 6 assumes a one-half week's demand adjustment to EOP, or

$$EOP = \overline{X} \cdot L + \sqrt{L} \cdot S_x \cdot Z + 0.5\ \overline{X}.$$

Simulations using CV's again ranging from 5 percent to 110 percent in increments of 5 percent produces cost relationships between inventory carrying cost and ordering cost similar to the results depicted in Figure 5, but with a much lower range of stockout rates than the series of simulations using no adjustment to EOP. This conclusion is consistent with the results discussed in association with Figures 1 and 2 where lead time, versus CV, was the parameter varied. Stockout rates (Figure 6-C) varied from approximately 22 percent (CV = 5%) to a low of 5.3 percent (CV = 35%). Consistent with results depicted in Figure 5-C, Figure 6-C indicates an extremely stable stockout rate pattern through the middle zone of CV values considered, here from 15 percent to 90 percent. Beyond 90 percent, a slight tendency exists for the stockout rate to again increase for these extremely high CV values.

Figure 7 is immediately interesting in that the carrying cost curve and ordering cost curve do not intersect anywhere over the range of CV values simulated. This result says that inventory carrying cost does not even approximate ordering cost, except for very low CV values, with the difference increasing as CV increases. In other words, the discrepancy between ordering cost and carrying costs is increasing as stockout rates are being brought into line with the desired service rate. The adjustment mechanism is again an addition to EOP as a portion of a week's demand for the previously discussed timing problem. Figure 7 assumes a three-quarter week's demand adjustment to EOP, or

$$EOP = \overline{X} \cdot L + \sqrt{L} \cdot S_x \cdot Z + 0.75\ \overline{X}.$$

The stockout rate range for Figure 7-C is interesting in that, relative to Figures 5-C and 6-C, a definite upward stockout rate trend is discernible for high CV values. The range of simulated stockout rates depicted in Figure 7-C is from 1.8 percent (CV = 25%) to 9.5 percent (CV = 105%). These results suggest that the user of EOP/EOQ logic should adjust EOP for both the timing of a placed order and standard deviation.

Figure 8 assumes a full week's demand adjustment to EOP, or

$$EOP = \overline{X} \cdot L + \sqrt{L} \cdot S_x \cdot Z + 1.0\ \overline{X}.$$

Again, carrying costs and ordering costs are only approximately equal for very low CV values and become orders of magnitude apart for high CV values. The upward drift in the stockout rate curve for high CV values (Figure 8-C) continues the trend suggested by Figure 6-C and which is much more apparent in Figure 7-C. The range of stockout rates is from 0 percent (CV = 5%) to 7.9 percent (CV = 105%).

In conclusion, a caution must be made to a potential user of the classical theory that adjustments to EOP may be in order due to both the timing of the placed order while also allowing for the standard deviation of weekly demand.

CONCLUSION

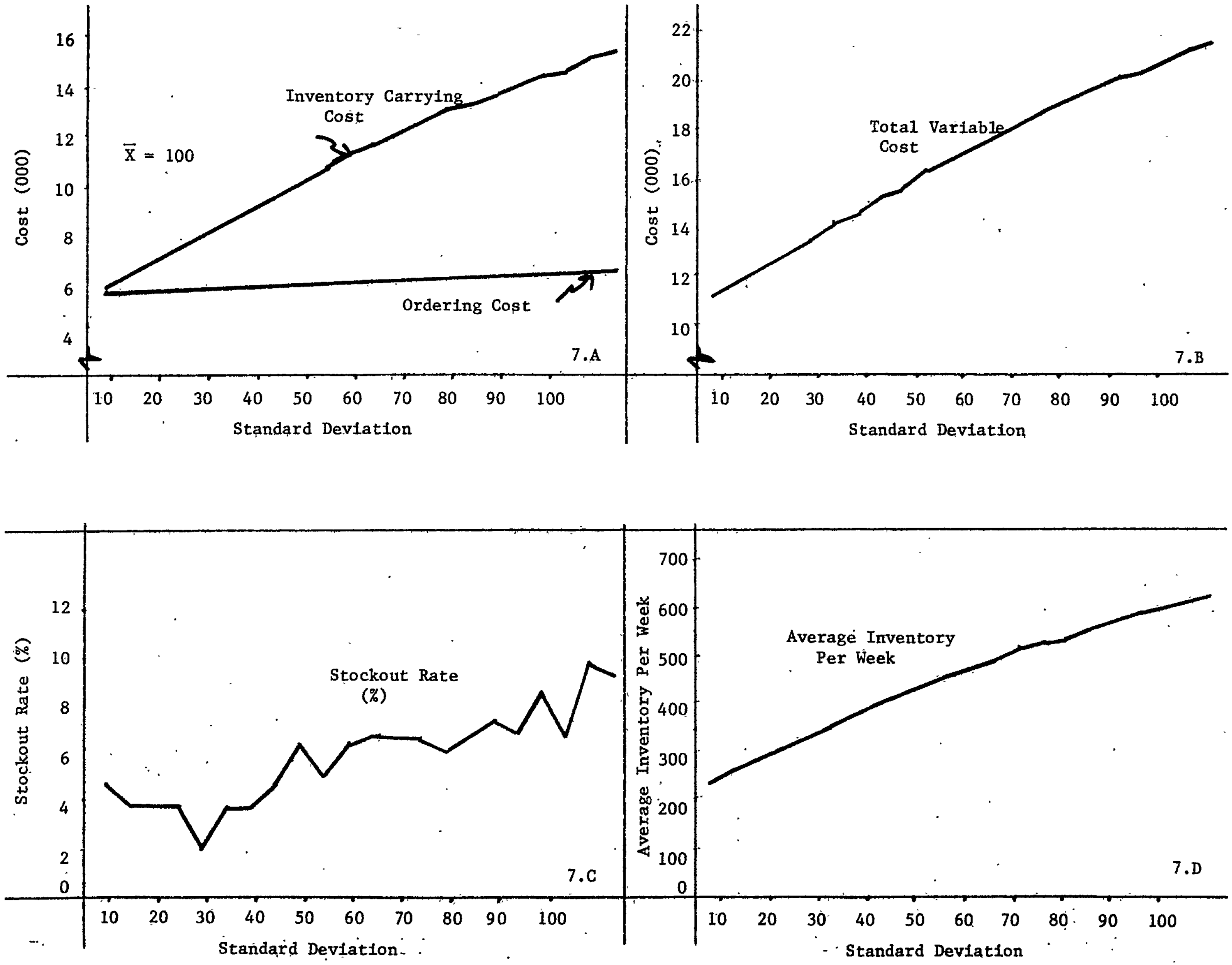
This simulation study was designed to look at how well EOQ/EOP logic performs under conditions of an assumed mean, standard deviation, lead time and cost parameters. An attempt was made to discern how well first-order exponential smoothing performs under conditions of varying demand.

Results obtained included the observation that a timing gap may exist between the point that an order should be released to the point when an order was actually released. This condition was simulated by releasing all orders at the end of a week recognizing that a release could have been at any point during the week. Figures 1 and 2 were presented to show how costs behave and stockout rates vary with no adjustment to EOP for any timing or order release gap (Figure 1) versus a one-half week adjustment to EOP (Figure 2).

Figure 3 presented general cost and stockout rate relationships for a series of EOP calculations based on assumptions of no adjustment to EOP to a full week's adjustment. As the adjustment to EOP increased, costs went up and stockout rates down with the tendency for carrying costs to outstrip order costs as lead time increased. It may be very misleading to presume the selection of some theoretical service rate (Z value) as employed in the EOP equation will necessarily give the user results as expected.

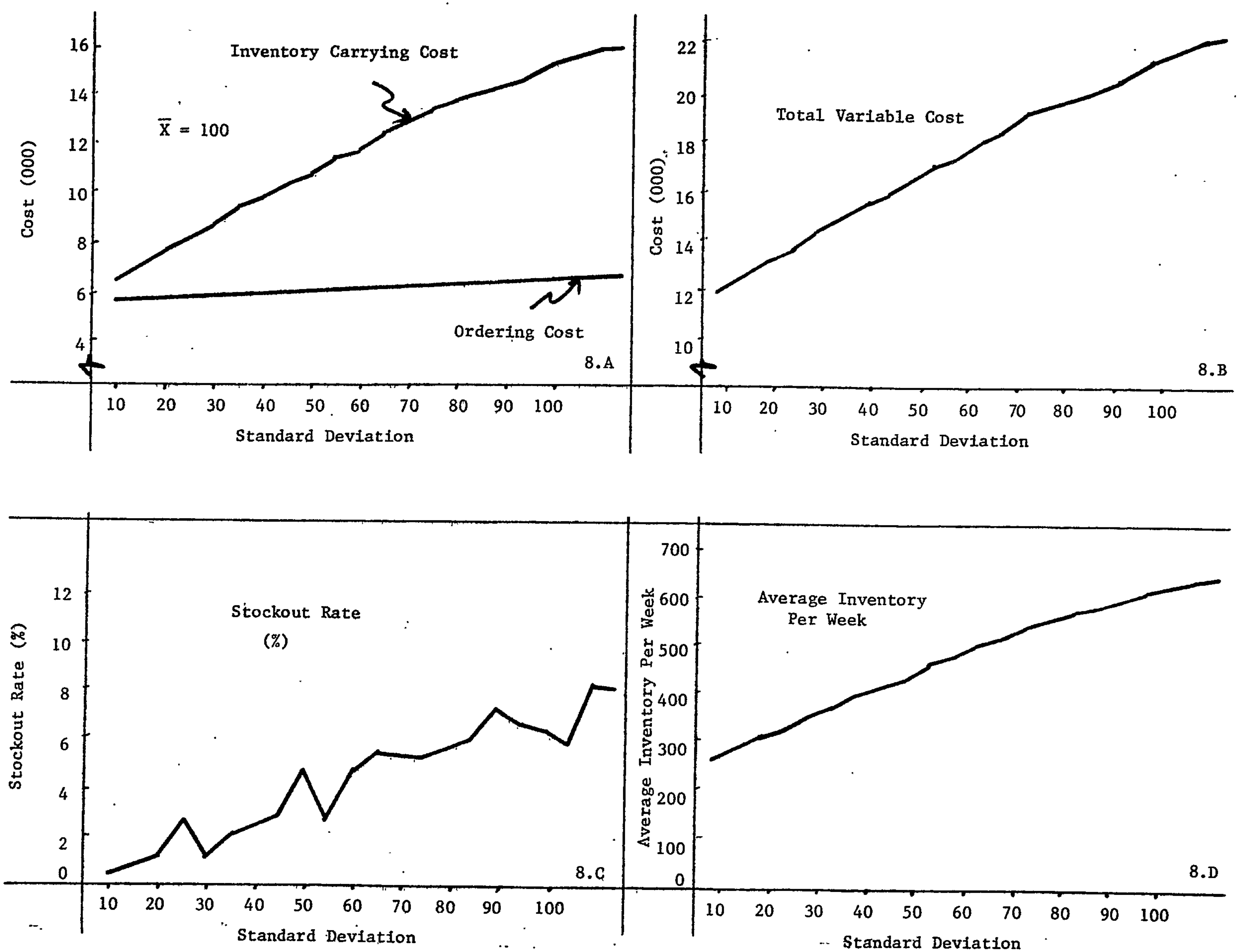
The efficiency of single-order exponential smoothing was examined as a

FIGURE 7



Three Quarters Week Adjustment to Order Point Calculation

FIGURE 8



Full Week Adjustment to Order Point Calculation function of the coefficient of variation (CV) and randomly generated demand data. It was learned that the efficiency of single-order smoothing as a forecasting tool does degrade as a function of CV -- however, possibly not sufficiently enough to detract a potential small business user who employs no formal forecasting procedures.

Finally, the relationship between cost and stockout rate was examined for a series of adjustments to the EOP calculation under conditions of varying demand. Figures 5 through 8 presented the graphic results. The general conclusion here is that some adjustment to EOP is necessary if in fact a timing difference between when an order should be placed and when the order is placed, exists. Also, inventory carrying costs tend to greatly outweigh ordering cost as a function of CV. Stockout rates tend to decrease absolutely as a function of the adjustment to EOP while also increasing for high CV values, independent of adjustment to EOP.

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