Proceedings of the 1983
Winter Simulation Conference
S. Roberts, J. Banks, B. Schmeiser (eds.)

INPUT PROCESS MODELS

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Time series models for positive-valued and discrete-valued input processes are discussed, with the emphasis on the simulation problems which arise in generating time series from these models.

1. SUMMARY

Simple, additive models have long been available for dependent sequences of continuous random variables when the marginal distribution of the X_i's is Gaussian. These autoregressive-moving average ARMA(p,q) processes are also easy to generate on computers. Generalizations to other marginal distributions, and in particular to the positive random variables which occur in Operations Analysis, have only recently begun to appear.

The first-order autoregressive process, AR(1), is defined as

$$X_i = \rho X_{i} + E_i ,$$

where $|\rho| < 1$ and the E_i are independent. If an E_i exists for $0 \leq \rho < 1$ such that the X_i have a given distribution, $F_X(x)$, then that random variable X is called Type-L or self-decomposable. Examples are the exponential and Gamma distributions. For the exponential case E_i has a very simple form (Gaver and Lewis, 1980) but the process is defective. Extensions to non-degenerate first-order autoregressive processes in the exponential case have been found and these processes (NEAR(1), Lawrance and Lewis, 1981) can be extended to second-order autoregressions, NEAR(2), (Lawrance and Lewis, 1983). Higher order autoregressive structures probably also exist. In the Gamma case the E_i can not be found explicitly, but a method for simulating it has been found by Lawrance (1982). The process is, however, still degenerate. A broader and better behaved process was derived by Lewis (1982) using the Beta-Gamma transformation, and a multiplicative process was given by McKenzie (1981).

Various other continuous extensions of autoregressive processes have been obtained. AR(1) models with mixed exponential marginal distributions exist, but not for all values of ρ . The NEAR(2) structure generalizes quite easily to the case of Laplace distributions. For the exponential case, at least ten first-order autoregressive processes are now known. A problem is that negative serial correlation is difficult to obtain.

For discrete random variables, the analog of the (Gaussian) first-order autoregressive process is the first-order Markov chain. However, it is over-parameterized, especially for statistical purposes, and is not simple to simulate if the state space is large.

A very simple mixture model, DAR(1), was introduced by Jacobs and Lewis (cf. 1983); it is an explicitly autoregressive discrete process which works for any marginal mass function Π . It is in fact a Markov chain whose transition matrix is determined by Π and a parameter ρ , which is the first-order serial correlation, with $0 < \rho < 1$. This model generalizes to two types of mixed autoregressive-moving average structures, DARMA(p,q) and NDARMA(p,q). A drawback is that sample paths tend to "stick", giving "long" runs of the same value. A notable result here is that for the Markovian DAR(2) and DAR(3) processes, the joint limiting distribution of pairs and triples of X_n's is known. Thus it is possible to simulate the stationary process for any marginal mass function Π .

A discrete analog for self-decomposable (continuous) random variables was given by Steutel and van Harn (1979) in the form

$$X_n = \alpha * X_{n-1} + G_n ,$$

where $\alpha * X_{n-1}$ is, conditional on $X_{n-1} = j$, a Binomial (j,α) random variable. For X_n with a Poisson distribution, G_n is Poisson; this is the process which would be obtained by sampling an $M|M|\infty$ queue at regular intervals. The form of G_n is also known for negative binomial random variables, and for compound Poisson random variables.

In addition, McKenzie has shown that discrete analogs of the NEAR(2) structure exist for geometric random variables, and that discrete analogs of the Beta-Gamma process exist for negative binomials. Moving average analogs can also be found. Methods for obtaining negative correlation in these processes will be discussed, as well as the problem of modelling inhomogeneities in discrete valued time series.

2. REFERENCES

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