SIMULATION OF TRANSIENT HEAT FLOW IN COMPOSITE CIRCULAR SYSTEMS

Gerald N. Pitts, Associate Professor
Computing and Information Sciences
Trinity University
715 Stadium Drive
San Antonio, Texas 78284
(512) 736-7480

and

Barry L. Bateman, Assistant Vice President
Maryland System
Adelphi, Maryland 20783

INTRODUCTION

In drilling and completing oil wells in thick viscous oil reservoirs, it is frequently found that the productivity is quite low. Large oil reserves have existed in these viscous fields and, hence, served as a challenge to develop commercial oil production methods.

Many examples of both casing and cement failures have been cited when doing a steam stimulation job. Special cements have been proposed, and heavy-duty casing may now be specified. It was known that the thermal properties of the cement sheath could be varied, and it was believed that one should study the effects of the cement thermal properties on the temperature distribution about the well bore.

Analytical solutions are available to show the transient temperature for the case of uniform homogeneous media without a change in thermal properties from the well out into the reservoir. However, in initiating this problem, it was soon learned that new methods must be developed in order to cope with the problem where the thermal properties of the cement sheath were different from the rock matrix itself.

In reviewing the work on transient temperatures when injecting hot fluids into wells, it seems that very little work has been done showing the temperature of the cement sheath and contiguous rock as hot fluid is injected down a cased hole. Hence, the purpose of this paper is to present a method which may be used for calculating the transient temperatures existing in composite circular systems, i.e., from the well out into the cement sheath and rock matrix for the case in which the thermal properties of the sheath are different from the rock matrix. Some quantitative results will be shown.

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MATHEMATICAL DEVELOPMENT

The Fourier equation which governs cylindrical heat flow is:

$$\frac{\partial T}{\partial \theta} = \alpha \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} \right] \quad (1)$$

In difference form, equation (1) becomes

$$\left.\frac{\Delta T}{\Delta \theta}\right|_{r=\text{constant}} = \alpha \left[\frac{\Delta^2 T}{\Delta r^2} + \frac{1}{r} \cdot \frac{\Delta T}{\Delta r} \right] \quad (2)$$

Equation (2) can be reduced to the following finite-difference form:

$$\frac{T_{r,\theta+\Delta\theta} - T_{r,\theta}}{\Delta \theta} = \alpha \left[\frac{T_{r+\Delta r,\theta} - 2T_{r,\theta} + T_{r-\Delta r,\theta}}{\Delta r^2} + \frac{1}{r} \cdot \frac{T_{r+\Delta r,\theta} - T_{r-\Delta r,\theta}}{2\Delta r} \right] \quad (3)$$

The temperature at radius (r) and time ($\theta+\Delta\theta$) can be obtained from equation (3) to give:

$$T_{r,\theta+\Delta\theta} = \frac{(\alpha\Delta\theta)(2r+\Delta r)\, T_{r+\Delta r,\theta}}{2r\Delta r^2} + \frac{(\Delta r^2 - 2\alpha\Delta\theta)\, T_{r,\theta}}{\Delta r^2}$$

$$+ \frac{(\alpha\Delta\theta)\ (2r-\Delta r)\ T_{r-\Delta r,\ \theta}}{2r\Delta r^2} \quad (4)$$

One should note that the equation for heat flow in a cylinder of homogeneous material is derived from equation (4) when $\Delta\theta = \Delta r^2/2\alpha$, i.e.,

$$T_{r,\theta+\Delta\theta} = \frac{1}{2}\left[\left(1-\frac{\Delta r}{2r}\right) T_{r-\Delta r,\ \theta} + \left(1+\frac{\Delta r}{2r}\right) T_{r+\Delta r,\ \theta}\right] \quad (5)$$

Equation (4) becomes the equation for two composite concentric cylinders when

$$\Delta\theta_i = \frac{\Theta_i \Delta r_i^2}{2\alpha}$$

where

$$\Theta_i = \frac{2\alpha_i \Delta\theta}{\Delta r_i^2}$$

in the usual form:

$$\frac{\Theta_i}{2} = \frac{\alpha\Delta\theta}{\Delta r_i^2}$$

Dropping subscript "i" and using this in equation (4), we have:

$$T_{r,\theta+\Delta\theta} = \frac{\Theta(2r+\Delta r)}{4r} \cdot T_{r+\Delta r,\ \theta} + T_{r,\theta} - \Theta T_{r,\ \theta} + \frac{\Theta(2r-\Delta r)}{4r} \cdot T_{r-\Delta r,\ \theta} \quad (6)$$

The preceding equation provides the temperature at all points in the system, except at the boundary between the two layers. To couple the cement-rock equations, it is necessary to set the cement-rock boundary or interface temperature the same, and it is necessary to equate the flux on both sides.

Under steady-state conditions, the heat flowing into and out of a circle must be equal. Using this, we have:

$$K_1 \ln\left(\frac{r_2}{r_B}\right)(T_B - T_1) = K_2 \ln\left(\frac{r_B}{r_1}\right)(T_2 - T_B)$$

Hence,

$$T_B = \frac{K_1 \ln\left(\frac{r_2}{r_B}\right) T_1 + K_2 \ln\left(\frac{r_B}{r_1}\right) T_2}{K_1 \ln\left(\frac{r_2}{r_B}\right) + K_2 \ln\left(\frac{r_B}{r_1}\right)}$$

This method forces the heat flux and temperature at the interface to be the same for both layers.

PROCEDURE

For this problem, a cylindrical geometry may be visualized representing the well bore surrounded by a thin layer of one set of thermal cement sheath properties followed by a large cylinder of a second set of thermal properties. This forms a composite concentric cylinder (Fig. 1). Initially, the system is at geothermal temperature. The well bore is suddenly heated and maintained at constant steam temperature. It is desired to study the temperature rise in the cement sheath and out into the rock.

Equation (6) is the same for both zone 1 and zone 2 with the exception of Θ, dimensionless time. Since it is desirable to have the time intervals equal in each layer, one can choose the ratio of Θ_1 to Θ_2 in such a manner that the times are equal. By further restricting the Θ_1 and Θ_2, we could have the Δr_i equal also, but this is undesirable due to the large difference in the widths of the two layers. The calculations are simplified by arbitrarily setting $\Theta_1 = 1$, and this yields

$$\Theta_2 = \frac{1}{\frac{\alpha_1}{\alpha_2} \cdot \frac{\Delta r_2^2}{\Delta r_1}}$$

RESULTS

Fig. 2 is presented for two purposes. The figure shows the transient dimensionless temperature rise in the cement sheath and rock for the case in which the thermal conductivities were 0.3 and 1.4 for the cement and rock, respectively. The cement and rock densities were 50 and 150 lb/cu ft., respectively, and the specific heats were the same at 0.2 Btu/lb-deg F. Each of the three curves was calculated three times, using three different grid sizes for calculations. This was accomplished by changing the r_1 of the two zones by varying the number of slices into which each zone was divided, i.e., the first zone into 2 slices, the second zone into 20 slices; then 4 slices by 40 slices; and finally 8 slices by 80 slices. Such a procedure tests the calculational techniques.

Each slicing method gave good results. For example, the difference in heat flux at the boundary was:

Time Difference	Dimensionless Time	Method
1.3	20.8	2X20 slices
0.8	20.8	4X40 slices
0.1	20.8	8X80 slices

Effect of Thermal Conductivity

Figs. 3,4, and 5 show the transient temperature for the case in which the thermal conductivity of the cement was 0.075, 0.15, and 0.3, respectively. The low conductivity might possibly be achieved by the use of certain insulative types of materials. The density of the cement and the rock was unchanged for all three figures. It will be seen from Fig.3 that at the end of approximately 31 hr, the temperature at the cement-rock interface had been raised to approximately 20 percent of the dimensionless temperature rise.

Fig. 4 shows a similar study for the case in which the cement thermal conductivity was 0.15. This is approximately 1/10 that of the rock. It will be seen that the cement is serving as an excellent insulative material. By increasing the thermal conductivity of the cement to 0.3, the temperature rise at the rock interface had raised to approximately 30 percent of the dimensionless temperature rise (Fig. 5).

A comparison of the effect of thermal conductivities is shown in Fig. 6. Fig.6 shows the case of the temperature rise where the thermal conductivities of the cement ranged from 0.15 to 0.3.

Effect of Cement Density

Figs. 7,8, and 9 show the case in which the thermal conductivity of the cement is maintained at 0.7, but the density of the cement varied from 25 to 100 lb/cu ft. At the end of approximately 45 to 50 percent for both cements (Figs. 7,8, and 9). The effect of the cement density on temperature seems to be quite small throughout the range studied.

Fig. 10 shows a cross plot of the data with the triangles, circles, and squares corresponding to the case for cement densities of 25, 50, and 100 lb/cu ft, all having the same thermal conductivity. It will be seen from this figure that the temperatures varied slightly and were almost independent of the cement density.

CONCLUSIONS

A method has been proposed for simulating heat flow in composite circular systems in which the temperature is suddenly changed to a constant temperature and maintained at this temperature for a long period of time. The method requires that the temperature at the interface be the same, and the heat flux on both sides of the interface must be the same. The computational method appears to be reasonably stable. The quantitative effects of cement thermal conductivity and cement density on the transient cement sheath temperatures have been shown for the first time.

With knowledge of the transient temperature, one may then initiate calculations on the thermal stresses. Hopefully, this will lead to the design of improved cementing practices for thermal operations.

NOMENCLATURE

A_1, A_2 = area of inner and outer zones- 2 rh, sq ft

c_p = unit heat capacity of solid, Btu/lb_m - deg F

T = temperature within the solid, a function of time and position, deg F

T_B = temperature at boundary between zone 1 and zone 2 F

T_1 = temperature at any inner zone point

T_2 = temperature at any outer zone point

h = unit height, ft

k = thermal conductivity of solid Btu/hr-ft-deg F

r = radius at any point within the cylinder, ft

r_1, r_2 = change in radius in inner and outer zones, ft

Θ = dimensionless time = $2\alpha_i \Delta\theta / \Delta r_i^2$

α = thermal diffusivity of solid $= k/\rho c_p$; sq ft/hr

θ = time, measured from instant at which inner surface temperature is suddenly increased, hr

ρ = density of solid lb_m/cu ft

SUBSCRIPTS

B = boundary between zone 1 and zone 2

1 = inner zone

2 = outer zone

i = denotes some local position within solid with respects to x coordinate

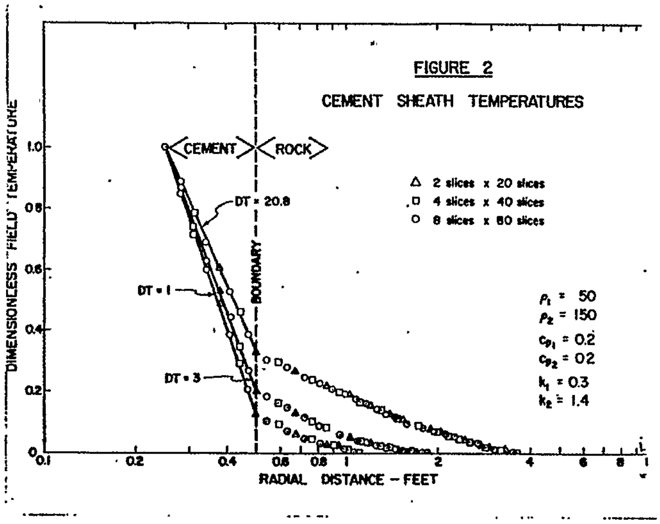

FIGURE 2
CEMENT SHEATH TEMPERATURES

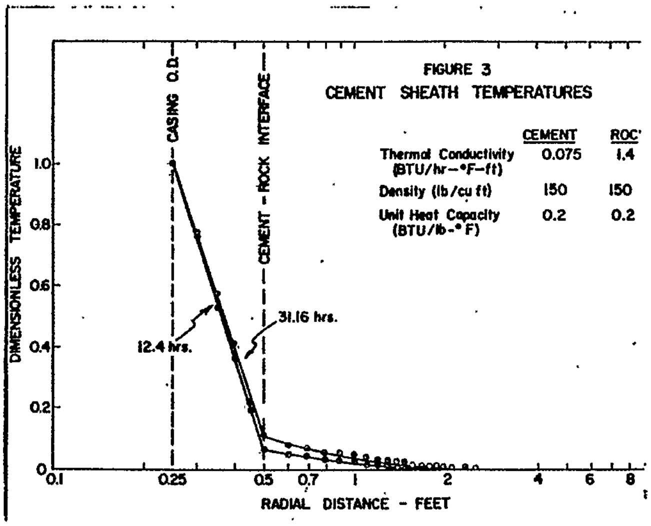

FIGURE 3
CEMENT SHEATH TEMPERATURES

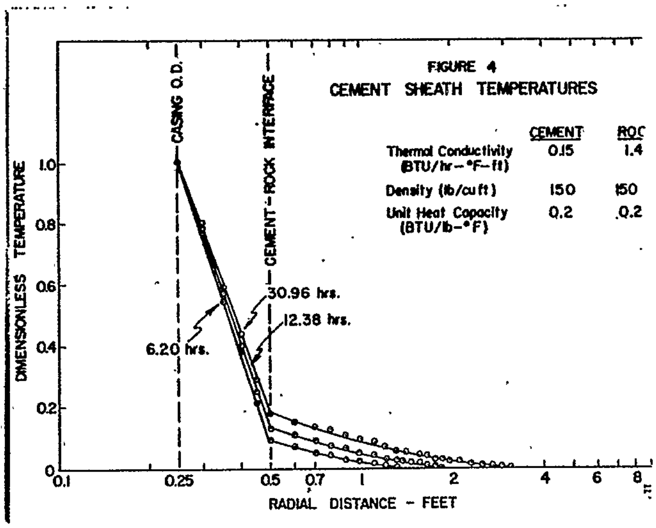

FIGURE 4
CEMENT SHEATH TEMPERATURES

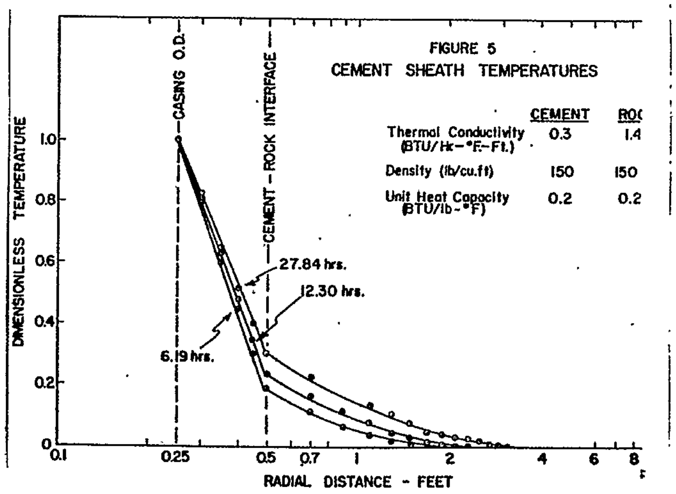

FIGURE 5
CEMENT SHEATH TEMPERATURES

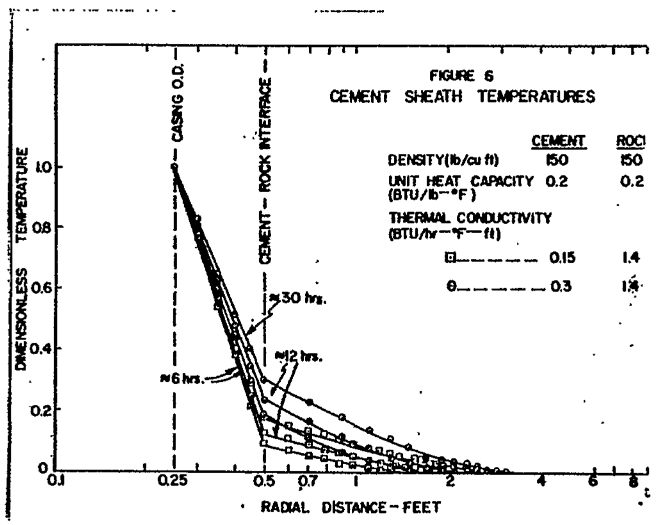

FIGURE 6
CEMENT SHEATH TEMPERATURES

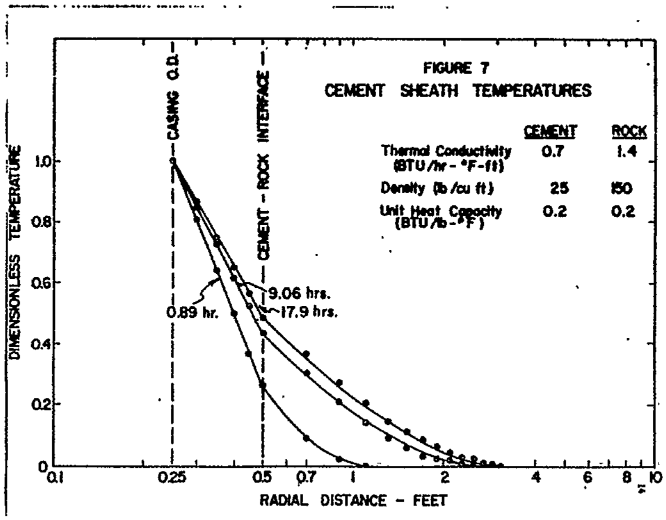

FIGURE 7
CEMENT SHEATH TEMPERATURES

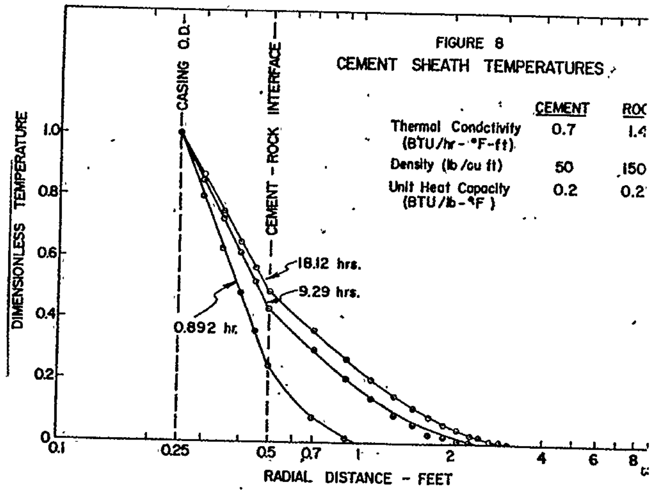

FIGURE 8
CEMENT SHEATH TEMPERATURES

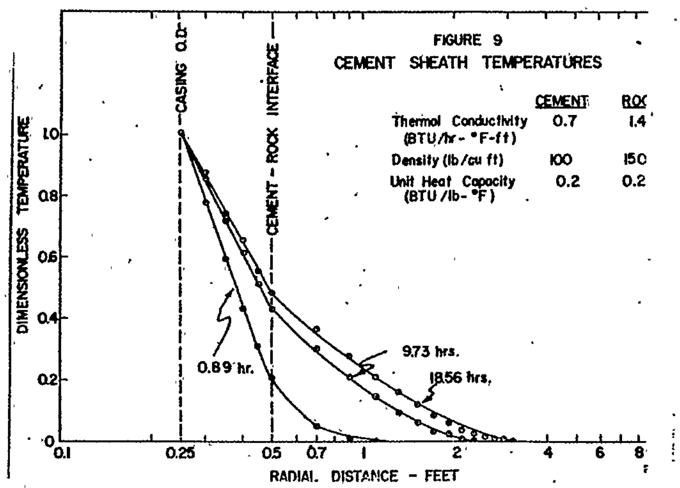

FIGURE 9
CEMENT SHEATH TEMPERATURES

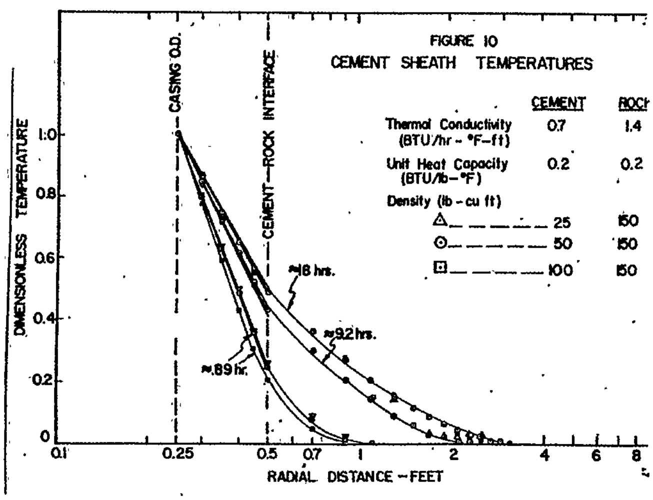

FIGURE 10
CEMENT SHEATH TEMPERATURES