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ANTITHETIC VARIATES AND QUASIRANDOM POINTS AS VARIANCE REDUCTION TECHNIQUES

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This talk describes how one can transform uniformly distributed quantities to achieve substantive variance reduction in Monte Carlo sampling experiments. An abbreviated historical account of the theory of antithetic variance and the theory of quasirandom points will be presented with emphasis on the identification of circumstances in which results for accelerated error convergence are known in both the univariate and multivariate cases for antithetic variates and quasirandom points. The presentation includes extensions of the theory of antithetic variates to the simulation of Markov chains and semi-Markov processes and the application of quasirandom points to the analysis of stochastic networks. As a special topic, the talk describes how these techniques might fare in the setting of parallel processing.

It has long been known that techniques exist for manipulating the elements in a sequence of uniformly distributed quantities to accelerate the rates at which errors of estimators that use these quantities converge to the parameters being estimated. In the simulation literature, the most notable of these techniques is that of antithetic variates introduced by Hammersley and Morton (1956). The most popular form of this technique concerns the inducement of negative correlation between two replications of the simulation experiment.

1. ROTATION AND REFLECTION SAMPLING

Recently Fishman and Huang (1980) showed that more general antithetic techniques called rotation sampling and reflection sampling can be used to improve the error convergence rate even further. The first part of this talk will describe rotation and reflection sampling and the conditions under which they prove useful in accelerating convergence. In particular, the presentation will focus on conditions under which these techniques are optimal. Examples for estimating the parameters of a Markov chain and of a semi-Markov process will be used to demonstrate how rotation and reflection sampling work in practice.

2. QUASIRANDOM POINTS

Although the foregoing techniques have great value in practice, they do not easily generalize to arbitrarily specified simulations. We illustrate this fact with an example. However, other sequences of quasirandom points exist that are free of the drawbacks of rotation and reflection sampling and appear to offer great convenience in accelerating error convergence. van der Corput (1935) describes the first such univariate sequence. Hammersley (1960) and Halton (1960) describe multivariate sequences. Two of the most important properties of these sequences are that they are non-random and that for sampling in N dimensions with K quasirandom points the absolute error of estimate has upper bound $O((\log K)^N/K)$. This bound is better than $O(1/K^{1/2})$ which ensues for the standard error with K independent replications based on random sampling.

The second part of this talk will describe the features of quasirandom points. Emphasis will be placed on the types of simulation that presumably can benefit from use of these points. The technique will be illustrated with examples of the estimation of network reliability measures and of the estimation of distribution functions in stochastic activity networks.

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