COMPUTER SIMULATION OF ROAD SURFACE PROFILES FOR A FOUR-WHEELED VEHICLE

R. J. Kavanagh

Electrical Engineering Department
University of New Brunswick
Fredericton, N.B., Canada

R. Ramanathan

Electrical Engineering Department
Virginia Polytechnic Inst. & State U.
Blacksburg, Virginia

SUMMARY

A computer method for simulating road surface profiles is described. The technique involves the generation and subsequent modification of pseudorandom binary sequences. The statistical properties of the profiles are similar to those of typical road surfaces. The profiles are particularly suitable for use in computer simulations of four-wheeled vehicles.

1. INTRODUCTION

Investigations into the design of vehicle suspension systems frequently require the computer simulation of a surface which is characteristic of those over which the vehicle is expected to travel. A four-wheeled vehicle experiences input perturbations at each of its wheels. When the vehicle is travelling along a road in a straight line, the perturbation at a rear wheel can be considered to be a time-delayed version of the perturbation experienced by the front wheel on the same side of the vehicle. Under these conditions, simulation of the road surface involves the generation of two functions of time together with delayed versions of these functions. The statistical properties of these functions should match, to a reasonable degree, those of a typical road surface. A practical road surface simulation should include provision for taking into account the vehicle speed, wheel-base and track width. Shinozuka and Jan [1] have described a method for the digital simulation of multidimensional or multivariate random processes and have suggested that it might be applicable to the simulation of highway surfaces. However their technique does not appear to be amenable to the ready variation of parameters such as vehicle speed and wheel-base.

This paper describes a contribution towards the achievement of a simulation as described above. The fundamental principle of the described method involves the use of pseudorandom sequences. The generated road profiles are, therefore, periodic rather than random as would be the case for an actual road profile. It is shown that the simulation technique has features which make it particularly attractive for hybrid or analogue computer optimization studies of vehicle suspension systems.

2. STATISTICAL PROPERTIES OF ROAD SURFACES

It is not easy to describe the statistical properties of road surfaces. The quality of road surfaces depends upon the class of the road, upon the surfacing material, and upon construction methods. Within a given road class the properties of the surfaces vary from one road to another. For any given road the surface quality varies along the length of the road. For suspension system design purposes, the best one can do is to simulate a surface which is characteristic of the roads of a given class. In so doing, it is necessary to neglect such considerations as variation in statistical properties along the length of the road and the effects of hills, banking and holes.

Many studies of the statistical properties of road surfaces, and of terrain in general, have been made in recent years. Among these, important

Proceedings of the 1982
Winter Simulation Conference
Highland * Chao * Madrigal, Editors

82CH1844-0/82/0000-0337 $00.75 © 1982 IEEE

contributions have been made by Robson et al [2,3], Cote et al [4], LaBarre et al [5] and Laker [6]. By far the greatest emphasis has been on studies of the statistical properties of the surface elevation along a single track. Consequently, most analytical or computer studies of vehicle suspension system optimization have been restricted to single wheel or bicycle-type systems. The studies by Thompson [7] and Dahlberg [8] are examples of the use of single track representations of road surfaces. From the standpoint of the total vehicle response, as well as that of passengers, this approach is incomplete in that the roll variable is ignored.

In order to achieve a more realistic simulation of the behaviour of a vehicle travelling on a road surface it is necessary to describe the statistical properties of the surface along two tracks or, more generally, over the surface as a whole. The work of Robson [2,3] is particularly relevant in this context. Robson has proposed that a road surface be represented by a homogeneous, isotropic, Gaussian random process. This implies that the autocorrelation functions computed along any pair of parallel tracks will be identical. In addition, the cross-correlation between the two parallel profiles will be given by

$$R_{LR}(d) = R\left(\sqrt{d^2 + 4b^2}\right) \qquad (1)$$

where R(d) is the autocorrelation function along each track and 2b is the distance between the two tracks. Adoption of this model permits the conceptual simulation of a road surface for use in conjunction with a vehicle of any specified track width. On the other hand, the practical generation of random signals with these properties is difficult.

An alternative approach to road surface simulation is to make use of the Parkhilovski model [9]. This model is less general than one having the isotropic property in that it only describes the properties of the surface along two parallel paths separated by a specific distance. The Parkhilovski model describes the two profiles in terms of two random variables: one is the mean elevation of the two profiles; the second is the slope across the two profiles. Both variables are functions of distance along the track. The two variables are assumed to be uncorrelated. The individual profile elevations are easily determined from a knowledge of the two variables as well as the track width. It is possible to express the autocorrelation function for each profile and the cross-correlation between the two profiles as follows:

$$R(d) = R_Y(d) + b^2 R_\theta(d)$$
$$R_{LR}(d) = R_{RL}(d) = R_Y(d) - b^2 R\ (d) \qquad (2)$$

where $R_Y(d)$ is the autocorrelation function for the mean elevation of the two profiles and $R_\theta(d)$ is the autocorrelation function for the slope across the two profiles. This model requires the generation of two uncorrelated random signals, each of which has specified statistical properties. The method to be described below involves a simpler model than the Parkhilovski model. Nevertheless, it is believed that the proposed technique could be modified to fit the Parkhilovski model if the more accurate model is considered to justify the additional complexity of the generation process.

In this paper a surface having the following properties is simulated:

(a) The first order probability distributions along both tracks are approximately Gaussian and are virtually identical.

(b) The joint probability density function for the two profiles is approximately Gaussian.

(c) The autocorrelation functions for the two profiles are identical.

(d) The cross-correlation function between the two profiles is small in comparison with the autocorrelation function for an individual profile, and is of the same order of magnitude as the cross-correlation function for a corresponding isotropic surface.

(e) The power density spectra for the two profiles have envelopes of the form

$$S(n) = \frac{A}{n^2 + K^2}$$

where n is the wave number and A and K are constants.

(f) The surface is homogeneous.

(g) The individual profiles are periodic functions.

In addition to generating two profiles with the above properties, the simulation generates time-delayed versions so that profiles corresponding to both front and rear wheels of a vehicle are available.

The proposed simulation differs from Robson's isotropic model in two respects. In the first place, property (g) implies that the power density spectra for the profiles are line spectra rather than continuous functions. This need be of no concern provided that the line spacing is small in comparison with the bandwidth of the vehicle system under test. A narrow spacing can be achieved by deriving the profiles from pseudo-random sequences with sufficiently long periods.

In the second place, property (d) implies that the surface simulation is not isotropic. In the case of an isotropic surface equation (1) applies and it is possible to compute the cross-correlation between the two profiles given the autocorrelation function for one profile. It follows that the cross-correlation function would be an even function and that it would have a similar functional form to that of the autocorrelation function. The cross-correlation function would depend upon the distance between the two tracks. As the track width becomes very small it is apparent that the cross-correlation function would tend to become identical to the autocorrelation function. On the other hand, as the track width increases the cross-correlation function would approach a constant. For profiles with zero mean values, that constant would be zero. For practical road surfaces and vehicle track widths the cross-correlation between the two profiles is likely to be rather small in comparison with the maximum value of the autocorrelation function. If this is the case the exact functional form for the cross-correlation function is of little consequence. Thus it is argued that, provided the magnitude of the cross-correlation function is similar to that for an isotropic surface with the same autocorrelation function, it is unnecessary for its functional form to match the isotropic surface function.

Road profile power density spectra can be modelled by expressions of the form [2]:

$$S(n) = \begin{cases} C\left[\dfrac{n}{n_o}\right]^{-w_1}, & n \leq n_o \\ C\left[\dfrac{n}{n_o}\right]^{-w_2}, & n \geq n_o \end{cases} \qquad (3)$$

where n is the wave number (i.e. number of cycles per unit distance), and C, n_o, w_1 and w_2 are constants. The values of w_1 and w_2 which give a reasonable fit to experimental data are generally about 2. The validity of this model is limited to wave numbers above a certain minimum value. This is because $S(n)$, as defined above, approaches infinity as n approaches zero. It is reasonable, therefore, to modify the model such that the spectrum approaches a constant value as n tends to zero. Consequently, it has been proposed [7] that the spectrum can be adequately modelled by the simple function

$$S(n) = \frac{A}{n^2 + K^2} \qquad (4)$$

where A is a constant which determines the variance of the road perturbations and K is a constant which determines the bandwidth of the spectrum.

For applications in which a vehicle is simulated on an analogue computer, the road profiles experienced by each wheel must be functions of time. Then the power density spectra become functions of a frequency in hertz rather than of wave number. The spectra in such cases will depend upon the vehicle velocity. It is a straightforward matter to transform a spectrum expressed in terms of wave number into one expressed as a function of frequency if the vehicle velocity is specified.

3. PRINCIPLE OF SIMULATION METHOD

The method involves the generation of pseudo-random sequences which are then modified in such a way that the resulting functions of time have the desired statistical properties described in the previous section.

A pseudo-random sequence is a series of numbers which is periodic and which has certain well-defined statistical properties. Pseudo-random binary sequences are particularly well known and there exists a substantial body of literature describing their theory [10, 11]. These sequences are often employed in place of true random sequences because they are easy to generate, they have precisely defined statistical properties, and their periodicity permits repetition of experiments without the variability of measurements associated with finite duration tests involving random sequences. The use of pseudo-random sequences for road profile simulation has been suggested previously by Kavanagh [12] for a two-wheel vehicle simulation and by Laker [6] for a one-wheel simulation.

The generation of a signal corresponding to the profile experienced by one wheel of a vehicle is accomplished by means of a system as illustrated in Figure 1. The first block shown is the pseudo-random sequence generator. An

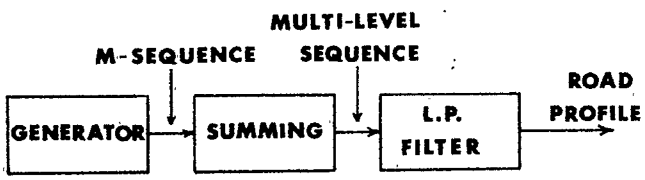

Figure 1

BASIC METHOD FOR GENERATING A ROAD PROFILE

example of this type of generator is shown in Figure 2. In this example, a seven-stage shift register is provided with feedback in such a way that the input to the first stage is the modulo-2 sum of the contents of the sixth and seventh stages of the register. The contents are the binary digits 0 or 1. The register is stimulated by a train of clock pulses which occur at regular intervals. On receipt of a clock pulse the contents of the ith stage of the register are shifted into the (i + 1)th

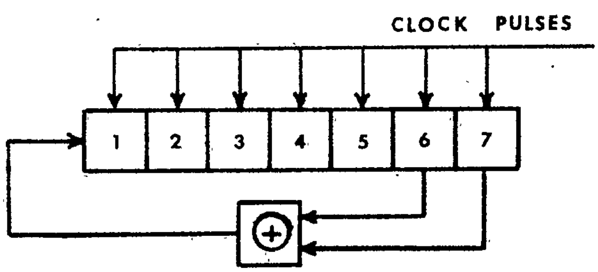

Figure 2

EXAMPLE OF A SHIFT REGISTER WITH FEEDBACK

stage. The result of supplying a continuous train of clock pulses to the register is that the contents of any one stage of the register form a binary sequence whose period depends upon the particular combination of stages from which feedback is taken. In this example the period is 127 binary digits, which is the maximum period which is obtainable from a seven-stage shift register. The sequence generated in this case is known as a maximum length sequence, or m-sequence. For a shift register with t stages the length of an m-sequence is 2^t-1 binary digits. Clearly, as t increases, m-sequences of considerable length are obtainable. It should be noted that, in general, for a given size shift register there is more than one set of feedback connections which will yield m-sequences. These sequences will be distinct but will have many identical statistical properties.

The power density spectrum for a pseudo-random binary sequence is a line spectrum with lines located at multiples of the fundamental frequency. In the particular case of an m-sequence with binary digit levels of +1 and -1 the spectrum is given by [13]:

$$\Phi(f) = \frac{1}{p^2}\,\delta(f) + \sum_{\substack{k=-\infty \\ \neq 0}}^{\infty} \frac{p+1}{p} \left[\frac{\sin\frac{k\pi}{p}}{\frac{k\pi}{p}}\right] \delta\left(f - \frac{kf_c}{p}\right) \qquad (5)$$

where f is the frequency in hertz, f_c is the clock frequency in hertz, p is the sequence length in binary digits, and $\delta(\cdot)$ is the Dirac delta function. All m-sequences of a given length have the same spectrum.

The second block in Figure 1 depicts the operation of adding together the contents of all stages of the shift register. When the shift-register is generating an m-sequence, and the addition operation assigns equal weights to the contents of all stages, the result is a multi-level pseudo-random sequence whose probability distribution is very nearly binomial. (The only discrepancy is that the level corresponding to the situation where all stages contain a "0" never occurs.) As the number of stages in the shift register increases the probability distribution for the multi-level sequence approaches a Gaussian distribution.

An effect of the summing operation is that the power density spectrum for the multi-level sequence is no longer given by equation (5). It is, however, easy to calculate the new spectrum. Noting that the sequence appearing in the ith stage of the register is identical to that appearing in the (i-1)th stage except for a delay of $1/f_c$ seconds, it follows that the transfer function relating the multi-level sequence to the m-sequence appearing in the first stage of the register is given by

$$H(f) = \sum_{i=1}^{t} \varepsilon^{-j2\pi(i-1)f/f_c} \qquad (6)$$

where t is the number of shift register stages and

$$j = \sqrt{-1} .$$

The power density spectrum for the multi-level sequence can now be determined by means of equations (5) and (6).

The function of the third block shown in Figure 1 is to shape the power density spectrum to the desired form and to convert the step-type multi-level sequence into a continuously variable function typical of a road profile. This is accomplished by means of an analogue low-pass filter. The output power density function is, again, easily determined from a knowledge of the spectrum of the multi-level sequence and of the filter transfer function. Thus the filter transfer function which gives an acceptable match to the desired output spectrum can be obtained.

Unfortunately, in addition to transforming the power density spectrum, the low-pass filter has an effect upon the probability density function of the signal. There is no general result available which relates the filter output probability density function to the input probability density function. It is a fact that if the input to a linear filter is a Gaussian process then the output signal also is a Gaussian process. That fact cannot be applied to this situation however because, although the first order density function for the filter input is very nearly Gaussian, no such assumption can be made about the higher order probability density functions. The input variable is not a Gaussian process. Some experimental evidence exists concerning the effect upon the probability density function of filtering an m-sequence [14]. It has been shown that the effect of a low-pass filter is to tend to skew the probability density function. For a given length of sequence, the amount of skew increases as the filter bandwidth decreases. In view of this fact the filter bandwidth cannot be too narrow and so it may not be possible to shape the output power density spectrum as precisely as desired.

In order to simulate the elevation along two tracks of a road surface it is necessary to use two generation processes of the type illustrated in Figure 1. The requirement for identical power densities and probability distributions for the two profiles can be met by using identical length shift registers, summing operations and low-pass filters. However, in order to meet the requirement that the cross-correlation between the profiles should be low, it is necessary to generate different m-sequences in the two shift-registers. This can be accomplished by using different feedback connections in the two registers. Tables of feedback connections for m-sequences of a given length are given by Golomb [10].

The choice of feedback connections can influence the joint probability density function for the two generated profiles as well as the cross-correlation function. Information concerning the cross-correlation function for two different m-sequences of the same length is very sparse. Briggs and Godfrey [15] have shown that, for two m-sequences of the same period, p, and having binary digit levels of +1 and -1, the mean value of the cross-correlation function is $1/p^2$ and the variance of the cross-correlation function is approximately $1/p$. Thus, for example, if $p=2047$, the mean value and variance of the cross-correlation function are $2 \cdot 4 \times 10^{-7}$ and $4 \cdot 9 \times 10^{-4}$ respectively. The auto-correlation function for each of the two m-sequences will have a maximum value of 1. It appears likely, therefore, that the values of the cross-correlation function will be relatively very small. If these two essentially uncorrelated sequences are subsequently processed identically in the manner depicted in Figure 1 it is reasonable to assume that the resulting variables will also have very low cross-correlation.

Virtually nothing is known about the joint probability density functions for two different filtered m-sequences. One can conjecture that the processes of Figure 1 applied to two different but equal length sequences will tend to cause the joint probability density function to have a roughly Gaussian shape, provided that the filter bandwidths are not excessively small.

In summary, therefore, the best one can do in arriving at two satisfactory sets of shift register feedback connections is to select them at random from those available and then measure the resulting correlation and probability functions to determine whether acceptable results are obtained. The evidence resulting from this particular investigation indicates that the choice of feedback connections is not critical.

The generation of time delayed variables corresponding to profiles at the rear wheels of a vehicle is a simple matter. Two methods are available and the choice depends upon the actual way in which the system of Figure 1 is realized. If the shift register is, in fact, an electronic device (e.g. an integrated circuit) time delayed sequences can be obtained by adding together, modulo -2, the contents of certain stages of the register. This method makes use of the so-called "shift-and-add" property of m-sequences [10] and its application for this purpose has been described elsewhere [12]. Alternatively, if the shift register is simulated by suitably programming a digital computer, it is simpler to produce delayed sequences by temporary storage in a "queue". The latter method has been used in this study. Whichever method is used, it is necessary to perform identical summing and filtering operations on the delayed sequences as were applied to the undelayed sequences.

The general method used to generate the four profile signals is as shown in Figure 3.

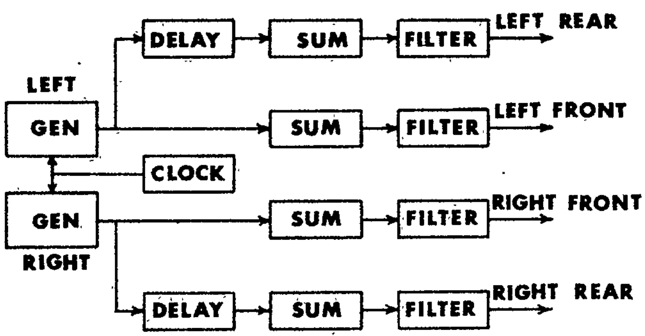

Figure 3

GENERATION SCHEME FOR FOUR PROFILES

4. DETAILS OF PROFILE GENERATION

There are two basic methods for realizing the generation system outlined in Figure 3. One method accomplishes all operations except the low-pass filtering by means of a digital computer. The second method involves the realization of each block by means of electronic hardware. Details of the design of the latter type of system for a two-wheel (one front and one rear) simulation have been described previously [12]. Details of a four-wheel simulation using the digital computer method will now be given.

It has been assumed that the envelope of the power density spectrum is of the form given in equation (4). For a vehicle speed of 50 km/h the spectrum, as a function of frequency, has been taken to be

$$S_{50}(f) = \frac{A}{25 + f^2} \qquad (7)$$

where A is an adjustable parameter which determines the mean square value of the profile perturbations (i.e. the roughness of the road). It should be noted that if the vehicle travels over the same surface at a different speed the spectrum must be changed accordingly. Thus, for example, if the vehicle speed is 100 km/h the spectrum will be

$$S_{100}(f) = \frac{A/2}{100 + f^2} \qquad (8)$$

It has been assumed that the vehicle wheelbase is 2·44m and that the track width is 1·37m. Then, at 50 km/h, the time delay between front and rear wheel profiles is 0·176s. The digital computer programme provides a "queue" which simulates this time delay. The value of the time-delay can easily be changed for use in the case of other speeds or wheelbases.

The choices of the shift register length, feedback connections, clock frequency and filter time constant are influenced by the following considerations:

(a) the number of shift register stages should be as large as possible in order that the probability distribution of the multi-level sequence is almost Gaussian.

(b) the sequence period should be large in comparison with the dominant time constant of the vehicle dynamics (or in other words, the line spacing in the power density spectrum should be small in comparison with the vehicle system bandwidth). However the period should not be so large as to lead to excessive computation time when making measurements over a complete period.

(c) the m-sequence spectrum $\Phi(f)$, has nulls at frequencies which are multiples of f_c. The frequency response function $H(f)$ has nulls at frequencies which are multiples of f_c/t. The values of f_c and t must be chosen such that the envelope of the spectrum of the multi-level sequence is greater than the value of $S_{50}(f)$ throughout the frequency range of interest.

(d) the time constant of the low-pass filter must be as small as possible in order that the filter does not cause an unacceptable skew in the output probability distribution. However, the time constant must also give rise to an acceptable output spectrum.

Consideration of these factors leads to conflicting requirements on the system parameters and, so, compromises are necessary. In this case, after digital computation of spectra corresponding to several possible designs, the parameters were chosen to be as follows:

$t = 11$, $p = 2047$, $f_c = 204{\cdot}7$ Hz

sequence period = 10s

filter time constant = 0·02s

The feedback connections for the two shift registers were chosen at random from among the set of such connections which yield m-sequences for an 11-stage shift register [10]. For the left wheel profile, feedback was taken from stages 2 and 11 of the register. For the right wheel profile, feedback was taken from stages 1, 2, 4 and 11.

In this investigation, the two shift registers, the feedback connections, the clock generator, the summing operations, and the time-delay operations have been accomplished by means of a digital computer simulation. The computer programme includes a capability to select the number of stages and the feedback connections of the shift registers, the clock frequency and the time delays. Thus, although the profiles generated in this particular study pertained to one particular wheelbase and speed, it is a simple matter to modify the profiles to correspond to other wheelbases and speeds.

For use as inputs to an analogue computer simulation of a vehicle, it is necessary to convert the four multi-level sequences to analogue form by means of digital-analogue converters. The low-pass filtering operations are then carried out by means of first-order operational amplifier filter circuits. The time constants of these filters can easily be changed when other vehicle speeds are simulated.

For timing purposes it is necessary to know when each period of the profiles is completed. This is done by sensing the occurrence of the particular state when a "1" is present in each stage of one of the shift registers. This stage, of necessity, occurs only once per period of the m-sequence.

5. PROPERTIES OF SIMULATED PROFILES

A sample of the simulated two-track profile is shown in Figure 4. The time delay (or displacement) between corresponding front and rear wheel profiles is apparent.

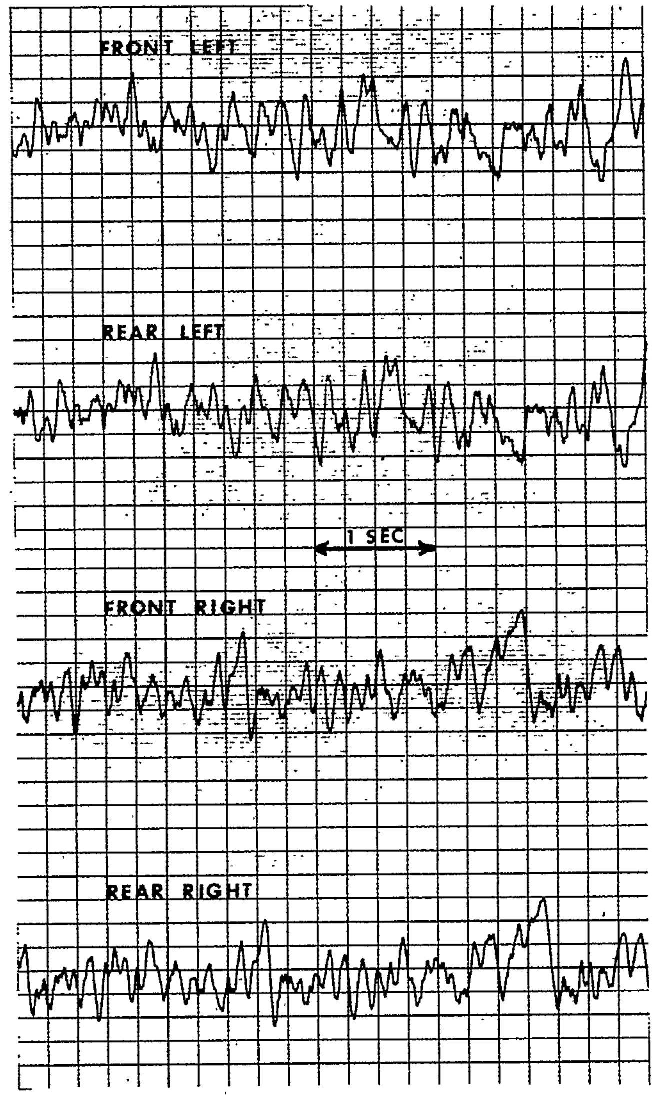

Figure 4

SAMPLE OF THE SIMULATED TWO-TRACK PROFILE

The measured probability distribution function for the left track is shown in Figure 5. The corresponding distribution function for the right track is virtually indistinguishable from that for the left track and so is not shown. The distribution functions for corresponding front and rear wheel profiles are, by the nature of the genera-

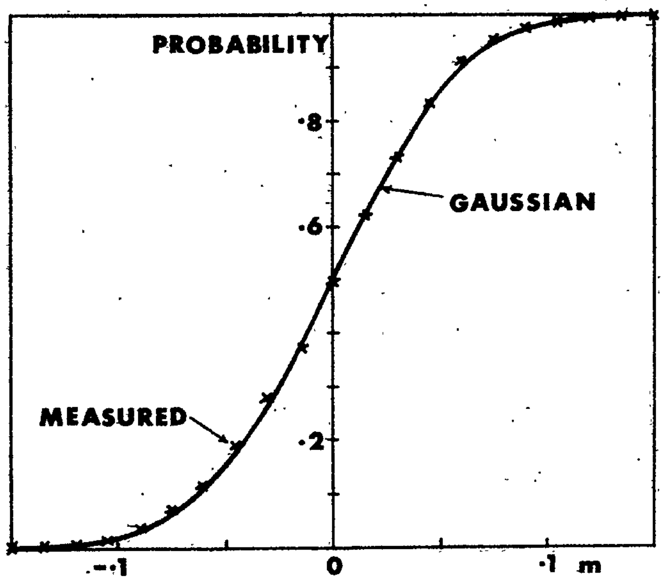

Figure 5

LEFT TRACK PROBABILITY DISTRIBUTION

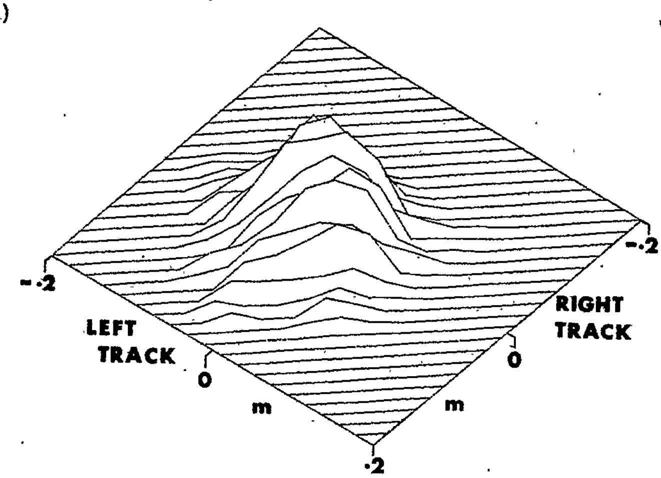

Figure 6

JOINT PROBABILITY DENSITY FUNCTION FOR LEFT AND RIGHT TRACKS

tion process, identical. Also shown in Figure 5 is the Gaussian probability distribution function for a random variable having a mean value and a standard deviation which are identical to those of the generated profile. It is apparent that the distribution function for the generated profile closely approximates the Gaussian distribution. A slight amount of skew can be observed. A computer generated perspective plot of the joint probability density function for the left and right wheel profiles is shown in Figure 6. There is a reasonable similarity to the Gaussian shape.

The autocorrelation functions for the left and right profiles are necessarily identical by virtue of the generation process. The measured autocorrelation function is shown in Figure 7. The cross-correlation function between the left and right profiles is shown in Figure 8. Also shown in Figure 8 is the cross-correlation function which an isotropic road surface would possess if its autocorrelation function were as shown in Figure 7. It is seen that, while the two functions are not identical, the magnitude of the generated profile cross-correlation is similar to that for an isotropic surface.

The power density spectra for the two profiles are necessarily identical.

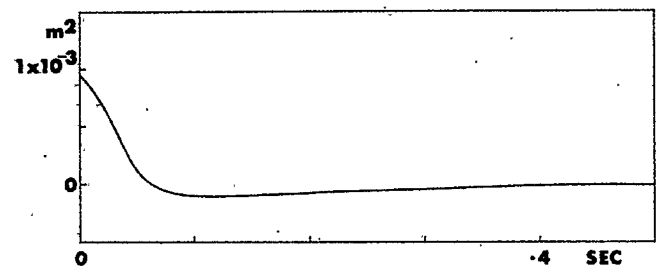

Figure 7

MEASURED AUTOCORRELATION FUNCTION

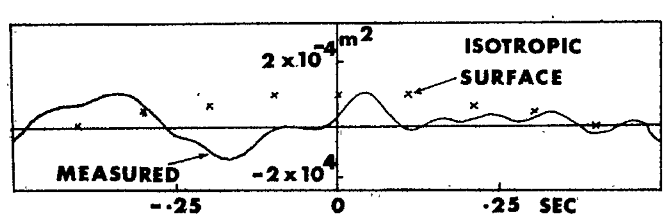

Figure 8

CROSS-CORRELATION BETWEEN LEFT AND RIGHT PROFILES

The envelope of the generated spectrum is shown in Figure 9 together with the spectrum specified by equation (7). The two functions are normalized with respect to their zero frequency values. The half-power frequency of the generated profiles is 5·5 Hz in comparison with the desired value of 5 Hz. The two spectra agree fairly well up to twice the half-power frequency. As this range contains 70% of the power in the signals, the deviation above this range will have relatively little influence upon any vehicle simulation which uses these generated profiles.

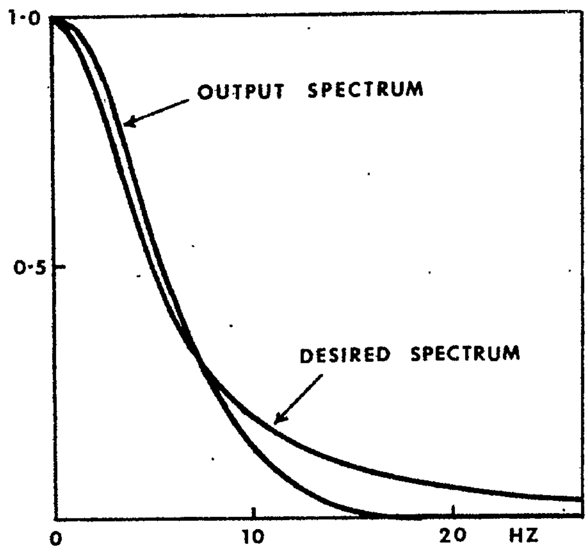

Figure 9

NORMALIZED OUTPUT AND DESIRED SPECTRA

6. DISCUSSION

The technique which has been described in this paper results in simulated profiles having statistical properties similar to those which have been measured for typical road surfaces. The digital computer simulation allows the investigator to specify vehicle speed and wheelbase, the degree of surface roughness and the bandwidth of the profile spectrum.

The method has a number of advantages over the use of a special purpose generator [12]. They are as follows:

(a) there are no hardware limitations involved in selecting the shift register feedback connections.

(b) the number of shift register stages and the feedback connections can easily be changed as the need arises.

(c) delayed signals are easily generated and the amount of the delay is readily changeable.

In addition, the method has an advantage over methods which generate truly random profiles in that the periodicity of the pseudo-random profiles is well suited to making repetitive measurements of simulated vehicle performance indices while adjustments are made to suspension system parameters.

The principal limitation of the method is the fact that, although the cross-correlation between the two profiles is small, its precise form cannot be controlled. In the event that it is considered desirable to control this function, it is suggested that the technique described in this paper could be modified so that a Parkhilovski model is generated. To do so it would be necessary to generate signals representing the mean elevation, y, and the slope, θ, using two shift registers connected so as to yield sequences of equal period but negligible cross-correlation. The delay, summing and filtering operations would be required as before. Finally, the left and right profile signals would be obtained by appropriately combining the filtered mean elevation and slope signals. This scheme would be somewhat more complex than that described in this paper. Furthermore, an investigation into the ability to match satisfactorily the required spectral densities and probability densities would be needed. It appears, however, that such a method is feasible.

The profile simulation technique described in this paper has been used in a study of the optimization of suspension system parameters for a vehicle having coupled active suspension units [16]. It is believed that the method may be of interest to others involved in vehicle optimization studies, particularly where hybrid or analogue computer techniques are employed.

BIBLIOGRAPHY

1. Shinozuka, M. and Jan, C.-M., "Digital simulation of random processes and its applications", Journal of Sound and Vibration, 25, 1972, 111-128.

2. Dodds, C.J. and Robson, J.D., "The description of road surface roughness", Journal of Sound and Vibration 31, 1973, 175-183.

3. Kamash, K.M.A. and Robson, J.D., "The application of isotropy in road surface modelling", Journal of Sound and Vibration, 57, 1978, 89-100.

4. Cote, L.J., Kozin, F. and Bogdanoff, J.L., "Introduction to a statistical theory of land locomotion - II. Ground roughness", Journal of Terramechanics, 2, 1965, 17-27.

5. LaBarre, R.P., Forbes, R.T. and Andrew, S., "The measurement and analysis of road surface roughness", Motor Industry Research Association Report No. 1970/5, 1970.

6. Laker, I.B., "The dynamic response of a single wheel suspension to computer generated profiles", Transport and Road Research Laboratory, Crowthorne, Supplementary Report 360, 1978.

7. Thompson, A.G., "Optimum damping in a randomly excited non-linear suspension" Proceedings of the Institution of Mechanical Engineers, 184 (2A), 1969-70, 169-184.

8. Dahlberg, T., "Ride comfort and road holding of a 2-DOF vehicle travelling on a randomly profiled road", Journal of Sound and Vibration, 58, 1978, 179-187.

9. Robson, J.D., "The role of the Parkhilovski model in road description", Vehicle System Dynamics, 7, 1978, 153-162.

10. Golomb, S.W., "Shift Register Sequences" Holden-Day Inc., San Francisco, 1967.

11. Hoffmann de Visme, G., "Binary Sequences", English Universities Press, London, 1971.

12. Kavanagh, R.J., "Simulation of road surfaces for use in analog computer studies of vehicle suspension systems", Proceedings of the Twentieth Midwest Symposium on Circuits and Systems, 1977, 269-273.

13. Cumming, I.G., "Autocorrelation function and spectrum of a filtered pseudorandom binary sequence", Proceedings of the Institution of Electrical Engineers, 114, 1967, 1360-1362.

14. Gilson, R.P., "Some results of amplitude distribution experiments on shift register generated pseudo-random noise" IEEE Transactions on Electronic Computers, EC-15, 1966, 926-927.

15. Briggs, P.A.N. and Godfrey, K.R., "Pseudorandom signals for the dynamic analysis of multivariable systems", Proceedings of the Institution of Electrical Engineers, 113, 1966, 1259-1267.

16. Ramanathan, R., "Simulation and optimization of a four-wheeled vehicle moving on a rough surface", M.Sc.E. Thesis Electrical Engineering Department, University of New Brunswick, Fredericton Canada, 1979.

ACKNOWLEDGEMENT

This work was supported, in part, by a research grant from the Natural Sciences and Engineering Research Council of Canada.