Proceedings of the 1986 Winter Simulation Conference
J. Wilson, J. Henriksen, S. Roberts (eds.)

MODEL EVOLUTION: A ROTARY INDEX TABLE CASE HISTORY

A. Alan B. Pritsker
Pritsker & Associates, Inc.
West Lafayette, IN 47906

ABSTRACT

In this paper, an indexed rotary table employed in the assembly of instrument clusters for automobiles is modeled. The purpose of the paper is to illustrate the procedures involved in model development. The evolution and rationale behind various SLAM II® models of the indexed rotary table are described. The paper demonstrates that alternative modeling concepts and viewpoints are important, and that modeling procedures and analysis can lead to a greater understanding of the system under study.

1. INTRODUCTION

At a P&A SLAM II seminar, two attendees from an automotive company asked me how to model an indexed rotary table that they were studying. A description of the rotary table is given below.

An indexed rotary table serves eight assembly centers A1 through A8 as depicted in Figure 1. A part enters the table for Operation 1 at A1. The table rotates only when all eight assembly stations have finished their current operation. The assembly time for each station is uniformly distributed between 3 and 6 minutes. The indexing of the table (rotation) takes one minute. It is desired to determine the utilization of each assembly station in a 50-part production run and to estimate the length of the production run. Assume that the table is completely loaded initially.

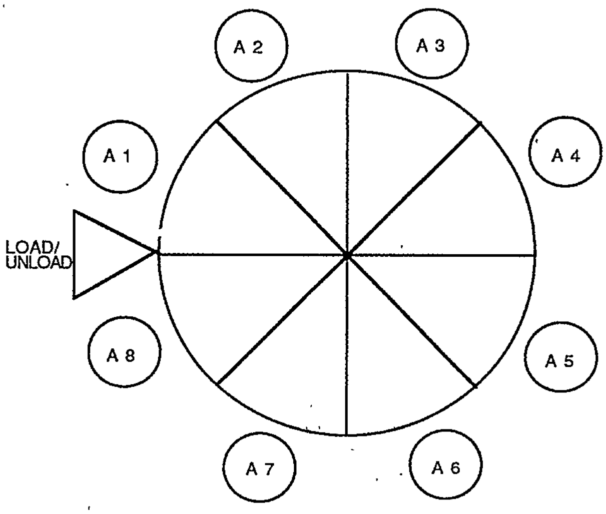

Figure 1. A Rotary Index Table.

2. MODEL 1

When beginning a modeling effort, the tendency is to represent the system as close as possible by modeling each element of the system. For the first model, each assembly station is modeled as a service activity and parts requiring the eight operations are modeled as entities. Attribute 1 of the entity is defined as the current operation number at which the part is being processed. The first model for this situation is shown in Figure 2.

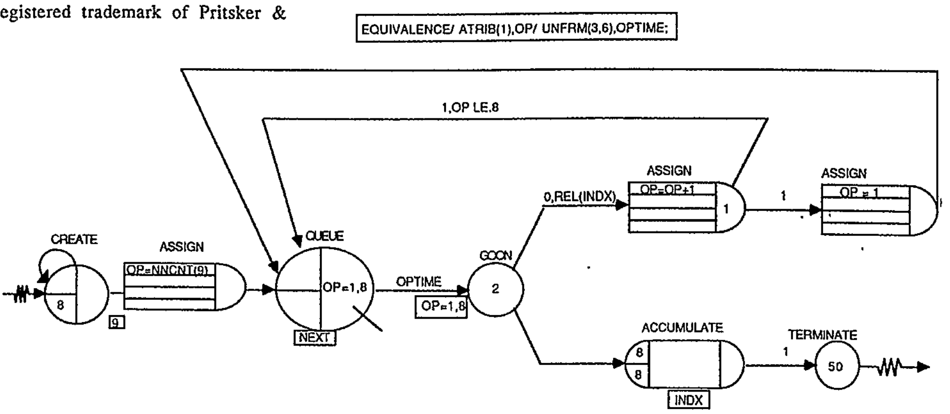

Figure 2. MODEL 1

SLAM II is a registered trademark of Pritsker & Associates, Inc.

At the CREATE node, eight entities are created that are routed to an ASSIGN node by traversing activity 9. At the ASSIGN node, the operation number where the entity is initially located is put into attribute 1 which is equivalenced to the letters OP. The entity is then sent to QUEUE node NEXT where it waits in file OP to be served by a service activity that performs its desired operation. The time to perform this operation is OPTIM which is equivalenced to a sample from a uniform distribution between 3 and 6.

In this situation, the QUEUE node-service activity concept of SLAM II models the eight assembly stations with each station having its own queue. Thus, all eight entities representing parts are in process at the same time. As soon as one of the parts completes processing, it is routed to a GOON node which sends it over an activity whose duration is indeterminant and specified as REL(INDX). This indicates that the part entity is to be released when the node whose label is INDX is released. An entity representing a signal is also sent from the GOON node to the ACCUMULATE node INDX which requires eight input signals before it is released.

When the ACCUMULATE node is released, all parts on the table have been completed and the rotary table can be indexed which takes 1 time unit. Since the production run is specified as 50 parts, a TERM node with a requirement of 50 is placed at the end of the path to model the completion of the production run. When the node INDX is released, all entities in the activity whose duration was indeterminant are routed to the ASSIGN node at the end of the activity. At the ASSIGN node, the operation number is increased by 1 and the part entity is routed to the next service activity if the operation number is less than or equal to 8. If the operation number is greater than 8 then the operation number is reset to 1 and one entity representing a new part is routed back to the QUEUE node NEXT.

This completes the first model of the rotary index table. As mentioned above, the model coincides with the physical characteristics of the system. (Actually 8 queues and 8 service activities could have been used on the network to display a greater correspondence between the model and the system.) The SLAM II input statements for this model are shown in Figure 3 and a portion of the SLAM II Summary Report is shown in Figure 4. The time to produce the 50 parts is seen to be 333.3 time units. The average of the 8 assembly station utilization values is 6.732.

```
GEN,PRITSKER,ROTARY_INDEX_TABLE,8/1/86;
LIMITS,8,9,10;
;
;  LET OP = ATTRIBUTE 1
;  LET OPTIME = SERVICE TIME
;
EQUIVALENCE/ATRIB(1),OP/UNFRM(3,6),OPTIME;
NETWORK;
      CREATE,0,,,8;               CREATE 8 ENTITIES
         ACT/9;
      ASSIGN,OP=NNCNT(9);         ASSIGN OP NUMBER TO ENTITY
NEXT  QUEUE(OP=1,8);              WAIT FOR STATION OP
         ACT/OP=1,8,OPTIME;       PERFORM ASSEMBLY
      GOON,2;
         ACT,REL(INDX),,OPAS;     WAIT FOR ALL 8 TO COMPLETE
         ACT,,,INDX;              SIGNAL ONE MORE ASSEMBLED
INDX  ACCUM,8,8;                  COUNT TO 8 BEFORE RELEASING
         ACT,1;                   ROTATE
OUT   TERM,50;                    FINISH PRODUCTION RUN OF 50
OPAS  ASSIGN,OP=OP+1,1;           SEQUENCE OP NUMBER
         ACT,1,OP.LE.8,NEXT;      ROTATE TO NEXT STATION
         ACT,1;
      ASSIGN,OP=1;                TAKE 1 OUT
         ACT,,,NEXT;              PUT 1 TO OP1
      END;
FIN;
```

Figure 3. Network Statements for MODEL 1

3. MODEL 2

In reviewing MODEL 1, it is observed that there is no need to change the operation number as the entities are indexed around the table. Thus, the ASSIGN nodes that index the OP number can be eliminated from the model. Further, it is only necessary to maintain the eight parts on the table and this can be accomplished with the SLAM II BATCH node. This eliminates the need for the

```
                                   S L A M   I I   S U M M A R Y   R E P O R T

                              SIMULATION PROJECT ROTARY_INDEX_TABLE        BY PRITSKER

                              DATE  8/ 1/1986                              RUN NUMBER    1 OF    1

                              CURRENT TIME   0.3333E+03
                              STATISTICAL ARRAYS CLEARED AT TIME  0.0000E+00

                                   **SERVICE ACTIVITY STATISTICS**

ACTIVITY  START NODE OR    SERVER    AVERAGE      STANDARD   CURRENT      AVERAGE   MAXIMUM IDLE  MAXIMUM BUSY  ENTITY
INDEX     ACTIVITY LABEL   CAPACITY  UTILIZATION  DEVIATION  UTILIZATION  BLOCKAGE  TIME/SERVERS  TIME/SERVERS  COUNT

   1      NEXT QUEUE          1         0.6749     0.4684        0         0.0000      3.6334        5.9260       50
   2      NEXT QUEUE          1         0.6763     0.4679        0         0.0000      3.8096        5.8623       50
   3      NEXT QUEUE          1         0.6745     0.4686        0         0.0000      3.5869        5.9613       50
   4      NEXT QUEUE          1         0.6626     0.4728        0         0.0000      3.6620        5.9486       50
   5      NEXT QUEUE          1         0.6669     0.4713        0         0.0000      3.6352        5.9619       50
   6      NEXT QUEUE          1         0.6803     0.4663        0         0.0000      3.7798        5.9880       50
   7      NEXT QUEUE          1         0.6748     0.4685        0         0.0000      3.8719        5.9911       50
   8      NEXT QUEUE          1         0.6754     0.4682        0         0.0000      3.5765        5.9763       50
```

Figure 4. SLAM II Outputs for MODEL 1

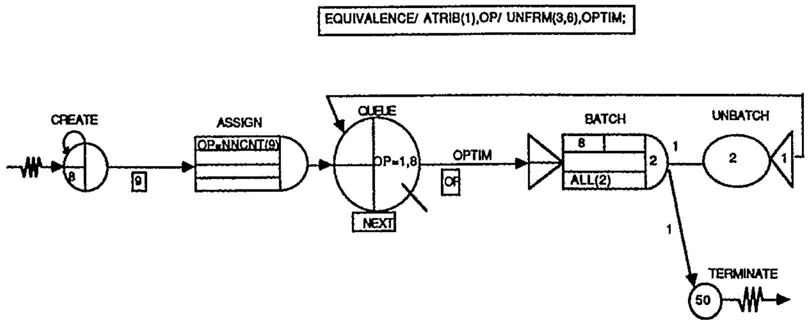

Figure 5. MODEL 2

REL(INDX) indeterminant activity duration in the model. The revised model, MODEL 2, is shown in Figure 5. In Figure 5, a BATCH node is used to cause the eight part entities to be put together into a batched entity. Attribute 2 of the batched entity is used to point to the original part entities that are in the batch. At an UNBATCH node, attribute 2 is referenced and each of the original part entities are routed back to the QUEUE node NEXT along the branch following the UNBATCH node.

4. MODEL 3

From MODEL 2, it is seen that each of the assembly stations performs an identical function and thus, a model could represent each station as an identical service activity. When this is done, an operation number need not be assigned to a part entity. A SLAM II model with this characteristic is shown in Figure 6. One entity is created and routed to an ASSIGN node where attribute 1 is set equal to 8. The entity is then routed to an UNBATCH node which splits the one entity into the number of entities according to the value of attribute 1. Each of these parts is routed to a QUEUE node that is followed by service activity 1 which represents eight identical parallel assembly stations. An ACCUMULATE node then accumulates eight entities. When eight entities are accumulated, the table can rotate and an entity is routed to a TERM node to count as one of the fifty parts produced. An entity is also routed to the UNBATCH node which causes eight new entities to be sent to the QUEUE node to restart the cycle.

5. MODEL 4

In MODEL 3, the QUEUE node does not perform a queueing function and can be removed from the network. The resulting model is shown in Figure 7. In MODELS 3 and 4 the utilization of all eight assembly stations is obtained as a group. Since all the assembly stations are equivalent, an estimate of the utilization of a single assembly station is one eighth of the utilization of all centers.

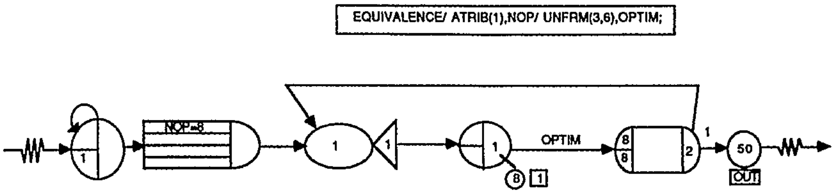

Figure 6. MODEL 3

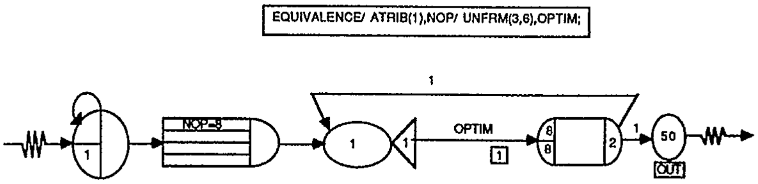

Figure 7. MODEL 4

6. A DIFFERENT VIEW

In reviewing MODEL 4, it is obvious that there is no queueing time as there are no QUEUE nodes in the network. Under such a situation, it is possible to perform an analysis to obtain directly the statistics desired from the model. From MODEL 4, it is seen that the rotary table cycle time is the largest OPTIM plus one where OPTIM is a uniformly distributed random variable between 3 and 6. Let Y_i represent this random variable for the i^{th} assembly station. Then the table cycle time, C, is a random variable which has the following equation:

$$C = \max[Y_1, Y_2, \ldots, Y_8] + 1$$

where $P[Y_i \leq t] = \frac{t - A}{B - A}$ for all i with A=3 and B=6.

Using basic concepts of probability theory, we have

$$\begin{aligned} P[\max[Y_1, Y_2, \ldots, Y_8] \leq t] \\ &= P[Y_1 \leq t, Y_2 \leq t, \ldots, Y_8 \leq t]. \\ &= P[Y_1 \leq t]P[Y_2 \leq t] \ldots P[Y_8 \leq t] \\ &= (P[Y_i \leq t])^8 \\ &= \left(\frac{t - A}{B - A}\right)^8 \end{aligned}$$

since the Y_i are uniform and iid.

7. MODEL 5

From the above analysis, we have derived a distribution function for the time to perform the operations on eight parallel stations. Since we have the cumulative distribution of this time, a single sample can be used to represent the activity time. The SLAM II network model with the operation time for all eight assembly stations defined as OPTIM8 is shown in Figure 8. At the CREATE node, a single entity is created and the time for activity 1 is the maximum of the eight operation times. At the end of the activity, a signal is sent to the TERM node and an entity is returned to the CREATE node to begin another cycle. The variable OPTIM8 is equivalenced to USERF(1) and the modeler writes the FORTRAN function USERF to obtain a sample from the cumulative distribution function derived in Section 6.

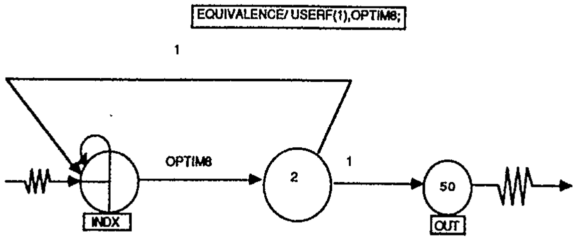

Figure 8. MODEL 5

To obtain a sample from a cumulative distribution function, we use the inverse transformation theorem, that is, we set a random number, r, equal to the cumulative distribution and solve for the variable of interest. This results in the following equation

$$r = \left(\frac{t - A}{B - A}\right)^8$$

Solving for t yields

$$t = \sqrt[8]{r} * (B - A) + A$$

Substituting DRAND(1) for the random number and A=3 and B=6 results in the following code for function USERF(I).

```
FUNCTION USERF(I)
USERF = DRAND(1)**0.125 * 3. + 3.
RETURN
END
```

8. MODEL 6

The problem statement requests the utilization of each assembly station which is equivalent to the fraction of time in each cycle that an assembly station is in use. The expected cycle time can be computed using the results of Section 6 as follows.

$$E[C] = E[MAX[Y_1, Y_2, \ldots, Y_8] + 1]$$

$$= \int_A^B t\left(\frac{8}{B-A}\right)\left(\frac{t-A}{B-A}\right)^7 dt + 1$$

$$= \int_3^6 0.0012193t\,(t-3)^7\,dt + 1$$

This integral can be solved using the continuous capabilities of SLAM II. The code required to solve the integral representing the expected time to complete 8 parallel operations is presented in Figure 9. The expected cycle time is the value of SS(1) at the end of the run, 5.667 minutes, plus the rotation time. Thus, the expected cycle time is 6.667 minutes. An approximation to the station utilization is the expected operation time 4.5 minutes divided by the expected cycle time or 0.675.

```
 SUBROUTINE STATE
 COMMON/SCOM1/ATRIB(100),DD(100),DDL(100),DTNOW,II,MFA,MSTOP,NCLNR
1,NCRDR,NPRNT,NNRUN,NNSET,NTAPE,SS(100),SSL(100),TNEXT,TNOW,XX(100)
 DD(1)=0.0012193*TNOW*(TNOW-3.)**7
 RETURN
 END
```

```
GEN,PRITSKER,CONT_ROT_IND,8/1/86;
CONT,1,0,0,1;
INIT,3,6;
FIN;
```

```
                 S L A M   I I   S U M M A R Y   R E P O R T

          SIMULATION PROJECT CONT_ROT_IND            BY PRITSKER

          DATE  8/ 1/1986                            RUN NUMBER    1 OF    1

          CURRENT TIME   0.6000E+01
          STATISTICAL ARRAYS CLEARED AT TIME  0.3000E+01


          **STATE AND DERIVATIVE VARIABLES**

 (I)         SS(I)          DD(I)
  1        0.5667E+01     0.1600E+02
```

Figure 9. MODEL 6

The integral given above can be evaluated using integration by parts to yield a value of 5 2/3. The expected time to produce 50 parts is 50 cycle times or 333 1/3 time units.

9. DISCUSSION

The problem presented in this paper is a real one. Engineers were actively involved in the design of a rotary index table. The models presented in this paper were not used to solve that problem. They did provide a starting point for a solution. The models for the problem solution became more complex through the addition of more factors and additional control strategies. It was interesting to note that the models presented in this paper seemed to get simpler but each simplification required additional insight on the part of the modeler. Any one of the models could have been the starting point. The simplest model is not necessarily the best. From experience we know that models tend to grow and expand to meet new system requirements or to include additional operational procedures. Embellishing MODEL 1 is a lot easier than embellishing MODEL 4, 5 or 6. The criteria for a good model is not necessarily a small number of nodes nor a high level of preciseness in the answer. In many cases, a good model is one that represents the system sufficiently to meet the purpose for modeling and is timely and extendable. Furthermore, a good model is one that is understandable, that is, it can be communicated and documented.

In the model evolution presented in this paper, insight and information was gleaned from each preceding model. This is a positive aspect of digging into a model and understanding how the model elements relate to the system. Network models are particularly well suited for this type of understanding. It is hoped that this paper has illustrated the relationship between modeling and design. A major conjecture stemming from the paper is that modeling is a difficult process because we do not have measurable criteria for evaluating the goodness of a model.

AUTHOR'S BIOGRAPHY

A. ALAN B. PRITSKER is President of Pritsker & Associates,Inc. He graduated from Columbia University with a BSEE and MSIE. In 1961, he obtained a Ph.D. from The Ohio State University. From 1956 through 1962, Dr. Pritsker worked for Battelle Memorial Institute in Columbus, Ohio. From 1962 through 1981, he was a Professor of Industrial Engineering at Arizona State University, VPI, and Purdue University.

Dr. Pritsker has published over 100 technical papers and written six books. He is a Fellow of AIIE and the holder of AIIE's Distinguished Research Award and Innovative Achievement Award. In 1981 he was the Indiana Academy of Science Speaker-of-the-Year. Dr. Pritsker is a member of the National Academy of Engineering.

A. Alan B. Pritsker
Pritsker & Associates, Inc.
P.O. Box 2413
West Lafayette, IN 47906
(317) 463-5557