SIMULATING LOW DEMAND ASSET AVAILABILITY IN BASE SUPPLY WITH Q-GERT

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Abstract

Many widely used analytic inventory models include the assumption that item lead times are randomly and independently distributed about a stationary mean value. The models use a generalized form of Palm's Theorem to compute expected item backorders as a measure of system performance. In actual practice, most large scale inventory systems do not conform to this assumption; demands which create backorders lead to expedited item delivery times. This research investigates the significance of the errors which are generated when the analytic model is used to represent such an inventory system. The methodology involved simulating the inventory system with Pritsker's Q-GERT.

INTRODUCTION

A number of inventory models have been developed which attempt to allocate an investment budget among a set of inventory items so as to optimize some measure of inventory system performance (5,7). Generally, these models measure system performance in terms of backorders, that is, the time-weighted average number of outstanding shortages. The theoretic models used to compute the estimate usually include the assumption that the resupply time (or lead time) for an item is a random variable which is independently distributed about a stationary mean. In practical systems, this assumption is frequently invalid. In many such systems, item demands which are backorders are handled on an expedited basis, greatly reducing the duration of the backorder. This complicating factor is extremely difficult to handle analytically. The purpose of this research is to simulate the performance of an inventory system with expedited backorders to determine whether the assumptions of the theoretic models lead to significant errors in estimating item backorders.

BACKGROUND

It can be shown that an optimal inventory policy for low-demand, high cost items is one-for-one replacement, that is, an (S, S-1) inventory policy (4). The expected backorders associated with an inventory level of size S in such a system can be estimated if the following assumptions are made:

1. Demand is an independent, stationary Poisson process, where the demand rate is λ units per unit time.
2. Inventory is replenished on a one-for-one basis, with no delaying or grouping of orders.
3. Item lead time, or resupply time, is a random variable which is independently distributed about a stationary mean, say τ time units.
4. All demands which occur when stock is zero are backordered until replenishment stock arrives.

Under these conditions it can be shown that the number of items in resupply, or the number of outstanding replenishment orders, at a random point in time is a random variable which is Poisson distributed with a mean of $\lambda\tau$. This is an extension of Palm's Theorem (2). It follows, then, that the probability of X outstanding replenishment orders is:

$$p(X|\lambda\tau) = \frac{e^{-\lambda\tau}(\lambda\tau)^X}{X!}$$

The expected number of backorders, or time-weighted average number of shortages, can be computed as:

$$E(B) = \sum_{X=S+1}^{\infty} (X-S)p(X|\lambda\tau)$$

This logic forms the basis for two inventory algorithms in wide use in the United States Air Force, METRIC (Multi-Echelon Technique for Recoverable Item Control), (7), and Mod-METRIC (5), as well as many other stockage models and studies; for example, that of Duke and Elmore (3).

PROBLEM STATEMENT

This research was triggered by an application of the inventory theoretic model to inventories of spare parts in the United States Air Force. In this system, item replenishment orders which are intended to replenish stock are handled on a routine basis and normally have lead times of 20 to 30 days. However, if the replenishment item is needed to satisfy a backorder, that is, the spare part is missing from an aircraft or missile, then the replenishment order

Proceedings of the 1982 Winter Simulation Conference
Highland * Chao * Madrigal, Editors

82CH1844-0/82/0000-0063 $00.75 © 1982 IEEE

is given priority and is expedited. An expedited resupply time would last from 5 to 10 days. The purpose of expediting, quite obviously, is to reduce the duration of backorders. An inventory model that ignores expediting which in fact takes place will overestimate the expected number of backorders, that is, it will underestimate system performance.

Since these theoretic estimates of item backorders are, in fact, used to allocate an investment budget, it is important to determine whether the error introduced in the model by this simplifying assumption is significant or trivial. The approach of this research was to simulate the performance of an inventory system in which backorders are expedited and to compare the simulation results with those predicted by the theoretic model. It is clear that the theoretic model will overestimate item backorders, the question is, how badly?

SYSTEM DESCRIPTION

This simulation describes the flow of a single item, two-echelon inventory system consisting of a base supply which acts as a distributor to the consumer and a depot (or central supply) which fills orders from base supply. A one-for-one inventory reorder policy is used. For every demand placed upon base supply by the consumer, an order for a replacement is submitted to the depot. If base supply has stock on hand when an order is received, then the replacement order is satisfied using the routine resupply time. If the part is not on hand, resulting in a backorder, then the expedient resupply time is used. It is assumed that the depot always has always has stock on hand. The amount of on-hand stock authorized for base supply at any one time is set at one unit. The critical variable under consideration is the number of backorders experienced by the system.

Demands and resupply times are considered stochastic. A simple Poisson probability distribution is used to predict the probability of failures (demands). Thus, the time between demands is represented by an exponential probability distribution. Five values are used for time between demands of integer values between one and five units per year. Resupply time is represented by two normal probability distributions representing the expedited and routine times. The mean expedited resupply time is ten days, and the mean routine resupply is thirty days. These values reflect actual trends within the United States Air Force. The difference between the means is created by established physical distribution standards reflecting an order's criticality.

SIMULATION MODEL

A Q-GERT model used to analyze the system is presented in Figure 1. It may be divided into three sections. The first section initiates the demand and assigns an attribute according to the status of the base supply inventory. The second section generates the order to the depot and its supporting paperwork. The final section assembles the property and paperwork.

The first section is composed of nodes ten and eleven. Node ten represents the consumer function, where activity one simulates the initiation of demands. The user function at node eleven assigns a value to attribute one based upon the amount of inventory on hand at base supply (node fifteen). If an asset is on hand, then the attribute is assigned the value one. If there are no assets on hand, then the attribute is assigned the value two.

Activity three begins the second section by relaying the transaction to node twelve where deterministic branching sends the transaction to node five (order queue) and node thirteen. The conditional, take-first branching at node thirteen compares attribute one's value to two constant standards. If the attribute's value equals one, the routine resupply time (activity six) is selected. If the attribute's value equals two, the expedited resupply time (activity seven) is chosen.

The final section of the Q-GERT model assembles the paperwork with the property (node nine), and forwards the transaction to node twenty. This final process represents delivering the requested part to maintenance. The computer program for the Q-GERT NETWORK is presented in Figure 2.

Figure 2

Q-GERT PROGRAM

```
      FUNCTION UF(IFN)
      COMMON/QVAR/NDE,NFTBU(100),NREL(100),NRELP(100),
     +NRUN,NRUNS,NTC(100),PARAM(100,4),TBEG,TNOW
      GO TO (1),IFN
      UF=1.0
      IF (NREL(15).GT.0) UF=1
      IF (NREL(15).EQ.1) UF=2
      RETURN
      END
*EOR
GEN,PANKOPETE,DATA,3,2,1982,,1,100,,100,,,1*
SOU,10/SOURCE,0,1,D,M*
REG,11/USERFUNC,1,1,D*
REG,12/DIVIDER,1,1,D*
QUE,5/ORDER-Q,0,,D,F,(10)9*
QUE,15/ASSET-Q,1,,D,F,(10)9*
SEL,9,ASM,,,,5,15*
SIN,20/SINK,1,1,D,I*
REG,13/COND,1,1,F*
REG,14/TEST,1,1,F*
VAS,11,1,UF,1*
ACT,10,11,,,2*
ACT,10,10,EX,1,1*
ACT,11,12,,,3*
ACT,12,5,,,4*
ACT,12,13,,,5*
ACT,13,14,NO,2,6,,,A2.EQ.1*
ACT,14,15,CO,0.0*
ACT,13,15,NO,3,7,,,A2.EQ.2*
ACT,9,20,CO,0.01,8,1*
PAR,1,365,0,1460*
PAR,2,30.,10.,50.,10.*
PAR,3,10.,6.,14.,2.*
FIN*
```

A transaction in Q-node 5 represents a demand which can not be satisfied because no property is

Figure 1

Q-QERT NETWORK

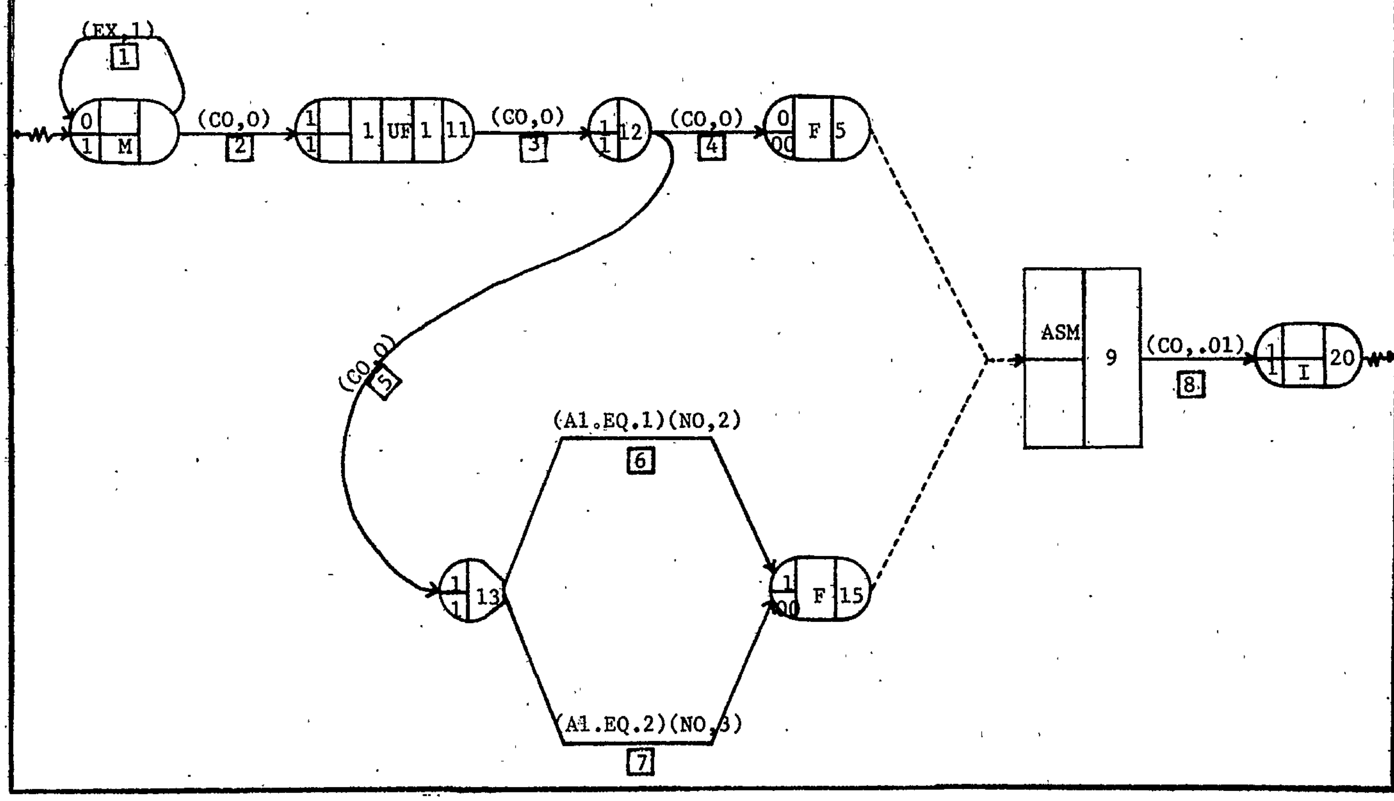

Table 1

THEORETICAL AND SIMULATED NUMBER OF BACKORDERS

λ (Orders Per Year)	Theoretic	Average	Simulated Std. Error
1/365	0.00329	0.0019	0.0001
2/365	0.01280	0.0066	0.0002
3/365	0.02805	0.0139	0.0003
4/365	0.04858	0.0235	0.0005
5/365	0.07690	0.0347	0.0007

Table 2

THEORETICAL AND SIMULATED NUMBER OF BACKORDERS
(Sensitivity Analysis)

λ (Orders Per Year)	Theoretic	Average	Simulated Std. Error
1/730	0.00083	0.0005	0.0000
1/1095	0.00037	0.0002	0.0000
1/1460	0.00021	0.0001	0.0000

available to the base; that is, the transaction is a backorder. The time-weighted average number of transactions in Q-node 5 therefore represents the expected number of backorders in the system.

MODEL VALIDATION

To validate the Q-GERT model for the system under study, the simulation results, using a moderate demand rate, were compared to the theoretical model, assuming that the theoretical model's assumptions were valid. The user function (node 11) were modified to always set the attribute's value to one to correspond to the assumption of the theoretical model that routine resupply time satisfactorily represents the availability of low-demand assets. Knowing that the stock level was fixed at one and resupply time was ten days, the mean number of failures (demand) was set at ten per year. The average number of backorders from thirty replications was .0353 backorders per year, with a standard deviation of .0018. The theoretical expected backorder formula was used to confirm this result:

$$E(B) = \sum_{X=s+1}^{\infty} \frac{(X-s)e^{-\lambda\tau}(\lambda\tau)^X}{X!} = .034231$$

A 95 percent confidence interval about the sample mean generated by the simulation program, 0.0317-0.0389, included the theoretical number of backorders. This validated the Q-GERT network's operation in relation to the theoretical model.

DISCUSSION

One hundred runs of the simulation program, each consisting of 100 demands were performed for each of the five values of the mean frequency of annual demand under consideration. The simulation output is summarized in Table 1. The data includes the theoretical expected backorder value, the simulation's average backorder value, and the standard error of the estimate of the simulation's average backorder value. A one-tailed large sample test of an hypothesis about a mean was performed on the data for each mean frequency of annual demand. The one sample test of an hypothesis about a mean was used in this study since the theoretical backorder value could not be considered a sample. Every test indicated, at the 95 percent confidence level, that a significant difference existed between the theoretical and the simulation means.

As can be seen from Table 1, as the demand frequency increases within the range of one and five orders per year, the difference between the theoretical and the simulation backorder means increased. To test the simulation's sensitivity to demand frequencies lower than the range of interest, three additional mean demand values were tested (unit/730 days, 1 unit/1095 days and 1 unit/1460 days). Table 2 lists the resulting theoretical and simulated number of backorders for the three additional mean demand values with all other simulation imputs unchanged. Since the standard errors of the average number of backorders was translated to below .0000 in all cases, no hypothesis test was performed. However, it can be suggested from direct observation that the divergence persists.

CONCLUSION

The assumption of a routine resupply time, as in the theoretic model, does not accurately represent the supply availability of a low demand time. In all tested cases, the theoretic model consistently provided a conservative estimate of the actual supply availability. It overestimated the average number of backorders that would be experienced by the system.

The significant difference between the models was strongly supported by the large sampe hypothesis test results, and subsequent sensitivity analysis. The simulation results indicate that many widely used inventory models should b reevaluated and specifically modified for use with low demand items when backorders are expedited.

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