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Alternatate methods for generating and analyzing the output series of frequency domain experiments

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ABSTRACT

In this paper we compare two different methods of spectral estimates for Frequency Domain Experiments and present some performance data for execution on a SUN 3/50 workstation. We also suggest an alternative method for generating the output series by replication and demonstrate that at least for some cases this is more efficient than using a single run of longer length.

1. INTRODUCTION

Frequency Domain Experiments (FDE) were recently introduced by Schruben and Cogliano [1987] as a means of factor screening. Essentially the method involves sinusoidally oscillating the input factors over their ranges of interest and then examining or testing the spectrum of the output series generated by the simulation model for 'peaks'. Spectral peaks at indicator frequencies signify that the corresponding input factor significantly affects the output of the simulation model.

Once the output series has been generated, the primary computational effort involves estimating the spectrum. There are two basically different approaches for estimating the spectrum and in this paper we compare these and suggest situations where one may be more useful than the other. We also suggest an alternative method of generating the output series which may be more efficient than using a single long run. The remainder of this paper is organized as follows:

Section 2: Alternative Methods of Estimating the Spectrum
Section 3: An Alternative Approach for Generating the Output Series
Section 4: Summary and Conclusions

2. ALTERNATIVE METHODS OF ESTIMATING THE SPECTRUM

Methods of estimating the spectrum fall under two basic classes:
a) Transforming the truncated autocovariance function, and
b) Smoothing the periodogram.

The methods falling under (a) basically involve taking a Fourier Transform of the truncated sample autocovariance function using a weighting procedure. Two of the popular weighting procedures are Tukey and Parzen windows. For the discussions that follow, we shall assume a Tukey window since both Tukey and Parzen windows give similar estimated spectrum for a given series [Chatfield, pp. 141]. Also, we shall not consider the Hanning and the Hamming windows separately since the former is equivalent to using the Tukey window and the latter is only slightly different from the former [Chatfield, pp. 143].

Alternative methods falling under (b) are based on smoothing the periodogram [Chatfield, p. 133] by simply grouping the periodogram ordinates in groups (of size m) and finding their average value. It is also possible to use nonuniform averaging procedures to smooth the periodogram. However, in this paper, we shall assume that the smoothing is uniform. We shall also assume that periodogram ordinates are estimated using a Fast Fourier Transform (FFT). Without the use of FFT, estimating periodogram ordinates are computationally very expensive and are impractical except for output series of very small length. For the purpose of this paper, we have used a FFT algorithm described in Oppenheim and Schafer (1975, pp. 331). This algorithm requires the length of the time series to be equal to 2^n, where n is a positive integer.

2.1. Desirable Properties of Spectrum Estimators

When using FDE, spectrum estimates should have the desirable properties of high resolution and low variance. For a given output series length, this combination makes it easier (both visually and statistically) to identify the spectral peaks at indicator frequencies. There is however, a trade off. High resolution can be achieved by using a high value of window size (M) for the Tukey window or a low value of 'm' used for smoothing the periodogram. In both cases, the variance increases. Figures 1 and 2 show how the resolution increases with Tukey window size. (Note numerical values of estimated spectrum in addition to plots.) In many cases, therefore, it is necessary to have a very long run length (to keep the variance low) and to use a large Tukey window 'M' or a small value of 'm' (to achieve a good resolution). For example, the authors have found that to detect the quadratic effect of

Estimated Spectrum	Frequency
3931.8623529	0.010
1221.6218419	0.020
663.0366565	0.030
349.4327954	0.040
241.5978550	0.050
151.3986663	0.060
141.1010009	0.070
123.7406392	0.080
115.5963025	0.090
107.9571918	0.100
75.0465849	0.110
60.3164878	0.120
44.2421036	0.130
45.1755197	0.140
35.4634683	0.150
32.4093003	0.160
40.3396055	0.170
36.0543801	0.180
26.6875776	0.190
17.6967391	0.200
22 9489869	0.210
17.7497550	0.220
18.1389505	0.230
15.9111238	0.240
18.2298355	0.250
14.2882357	0.260
12.3134764	0.270
13.7390175	0.280
12.2023374	0.290
11.2067182	0.300
13.6452216	0.310
9.6329941	0.320
11.9728296	0.330
10.5627212	0.340
13.8048156	0.350
10.2548247	0.360
10.7301360	0.370
10.5150998	0.380
10.4651026	0.390
11.2800522	0.400
11.5232388	0.410
7.8632184	0.420
10.0379776	0.430
8.6117590	0.440
7.9967489	0.450
9.3482479	0.460
8.8644199	0.470
7.4069451	0.480
8.7646474	0.490
7.4967492	0.500

Plot of Estimated Spectrum

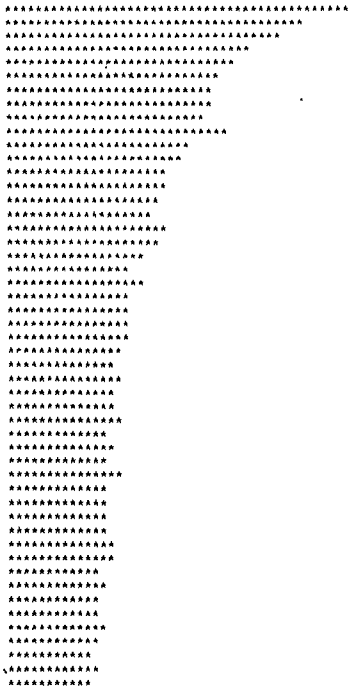

Figure 1

Run Length = 5000
Tukey Window Size = 100

Estimated Spectrum	Frequency
4266.8218548	0.010
1367.8901090	0.020
705.3273468	0.030
330.8818220	0.040
256.0468803	0.050
151.3395042	0.060
138.5939209	0.070
112.5013902	0.080
115.8986071	0.090
273.6435637	0.100
71.4016493	0.110
58.0032075	0.120
36.3272031	0.130
47.1538150	0.140
35.8303469	0.150
30.9369687	0.160
40.3827569	0.170
37.7112708	0.180
27.9072179	0.190
15.8536001	0.200
24.8380552	0.210
16.1747600	0.220
15.6337146	0.230
13.4462594	0.240
18.3543066	0.250
14.5590483	0.260
11.7005221	0.270
13.2887626	0.280
12.5218335	0.290
11.5409121	0.300
14.5359390	0.310
7.9071824	0.320
12.3732085	0.330
8.9060949	0.340
14.7463506	0.350
8.1680304	0.360
8.7284737	0.370
10.0359435	0.380
9.8684730	0.390
9.1500075	0.400
10.9452359	0.410
6.6392277	0.420
11.5381767	0.430
9.4207888	0.440
8.3187525	0.450
11.4029967	0.460
9.3345468	0.470
7.1830713	0.480
9.3001683	0.490
5.5940286	0.500

Plot of Estimated Spectrum

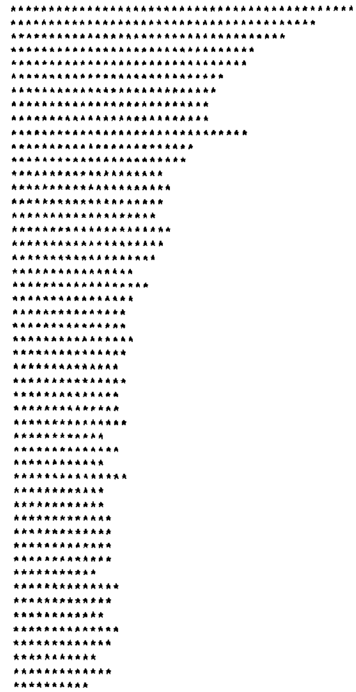

Figure 2

Run Length = 5000
Tukey Window Size = 200

Estimated Spectrum	Frequency
323.4174393	0.010
228.2678682	0.020
179.1994763	0.030
140.1541380	0.040
107.3753629	0.050
84.0939558	0.060
64.3450644	0.070
52.3364713	0.080
48.5587722	0.090
122.5059994	0.100
33.2019902	0.110
29.3248221	0.120
25.6748045	0.130
21.5577098	0.140
20.6115596	0.150
18.4629633	0.160
15.8741882	0.170
14.9654042	0.180
13.3902028	0.190
13.3424905	0.200
12.5019596	0.210
11.6652373	0.220
11.3704167	0.230
10.3976522	0.240
9!7927885	0.250
8.8214522	0.260
9.0546902	0.270
8.5830984	0.280
8.0032526	0.290
7.6956787	0.300
7.0148500	0.310
7.0310086	0.320
7.0339441	0.330
6.4210232	0.340
6.5360789	0.350
6.2834553	0.360
6.1872398	0.370
6.0514195	0.380
5.8399290	0.390
5.4743685	0.400
5.6284764	0.410
5.0428921	0.420
5.1314067	0.430
5.2419541	0.440
5.2126555	0.450
5.2604847	0.460
5.5902620	0.470
5.3211185	0.480
5.1385297	0.490
5.0333473	0.500

Plot of Estimated Spectrum

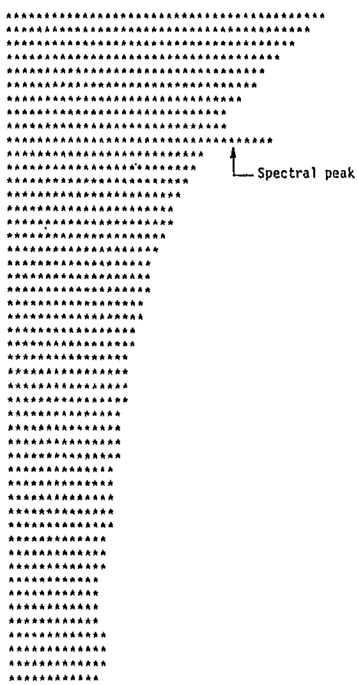

Figure 3

Run Length = 45,000 (single replication)
Tukey Window Size = 100

Estimated Spectrum	Frequency
28.4279046	0.010
22.5917671	0.020
18.0850313	0.030
17.1984263	0.040
12.9769738	0.050
8.9679996	0.060
7.8659411	0.070
5.7357734	0.080
4.8190604	0.090
81.1117400	0.100
4.4776492	0.110
3.0568095	0.120
2.2427064	0.130
3.0615496	0.140
2.3229396	0.150
2.2432309	0.160
2.1132352	0.170
2.1441363	0.180
1.7983663	0.190
1.5253140	0.200
1.6095402	0.210
1.2627694	0.220
1.4385566	0.230
1.2324623	0.240
1.0482170	0.250
0.9150065	0.260
0.7169593	0.270
0.8638366	0.280
1.0066517	0.290
0.9096633	0.300
0.7330950	0.310
0.6245009	0.320
0.7197212	0.330
0.7408404	0.340
0.7428818	0.350
0.7552007	0.360
0.7135710	0.370
0.5794899	0.380
0.6808164	0.390
0.6147074	0.400
0.7094569	0.410
0.6190146	0.420
0.5623098	0.430
0.4853313	0.440
0.5963107	0.450
0.4561782	0.460
0.5494564	0.470
0.5050825	0.480
0.5315962	0.490
0.6475668	0.500

Plot of Estimated Spectrum

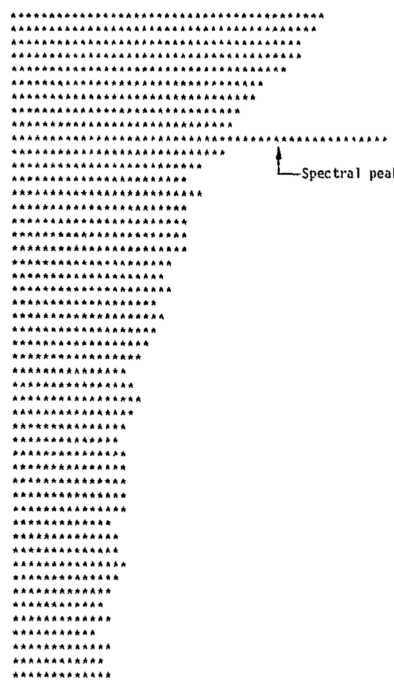

Figure 4

Run Length = 5000 (each point is the average from nine (9) replications)
Tukey Window Size = 100

service time on the mean waiting time for an M/M/1 queue, a run length in the order of one million observations and a Tukey window size of one thousand is necessary.

2.2. Computational Complexity

When transforming the truncated autocovariance function (using the Tukey window), the computational complexity $\simeq$ run length x Tukey window size (M). Therefore for a given run length the computation time increases in direct proportion to the Tukey window size. Using the smoothed periodogram (with the Fast Fourier Transform), the computational complexity $\simeq$ run length x $\log_2$(run length). (We are assuming that run length = 2^n, where n is an integer). Method (b) therefore is more efficient than method (a) as long as run length < 2^M, where M is the Tukey window size. In many cases of practical interest, driving frequencies must be very low (≤0.001) to detect higher order effects by FDE. In such situations, band width requirement (see Schruben and Cogliano) dictates that M > 1000. It therefore follows that method 'b' will be more efficient as long as run length < 2^{1000} (i.e., almost always).

It is apparent in method (a) that the computational complexity is determined by the required resolution since the Tukey window size is primarily decided by the required resolution. In method (b), the computational complexity can be considered as independent of the required resolution (since there appears no 'm' in the approximate expression for complexity). We wish to point out that Fast Fourier Transforms can be used for method (a) also. In that case, the computational complexity is largely independent of the Tukey window size, but one needs a transform of the original data and then a retransformation [Chatfield, pp. 149], thus potentially requiring twice as much computation as method (b).

2.3. Sample Results

Table 1 shows the CPU time required for estimating the spectra on a SUN 3/50 workstation using the alternative methods described above.

TABLE 1

Performance Data for Alternative Spectrum Estimation Methods

Case	Method (a)	Method (b)
1	Run length = 32,768 Tukey window size, M = 50 CPU time = 14 minutes	Run length = 32,768 m = 874 CPU time = 5 minutes
2	Run length = 65,536 Tukey window size, M = 100 CPU time = 40 minutes	Run length = 65,536 m = 874 CPU time = 11 minutes
3	Run length = 65,536 Tukey window size, M = 300 CPU time = 116 minutes	Run length = 65,536 m = 291 CPU time = 10 minutes

The output series for these estimations was generated from an M/M/1 queueing model with oscillating mean service time. Tukey window size M and m were chosen so that both methods generate a spectral estimate having approximately equal variance (see Chatfield pp. 154).

3. AN ALTERNATIVE APPROACH FOR GENERATING THE OUTPUT SERIES

The basic aim of the spectrum estimates is to detect sinusoidal patterns in an output series. In the presence of noise, the detection is made easier if a longer series (i.e., longer run length) is available. That is why spectral estimates improve as run length is increased. The other alternative is to reduce the noise. This can be easily done by replication. Preliminary investigation by the authors suggest that noise reduction by replication may be more efficient than using a long run length. Figures 3 and 4 show the estimated spectrum by using these two methods. As before, we used an M/M/1 queue with oscillating mean service time with an oscillation frequency = 0.1. Spectrum shown in Figure 3 is based on an output series of length 45,000. Spectrum shown in Figure 4 is based on an output series of length 5000. Each observation in this latter series is the average of nine (9) independently seeded replications each of length 5000. It can be easily seen that spectral peak in Figure 4 is much sharper than the spectral peak in Figure 3. (Note the numerical values as well as the plots.)

Two things can be noted from these figures. First, the difference between the spectral peak at frequency 0.1 and the level of spectrum immediately surrounding it is approximately the same (i.e., $\simeq$ 80) in both cases. This implies that strength of the input signal is equally well detected in both cases. Second, the noise (represented by the general level of the spectrum) is an order of magnitude lower for Figure 4. Since the statistical tests described in Schruben and Cogliano, and Sanchez are based on the ratio of the detected signal strength and the noise, it appears that the method of replication should give better statistical results.

4. SUMMARY AND CONCLUSIONS

In this paper we compared two alternative methods of spectrum estimation and showed that where high resolution is needed, it is preferable to use a smoothed periodogram using Fast Fourier Transform. We also suggested an alternative method of generating the output series by replication and showed that at least for some situations this is more efficient than increasing the run length.

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