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SIMULATING THE $(HE_3/HE_3/S):(PRP/\infty/\infty)$ QUEUING MODEL FOR A MAINTENANCE PROBLEM

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ABSTRACT

A new queuing model $(HE_3/HE_3/S):(PRP/\infty/\infty)$, in which both the interarrival times and the service times follow the hyperexponential distributions with three branches, has been justified for a machine maintenance problem.

Analysis of the problem is based on the data obtained from a large and complex manufacturing system. The GPSS model is constructed so that it collects desired queuing statistics under alternative conditions of a different number of servers. The essential purpose of this study is to model a priority queuing problem with multi-servers. The GPSS design can apply to waiting line problems where random variables of interest have large variations.

Key Words: $(HE_3/HE_3/S):(PRP/\infty/\infty)$, Queuing Model, GPSS, Hyperexponential Distribution

1. INTRODUCTION

In a rubber manufacturing system, a large number of machines are subject to potential breakdowns. Each machine breakdown is assigned a priority number for corrective maintenance according to its relative importance in production. In the present system, only one electrician repairs broken machines.

The interarrival times between two consecutive machine breakdowns and the electrician's service times to repair those broken machines are random variables of interest. In order to predict behaviors of these random variables, standard statistical analyses such as data collection, data analysis, and hypothesis testing have been conducted. As a result, it has been justified that both the interarrival times and the service times follow hyperexponential distributions with three branches [1].

The following information has been obtained:

(1) The cumulative probability distribution of interarrival time is

$$F(x) = \sum_{i=1}^{3} \alpha_i(1 - e^{-\lambda_i x}) = 1 - 0.15e^{-0.2553x} - 0.54e^{-0.907x} - 0.31e^{-0.5395x}$$

Subject to

$$\sum_{i=1}^{3} \alpha_i = 1$$

where x is the interarrival time between two consecutive machine breakdowns.

(2) The cumulative probability distribution of service time is

$$F(y) = \sum_{i=1}^{3} \beta_i(1 - e^{-\mu_i y}) = 1 - 0.155e^{-0.613y} - 0.69e^{-2.73y} - 0.155e^{-0.613y}$$

subject to

$$\sum_{i=1}^{3} \beta_i = 1$$

where y is the electrician's service time.

(3) Machine breakdowns are classified into three priority groups: high, middle, and low. The high priority arrivals consume 8.9% of total arrivals; the middle, 83.2%; and the low, 7.9%.

Characteristics of the $(HE_3/HE_3/1):(PRP/\infty/\infty)$ can be derived analytically [1]. However, when the number of electricians, S, is equal to or greater than two, it is very difficult if not impossible to obtain desired characteristics, by analytical approach, for the queuing model $(HE_3/HE_3/S):(PRP/\infty/\infty)$. Therefore, the efforts of simulating the $(HE_3/HE_3/S):(PRP/\infty/\infty)$ queuing model, where S takes different integers, have been made; subsequently, GPSS computer language has been chosen to perform the task.

2. DESIGN SIMULATION FOR THE $(HE_3/HE_3/S):(PRP/\infty/\infty)$ QUEUING MODEL

To construct simulation for the corresponding queuing model, several key links are attacked respectively. They are design of arrival pattern, design of service pattern, and implementation of preemptive-resume queue discipline.

2.1. CONSTRUCT ARRIVAL PATTERN AND SERVICE PATTERN

The interarrival times under investigation follow a hyperexponential distribution with three branches. However, in GPSS the processor can only generate uniformly distributed pseudo-random numbers. By means of other features in GPSS, such as functions, random variates from a nonuniform distribution can be produced, provided users have to design the process by themselves.

A hyperexponential distribution

$$F(x) = \sum_{i=1}^{3} \alpha_i(1 - e^{-\lambda_i x})$$

is regarded as a mixture of three exponential distributions with the ith exponential distribution assigned by a normalized proportion α_i. Exponentially distributed random variates can be directly obtained by using the continuous 24-point GPSS function developed by IBM [2]. Thus, generating a hyperexponentially distributed random variate can be achieved by using two uniformly distributed random numbers in GPSS. The first number determines the exponential distribution to be used. If this number is less than or equal to α_1, the first exponential distribution with parameter λ_1 will be used. If the number is greater than α_1 and less than or equal to $\alpha_1 + \alpha_2$, the second exponential distribution with parameter λ_2 will be used. If the number is greater than $\alpha_1 + \alpha_2$, then the third exponential distribution with parameter λ_3 will be used. Next, the second random number is converted to an equivalent value from the corresponding distribution. This value is then divided by the corresponding parameter, λ_i, to give a desired hyperexponential random variate. The task stated above can be accomplished by defining operands A and B in the GENERATE block as two functions using two random number generators independently.

In constructing the GPSS model, a transaction corresponds to a machine breakdown in the real situation, and a facility in the model corresponds to a server, that is, an electrician. The electrician's service time for each machine breakdown has been assigned to the transaction representing that breakdown once the job gets into the model. The value of the service time for the transaction is stored in its first parameter. Service times also follow a hyperexponential distribution but with distinct parameters. Determination of the service time for each transaction is in the same manner as that of the interarrival time.

2.2. PRIORITY CONSIDERATIONS

Each transaction that has just been put into the model has a priority value of zero that corresponds to the low priority in the corresponding queuing problem. After each transaction has been assigned a service time value, its priority is reconsidered by two subsequent statistical TRANSFER blocks. Each time a transaction enters the first TRANSFER block, a uniform random variable is fetched to determine whether or not the transaction will represent a high priority machine breakdown. The percentage of the high priority arrivals in total arrivals is 8.9%. Therefore, if the value of that fetched random number is less than 0.089, the transaction will be sent to exit to the PRIORITY block where priority 2, which means high priority, will be given to this transaction. Otherwise, the transaction can never be a member of the high priority group and will go to the next TRANSFER block where the transaction might have a chance of becoming a member of the middle priority group. The percentage of the middle priority arrivals in total arrivals is 83.2%. In the long run, 91.1% of the total arrivals pass the first TRANSFER block and enter the second TRANSFER block. As soon as a transaction gets into the second TRANSFER block, one more uniform random number is sampled. If the return value is less than 0.913 [0.832/(1 - 0.089)], the transaction will become a member of the middle priority group and then be sent the PRIORITY block where priority 1, meaning the middle priority, will be assigned to the transaction. Otherwise, the transaction will keep the priority value of zero and be a low priority member until it leaves the model.

2.3. IMPLEMENTATION OF THE PREEMPTIVE-RESUME QUEUE DISCIPLINE

In this model, there are three priority transactions representing different machine breakdowns.

Whenever a transaction with priority 2 requires service, it will be served immediately unless there is at least one transaction with the same priority already waiting. This can be implemented by the following logic.

Once a priority 2 transaction requests service, it enters a SELECT block to check whether a facility is available. The auxiliary operator NU is used in this block to check which facility is not in use. If a facility is available, the transaction will enter the sequential block to catch the facility available. If no facility is empty, the transaction will go to another SELECT block to check whether a facility is being used by a transaction with priority 0. The auxiliary operator NI in the SELECT block is used to find which facility is not interrupted by a transaction with priority 1 or 2. After testing, if there is such a facility, the transaction will be directed to that facility to preempt the transaction with the defined low priority. If the test does not indicate such a facility, the transaction with priority 2 will go to the third SELECT block, where the auxiliary operator MIN is used to find a facility that is occupied by a transaction with the minimal priority value.

It should be mentioned here that each time a facility is caught by a transaction the first time, the priority value of this transaction will be stored in the savevalue corresponding to this facility. In this case, the SELECT block actually checks priority values in these savevalues, and the resulting value can only be either 1 or 2. No matter what value it is, the transaction with priority 2 will go to the sequential TEST block to test that the minimal value is 1 (equivalent to priority 2, which is greater than the minimal value as designated in the model). If the test result is true, that is, if there is such a facility that is currently used by a transaction with priority 1, the transaction with priority 2 will be directed to that facility to preempt the transaction with the lower priority (priority 1). If the test result is opposite, that is, all facilities are occupied by transactions with priority 2, this transaction has to go to user chain HIGH to wait.

As soon as a transaction with priority 2 finishes service and enters the UNLINK block, which is used to remove one transaction from user chain HIGH as specified in the model, the unlinkee transaction will be directed to the facility just released and the unlinker transaction will leave the model. If no transaction is fetched from the specified user chain, the leaving transaction, by going through path MOUT, enters another UNLINK block related to user chain MID. In this block, the leaving transaction tries to fetch a priority 1 transaction from user chain MID. If the operation is successful, a transaction with priority 1 will be directed to the facility available and the leaving transaction will leave the model by going through the sequential block. If no such facility is fetched, the leaving transaction enters the third UNLINK block by going through path LOUT. In this block, it tries again to fetch a transaction but with priority 0. If there is such a transaction waiting in user chain LOW, it will be fetched to the idle facility. The leaving transaction will, however, leave the model regardless of whether a priority 0 transaction has been fetched.

Whenever a transaction with priority 1 (defined as middle priority) requests service, it enters the SELECT block where the auxiliary operator NU is used to find a facility not in use. If such a facility is found, the coming transaction will go to sequential blocks and catch the available facility. If such a facility is not available, the coming transaction will, by going through path MIN2, enter another SELECT block where the auxiliary operator NI is used to check which facility is occupied by a transaction with priority 0. If such a facility is found, the coming transaction will be directed to preempt the transaction using that facility. If no facility is available, that is, all facilities are currently occupied by transactions with either priority 1 or priority 2, the coming transaction will go to user chain MID to wait. As soon as a transaction with either priority 1 or priority 2 finishes service and enters the UNLINK block related to user chain MID, it will try to fetch a transaction from user chain MID. If the attempt is successful, the unlinker transaction will go to sequential blocks to leave the model, and the unlinkee transaction will be directed to the facility released by the unlinker transaction. If no transaction is waiting in user chain MID, the unlinker transaction will, through path LOUT, enter the UNLINK block related to user chain LOW, and the coming unlinker transaction will try to remove one waiting transaction from user chain LOW to the available facility. Then, the unlinker transaction goes out of the block and leaves the model regardless of the results of unlink attempt.

When a transaction with priority 0 requests service, a check of whether a facility is idle will be carried out. If a facility is idle, the coming transaction will catch the facility. Otherwise, the transaction will be sent to user chain LOW to wait. Whenever a transaction with any priority value enters the UNLINK block related to user chain LOW, one waiting transaction will be fetched to catch the idle facility.

2.4. CONSIDERATIONS OF ALTERNATIVE CONDITIONS

The main concern about alternative conditions is related to the number of facilities used in the model. The number of electricians currently used in this system is one. In order to evaluate the policy concerned, data in addition to those under one electrician condition have to be collected.

This requirement has been accomplished by indirectly defining the number of electricians in savevalue 15. Therefore, in this model, this savevalue is assigned a value of one, two, or three. In this way, the machine maintenance process is simulated three times under the condition of one, two, or three electricians.

2.5. DEFINITIONS AND STRUCTURE OF THE MODEL

The definitions of entities in the model are given below:

GPSS Entity	Definition or Explanation
(1) Transactions	
Model segment 1	A machine breakdown
Model segment 2	A timer
(2) Facilities	
1 or 1, 2 or 1, 2, 3	Facility or facilities used to simulate electricians. The number of facilities is indirectly defined in savevalue 15
(3) Functions	
ARRIV	Function describing the mean interarrival time for different equivalent arriving branches
SERVE	Function describing the mean service time for different equivalent service branches
XPDIS & 3	Exponential distribution function
(4) Queues	
1, 2, 3	The queues used to collect statistics for the waiting lines of transactions with low, middle, and high priorities, respectively
LINE	The queue used to gather statistics for the waiting line of all transactions
(5) Tables	
1, 2	The tables used to collect interarrival times and service times to verify the predicted patterns
(6) Savevalues	
1 or 1, 2 or 1, 2, 3	The savevalue or savevalues corresponding to facilities 1 or 1, 2, or 1, 2, 3, used to store the priority values of the transactions currently in use of facilities
11, 12, 13	The savevalues used to store the mean service times for equivalent service branches. By assigning the savevalues different values, the statistics of the system under different service rates can be collected.
14	The savevalue used to store the value of total simulation time
15	The savevalue used in indirectly defining the number of facilities

(7) User chains

LOW	The user chain on which the low priority (priority 0) transactions wait for service
MID	The user chain on which the middle priority transactions wait for service
HIGH	The user chain on which the high priority transactions wait for service

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The simulation model represented by GPSS block diagram is illustrated in Fig. 1. The simulation time unit used in this model is one-tenth of a minute. The corresponding program is given in the Appendix.

3. SIMULATION RESULTS

Given that the mean interarrival time of machine breakdowns is 105.76 minutes and the mean service time of electricians is 45.5 minutes, the maintenance process has been simulated one year for each condition so that the number of servers is defined as one, two, or three.

The desired statistics from the program output are summarized in Table 1.

Table 1. Characteristics of the $(HE_3/HE_3/S)$:(PRP/∞/∞) queuing model simulation.

Number of Servers	Average Queue Length				Overall Average Waiting Time (minutes)
	1	2	3	Line	
1	0.002	0.484	0.065	0.552	58.99
2	0	0.027	0.002	0.029	3.16
3	0	0.001	0	0.001	0.16

From the simulation result, it can be expected that if one electrician with the same service rate is added to the maintenance team, the average number of machine breakdowns waiting for service will be reduced from 0.552 to 0.029. Whenever costs of machine breakdowns and labor can be taken into account, the optimal decision can be made.

4. CONCLUSIONS

The importance of this study is in designing a simulation model that can be applied to many waiting line problems. Modeling the arrival pattern and the service pattern in this study uses the existing achievements and is easy to follow. The implementation of the preemptive-resume queue discipline is also valuable because conventional simulations in GPSS work efficiently with single-server queuing problems [3].

5. ACKNOWLEDGMENT

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6. APPENDIX

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*
*
       SIMULATE
*
 XPDIS EQU         3,Z
 LINE  EQU         4,Q
       RMULT       891,221,331,441
*
       INITIAL     X11,1011/X12,218/X13,1011/X14,5256000/
*                  X15,2
*      FUNCTION SPECIFICATIONS
*
 ARRIV FUNCTION    RN2,D3
.15,2350/.683,660/1,1110
*
 SERVE FUNCTION    RN3,E3
.155,X11/.845,X12/1,X13
*
 XPDIS FUNCTION    RN4,C24
0,0/.1,.104/.2,.222/.3,.355/.4,.509/.5,.69/.6,.915/.7,
1.2  .75,1.38/.8,1.6/.84,1.83/.88,2.12/.9,2.3/.92,2.52/
.94,2.81 .95,2.99/.96,3.2/.97,3.5/.98,3.9/.99,4.6/.995,
5.3/.998,6.2   .999,7/.9998,8
*
*      TABLE SPECIFICATIONS
*
 1     TABLE       IA,300,300,40
 2     TABLE       P1,300,300,40
*
*      MODEL SEGMENT 1
*
       GENERATE    FN$ARRIV,FN$XPDIS,,,,2,F
       TABULATE    1
       ASSIGN      1,FN$SERVE,3
       TABULATE    2
       TRANSFER    .089,,HIGH
       TRANSFER    .913,,MIDLE
       QUEUE       3
       QUEUE       LINE
 LIN   SELECT NU   2,1,X15,,,LCH
       SEIZE       P2
       DEPART      LINE
       DEPART      3
       SAVEVALUE   P2,PR
       ADVANCE     P1
       PRIORITY    PR,BUFFER
       RELEASE     P2
 LOUT  UNLINK      LOW,LIN,1
 BYBYE TERMINATE
*
 LCH   LINK        LOW,FIFO
*
 HIGH  PRIORITY    2
       QUEUE       1
       QUEUE       LINE
       SELECT NU   2,1,X15,,,HM
 HIN1  PREEMPT     P2,PR
       DEPART      LINE
       DEPART      1
       SAVEVALUE   P2,PR
       ADVANCE     P1
       PRIORITY    PR,BUFFER
       RETURN      P2
```

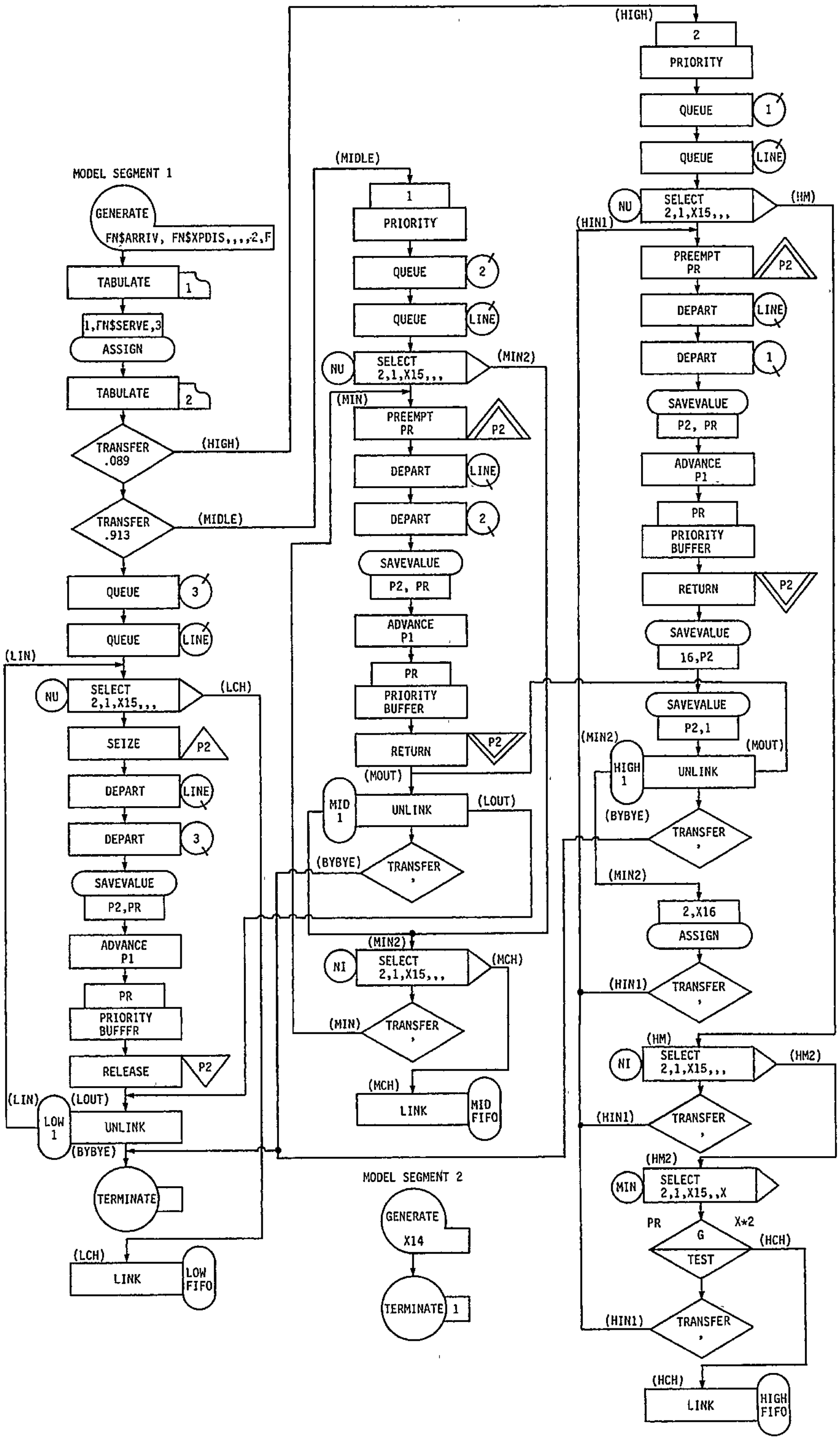

Fig. 1. GPSS block diagram of the $(HE_3/HE_3/S):(PRP/\infty/\infty)$ queuing model.

```
        SAVEVALUE  16,P2
        SAVEVALUE  P2,1
        UNLINK     HIGH,HIN2,1,,,MOUT
        TRANSFER   ,BYBYE
*
 HIN2   ASSIGN     2,X16
        TRANSFER   ,HIN1
*
 HM     SELECT NI  2,1,X15,,,HM2
        TRANSFER   ,HIN1
*
 HM2    SELECT MIN 2,1,X15,,X
        TEST G     PR,X*2,HCH
        TRANSFER   ,HIN1
*
 HCH    LINK       HIGH,FIFO
*
 MIDLE  PRIORITY   1
        QUEUE      2
        QUEUE      LINE
        SELECT NU  2,1,X15,,,MIN2
 MIN    PREEMPT    P2,PR
        DEPART     LINE
        DEPART     2
        SAVEVALUE  P2,PR
        ADVANCE    P1
        PRIORITY   PR,BUFFER
        RETURN     P2
 MOUT   UNLINK     MID,MIN2,1,,,LOUT
        TRANSFER   ,BYBYE
*
 MIN2   SELECT NI  2,1,X15,,,MCH
        TRANSFER   ,MIN
```

```
*
 MCH    LINK       MID,FIFO
*
*       MODEL SEGMENT 2
*
        GENERATE   X14
        TERMINATE  1
*
*       CONTROL CARDS
*
        START      1
        RMULT      891,221,331,441
        CLEAR      X11,X12,X13,X14
        INITIAL    X15,3
        START      1
        RMULT      891,221,331,441
        CLEAR      X11,X12,X13,X14
        INITIAL    X15,1
        START      1
        END
*
```

7. REFERENCES

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