Proceedings of the 1984
Winter Simulation Conference
S. Sheppard, U. Pooch, D. Pegden (eds.)

APPLICATION OF AN OPTIMIZATION PROCEDURE TO STEADY-STATE SIMULATION

Ardavan Nozari
Member of Technical Staff
AT&T Bell Laboratory
Murry Hill, NJ 07971

John S. Morris
Department of Business
University of Idaho
Moscow, ID 83843

Nozari and Morris (13) have proposed an optimization procedure for simulation experiments. In this paper we discuss how that method can be applied to simulation experiments. An example is provided to demonstrate the effectiveness of the method.

1. INTRODUCTION

Simulation of industrial systems often offers the only practical means of comparing alternative policy directions. For each set of decision variables a simulation experiment can be conducted in order to make a statistically valid statement about the expected response of the system. However, the decision maker is still faced with the problem of systematically evaluating the set of feasible input policies in order to select the best combination. For example, a materials manager attempts to select from a wide range of choices, the particular reorder point and order quantity policy which is expected to minimize costs. The general problem then may be represented as follows. Find the vector of decision variables ($\underset{\sim}{X}$) such that the expected simulation response ($\bar{f}(\underset{\sim}{X})$) is optimized.

Farrell et al (6) provide a survey of the techniques available for optimizing simulated systems. Procedures are classified as either Response Surface Methodology or direct search. The first, Response Surface Methodology, was initially proposed by Box and Wilson (3), and relies on fitting a linear or quadratic equation to the simulation response via regression analysis. Response Surface Methodology is applied to simulation experiments by several authors including Biles (1), Biles and Swain (2), Eldridge (5), Montgomery and Evans (11), and Smith (16).

The second category of approaches combines an optimization search procedure with a method for statistical comparison of two or more different systems. The work in this area includes Lefkowits and Schriber (9), Nelson and Kirsbergh (12), Pegden and Gately (14), Schmidt et al (15), and Smith (17). Nozari and Morris (13) propose combining a modification of the Hooke-Jeeves Pattern Search and the Dudewicz-Dalal method. The algorithm has been tested on a variety of two-variable functions with known expected optima. Results from these tests have been very encouraging. In this paper we apply the technique proposed by Nozari and Morris (13) to a steady-state simulation.

2. THE OPTIMIZATION PROCEDURE

The approach incorporates a modification of the Hooke-Jeeves Pattern Search and the Dudewicz-Dalal method for comparing system responses. The search procedure consists of a series of exploration and projection moves to regions defined by 9 policy vectors ($\underset{\sim}{X}$). If no improvement in the objective function is found in an exploratory move, a retraction step to the previous best point occurs. Finally, when the best point in a region is the center point, a contraction move is employed. The contraction move reduces the size of the region until the points in the region collapse to within a user-specified termination criterion.

Policies are evaluated and compared using the method developed by Dudewicz and Dalal (4). The technique involves a two-stage sampling from each of the systems and provides a weighted sample mean for each system. These weighted sample means are used as the criterion for selection of the best system. Law and Kelton (8) provide a discussion of this method and give one approach for calculating the weighted means.

The procedure requires that the user prespecify an acceptable probability of correct selection, P(CS), and a positive value which reflects the user's indifference between the top two policies. In particular, assuming that the objective is to select the system with the smallest expected response, the method assures $P(CS) \geq P^*$ provided that $\mu_{[k-1]} - \mu_{[k]} \leq \delta$
where:

- P^* is a user supplied value between 1/k and 1.
- δ is a user supplied indifference value between the best and next best system.
- $\mu_{[k]}$ is the mean response of the best system.
- $\mu_{[k-1]}$ is the mean response of the second best system.

For further discussion of the procedure see Nozari and Morris (13).

3. APPLICATION TO STEADY-STATE SIMULATION EXAMPLE

To apply the procedure to steady-state simulations we utilize the method of batch means with batch sizes determined by the procedure given by Fishman (7). For each proposed input policy we obtain a long simulation run. First, a portion of the initial observations are truncated to reduce the effects of initialization bias. The remaining observations are then grouped into batches containing only one observation. If the Von Neuman (18) test of independence fails, batch sizes are doubled and retested. The algorithm continues in this manner until the null hypothesis of independence can not be rejected, or the number of batches becomes less than or equal to 8. In the latter case, the simulation is restarted to obtain more observations and the procedure continues.

We note that the Dudewicz-Dalal procedure requires that observations be independent. Once an appropriate batch size is determined we treat the batch sample means as independent. In addition, the number of independent observations in the first stage must be the same for all systems. Therefore, if the number of batches remaining after the batch means test is less than this first - stage specification (usually n = 20 or n = 40), the simulation is continued until the necessary number of first stage batches has been collected. Note that this total includes the batches obtained for determining batch size.

Next, using these first stage batch means we apply the Dudewicz-Dalal procedure to determine the number of additional batches necessary for the second stage calculations. The simulation then continues until all second-stage observations are collected. Finally, a weighted average is calculated using the batch sample means from each stage. This weighted average is then used to compare alternative input policies and determine a search direction vis-a-vis the search procedure mentioned earlier.

As an example we applied the method to an (s, S) inventory system. The cost parameters and probability distribution for demand are given in Tables 1 and 2, respectively. The objective is to select the reorder point (s) and order quantity (S) combination which results in the lowest expected total cost.

Table 1
(s, S) Inventory Example Parameters

Parameter	Value
Carrying Cost/unit/period	\$ 2.50
Backorder Cost/unit/period	\$25.00
Order Cost/order	\$40.00
Beginning Inventory	3 units

Table 2
Probability Distribution for Demand

Demand (units)	Probability of Occurrence
1	.1
2	.4
3	.3
4	.1
5	.1

The results of runs from three different starting points are given in Table 3. The number of observations in each initial simulation run was 1,000; 100 of these observations were truncated. The Dudewicz-Dalal parameters used included a probability of correct selection of .9, an indifference value (δ) of 1.0 and the specified number of first stage observations equal to 20.

The program converges to the same solution, (s=0, S=10), when values are rounded to the nearest integer, providing support for the conclusion that a global expected optimum has been achieved. As the solution indicates costs savings can be substantial. In the case of the (s=5, S=5) policy, a 50% reduction in expected cost is obtained.

Table 3
Solutions to (s, S) Inventory Problem

Initial Values Starting Point			Termination Values		
s	S	TC*	s	S	TC*
5	5	49.05	0.0	10.0	24.70
0	5	29.21	0.2	9.9	24.72
5	10	35.59	0.1	9.9	24.67

* TC = expected total cost

4. CONCLUDING REMARKS

The optimization procedure applied in this paper provides an effective vehicle for analyzing problems with stochastic objective functions. The Dudewicz-Dalal procedure for comparing k systems simultaneously provides an improvement over other methods which do not use protected statistical tests. In addition, the batch means procedure for collecting independent observations makes the program relatively inexpensive to run. The inventory problem examined in the last section required less than 100 CPU seconds for each of the starting points and cost about \$10.00 to run.

REFERENCES

1. Biles, William E. Optimization of Multiple Response Simulation Models. West Bend, Indiana: University of Notre Dame, 1978.

2. Biles, William E. and Swain, James J. Optimization and Industrial Experimentation. New York: John Wiley and Sons, 1980.

3. Box, G. E. P. and Wilson, K. G. "On the Experimental Attainment of Optimal Conditions." Journal of the Royal Statistical Society. 13, 1951, 1-45.

4. Dudewicz, E. J. and Dalal, S. R. "Allocation of Measurements in Ranking and Selection with Unequal Variances." Sankhya, B37, 1975, 28-78.

5. Eldridge, David L. "Experimental Optimization of Statistical Simulation." Proceedings of the 1974 Winter Simulation Conference, 503-510.

6. Farrell, W. et al. Optimization Techniques for Computerized Simulation Models. California: CACI, INc. 1975.

7. Fishman, G. S. "Grouping Observations in Digital Simulation." Management Science, 24, 1978, 510-521.

8 . Law, A. M. and Kelton, W. D. Simulation Modeling and Analysis. New York: McGraw-Hill, 1982.

9. Lefkowits, R. M. and Schriber, T. J. "Use of an External Optimizing Algorithm with a GPSS Model." Proceedings of the 1971 Winter Simulation Conference.

10. Mirham, A. Simulation: Statistical Foundations and Methodology. New York: Academic Press, 1972.

11. Montgomery, D. C. and Evans, D. M. "Second-Order Response Surface Designs in Computer Simulation." Simulation, 25(6), 1975, 169-178.

12. Nelson, C. W. and Krisbergh, H. M. "A Search Procedure for Policy Oriented Simulations: Applications to Urban Dynamics." Management Science, 20(8), 1974, 1164-1174.

13. Nozari, Ardavan, and Morris, John S., "An Optimization Procedure for Simulation Models," Working paper, College of Business and Economics, University of Idaho, Moscow, Idaho.

14. Pegden, C. Dennis and Gately, M. P. "A Decision Optimization Module for SLAM." Simulation, January 1980, 18-25.

15. Schmidt, J. W. et al. "A Heuristic Elimination Method for Optimum Selection of Alternatives Through Simulation." Simulation, 21(5), 1973, 129-134.

16. Smith, Dennis E. Automated Response Surface Methodology in Digital Computer Simulation, Volumes I and II, Pennsylvania: Desmatics, Inc. 1975.

17. Smith, Dennis E. "Automated Optimum-Seeking Program for Digital Simulation." Simulation, July 1976, 27-31.

18. Von Neumann, J. "Distribution of the Ratio of the Mean Square Successive Difference to the Variance," Ann. Math. Stat., Vol. 12 (1941), pp. 367-395.