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SOLUTION TO THE INDEXING PROBLEM OF FREQUENCY DOMAIN SIMULATION EXPERIMENTS

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ABSTRACT

A frequency domain simulation experiment is one in which selected system parameters are oscillated sinusoidally to induce oscillations in one or more system statistics of interest. A spectral (Fourier) analysis of these induced oscillations is then performed. To perform this spectral analysis, all oscillation frequencies must be referenced to a common, independent variable—an oscillation index. In a discrete-event simulation, the global simulation clock is the most natural choice for the oscillation index. However, past efforts to reference all frequencies to the simulation clock generally yielded unsatisfactory results. The reason for these unsatisfactory results is explained in this paper and a new methodology which uses the simulation clock as the oscillation index is presented. Techniques for implementing this new methodology are demonstrated by performing a frequency domain simulation experiment for a network of queues.

1 INTRODUCTION

A frequency domain simulation experiment (FDE) is one in which selected system parameters are oscillated sinusoidally. Each selected parameter is assigned a unique frequency of oscillation. If the system's response is sensitive to a selected system parameter, then the sinusoidal oscillation of that parameter will—hopefully—induce similar oscillations in the response. If so, spectral (Fourier) analysis of these induced oscillations yields valuable information about the system.

FDEs were first introduced to discrete-event simulation in 1981 by Schruben et. al. [Schruben and Cogliano, 1981]. The objective was to perform input parameter sensitivity analysis for factor screening in complex simulations. Since then significant work has been done to develop FDE techniques; this includes oscillation amplitude considerations [Jacobson, 1989], driving frequency selection [Jacobson *et al.*, 1987], methods for generating and analyzing the output series [Som and Sargent, 1988], methods of flattening the noise spectrum [Buss, 1988] and methods for using the global simulation clock time as the oscillation index [Jacobson *et al.*, 1988]. Efforts have also been made to extend the FDE technique to metamodeling [Schruben and Cogliano, 1987], [Sanchez and Buss, 1987] and [Som *et al.*, 1987], to the design of algorithms for simulation optimization [Morrice and Schruben, 1989], [Schruben, 1986] and to gradient estimation [Jacobson, 1990]. However, in a recent work, Sargent et. al [Sargent and Som, 1988] have pointed out that even when performed correctly FDEs, as they currently exist, have limited application.

In a FDE, all frequencies are measured relative to a common independent variable—an oscillation index. In most of the available FDE literature [Buss, 1988], [Jacobson *et al.*, 1987]-[Jacobson, 1990], [Morrice and Schruben, 1989]-[Sanchez and Schruben, 1986], [Schruben and Cogliano, 1981]-[Som *et al.*, 1987], the systems analyzed have been quite simple. For these simple systems, choosing some *discrete* simulation parameter, e.g. job number in a single-server queue, as the oscillation index yields satisfactory results. However, in more complex systems, e.g. an open network of queues, jobs do not necessarily leave the system in their order of arrival and so, as observed in [Buss, 1988] and [Jacobson *et al.*, 1988], choosing a discrete simulation parameter as the oscillation index may, in this case, require re-ordering of the output before it can be analyzed. Other more complex systems, e.g. a closed network of queues, are easily envisioned for which there may not be any natural discrete simulation parameter to serve as the oscillation index.

An inherent feature of virtually all discrete-event simulations, is the need to keep track of time via a global simulation clock. For example, in a next-event simulation, events are scheduled in time and the sim-

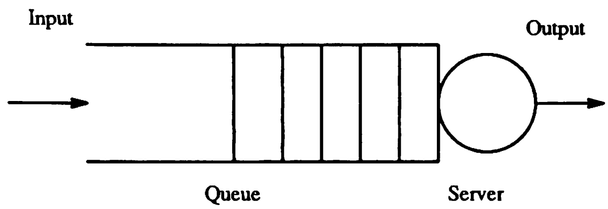

Figure 1: Single-Server FIFO Queue.

ulation clock is advanced from one event time to the next as the simulation progresses. In this manner the global simulation clock time becomes a natural *continuous* variable with respect to which all dynamic variables are referenced. Thus, the global simulation clock time is a natural choice for the FDE oscillation index. Attempts were first made by Jacobson et. al. [Jacobson *et al.*, 1988] to use the global simulation clock as the oscillation index. However, they concluded that the "··· global clock is not necessarily a good index for running frequency domain experiments" and a discrete oscillation index should be used whenever possible. We disagree.

In this paper, we first explain why past efforts to reference all frequencies to the global simulation clock generally yielded unsatisfactory results. Then we introduce a technique for performing FDEs which uses the simulation clock as the oscillation index. The technique is based upon histograming data sampled at random event times. This technique is versatile, efficiently implementable and resilient to noise. The technique is illustrated by performing a FDE for a network of queues.

2 THE FDE INDEXING PROBLEM

In a recent paper Jacobson et. al.[Jacobson *et al.*, 1988] suggested using the global simulation clock as the FDE index. They considered an M/M/1 queuing system [Figure 1] and oscillated the arrival rate (λ) and the service rate (ν) as:

$$\begin{aligned}\nu(t) &= \nu(0) + \alpha \sin(2\pi\omega_1 t)\\ \lambda(t) &= \lambda(0) + \alpha \sin(2\pi\omega_2 t)\end{aligned} \quad (1)$$

where $\nu(0)$, $\lambda(0)$ represent the nominal service and arrival rates respectively, α determines the amplitude of oscillation, ω_1 and ω_2 represent the frequency of oscillation of the service and arrival rates respectively and t is the global simulation clock time. The time spent by each customer in the system was selected as the response parameter of interest and the response power spectrum was calculated using the Blackman-Tukey technique for power spectral density estimation[Hardin, 1986]. Jacobson et. al. observed that the response power spectrum did not necessarily exhibit a distinct peak at the frequency of oscillation, even though the response is known to be sensitive to the arrival and service rates.

In [Jacobson *et al.*, 1988] Jacobson et. al. also discusses a FDE performed on a simple manufacturing assembly station using the global simulation clock time as the oscillation index. The response was a record of the difference between each completion time of the assemble operation and the corresponding arrival time of the latest arriving component part. The response power spectrum was estimated using once again the Blackman-Tukey technique. Results of this experiment also demonstrated that the use of the global simulation clock as the oscillation index did not yield well-defined peaks in the power spectrum. From these two experiments Jacobson et. al. concluded that the global simulation clock is not a suitable index for FDEs and that, instead, a discrete oscillation index should be used whenever possible.

We can explain the reason for the unsatisfactory results in [Jacobson *et al.*, 1988]. First observe that in order to perform a spectral analysis, samples should be taken at *equally spaced* time intervals. This can be demonstrated with three simple experiments based upon sampling the deterministic function $x(t) = \sin(2\pi\omega t)$ at times defined by $t_i = t_{i-1}+\epsilon_i$ for all i (with $t_0 = 0$).

In experiment 1, $x(t)$ is sampled at *equal* time increments, i.e. $\epsilon_i = 1$ for all i. In experiment 2, $x(t)$ is sampled at *slightly random* time increments i.e. ϵ_i is an iid sequence of random variables uniformly distributed between 0.0 and 2.0 for all i. In experiment 3, $x(t)$ is sampled at *random* time increments i.e ϵ_i is an iid sequence of random variables exponentially distributed with mean 1 for all i.

For each of these three experiments the average intersample time is 1. In each case the discrete Fourier transform (DFT) of the sampled data was calculated and the amplitude of the (complex) result was plotted against the corresponding frequency.

Figures 2a, 3a and 4a represent $x(t_i)$ versus the sample index i for experiments 1, 2 and 3 respectively; figures 2b, 3b and 4b represent the corresponding DFT energy spectrum. Figure 5 represents $x(t_i)$ versus t_i for the three experiments. From these figures it is observed that even when the sampled function is a deterministic function of time, spectral estimates deteriorate (the energy peaks become more smeared) as the randomness of the inter-sampling times increases. This illustrates why equally spaced samples

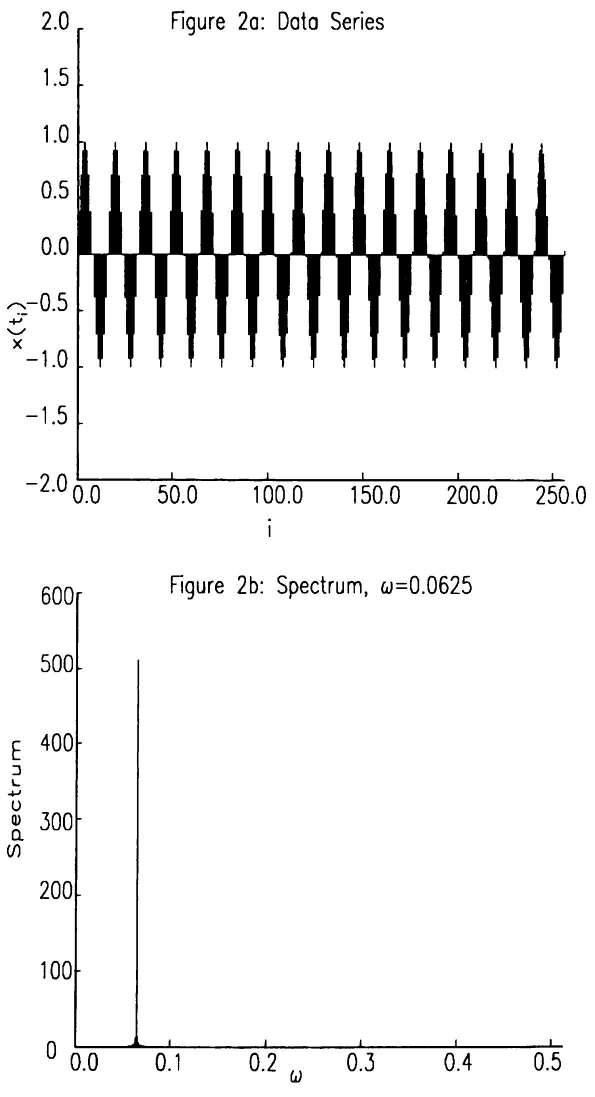

Figure 2: Data Series and corresponding spectrum obtained by sampling a sine wave at equal intervals; $\omega = 0.0625$, 1024 samples

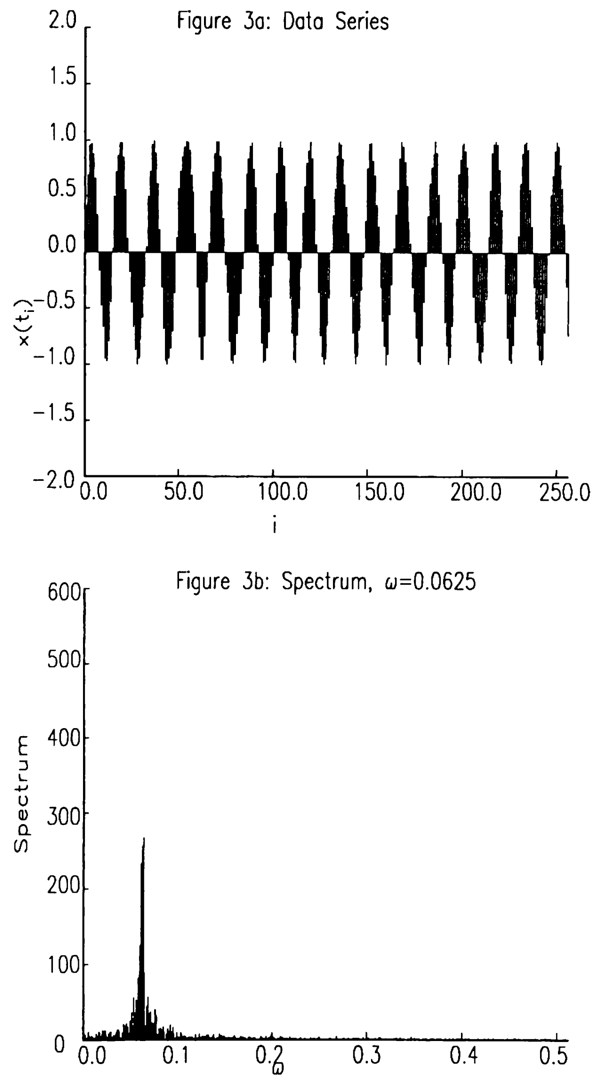

Figure 3: Data Series and corresponding spectrum obtained by sampling a sine wave at "slightly random" intervals; $\omega = 0.0625$, 1024 samples

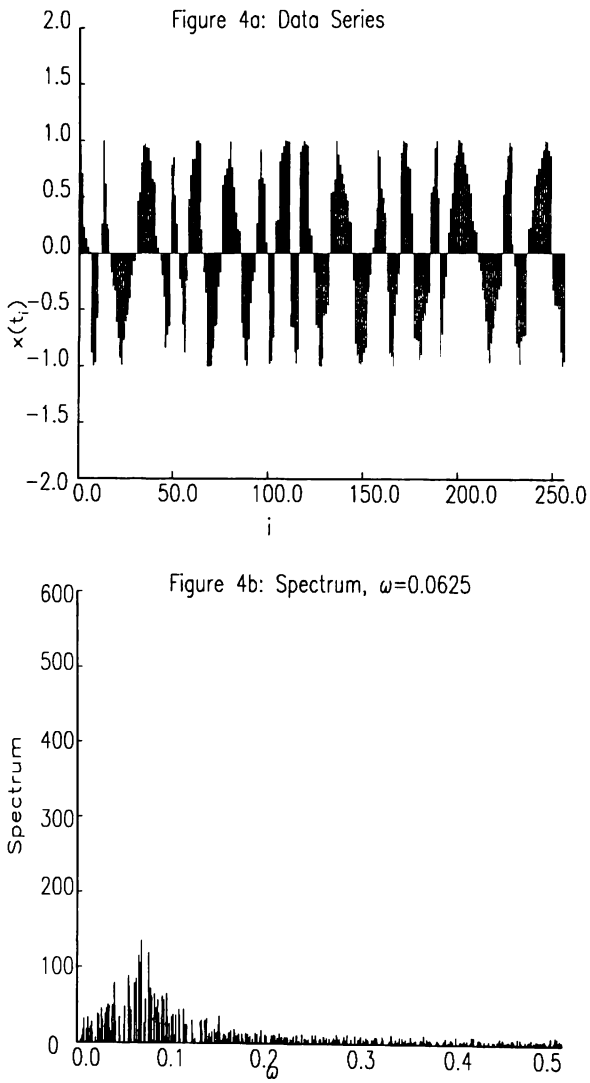

Figure 4: Data Series and corresponding spectrum obtained by sampling a sine wave at random intervals; $\omega = 0.0625$, 1024 samples

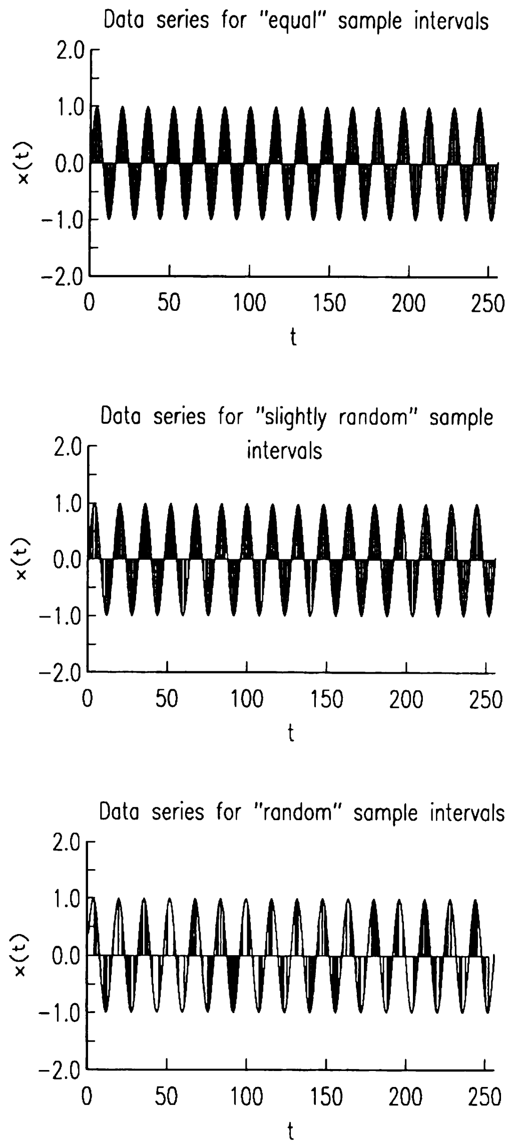

Figure 5: Data Series vs time for the 3 experiments

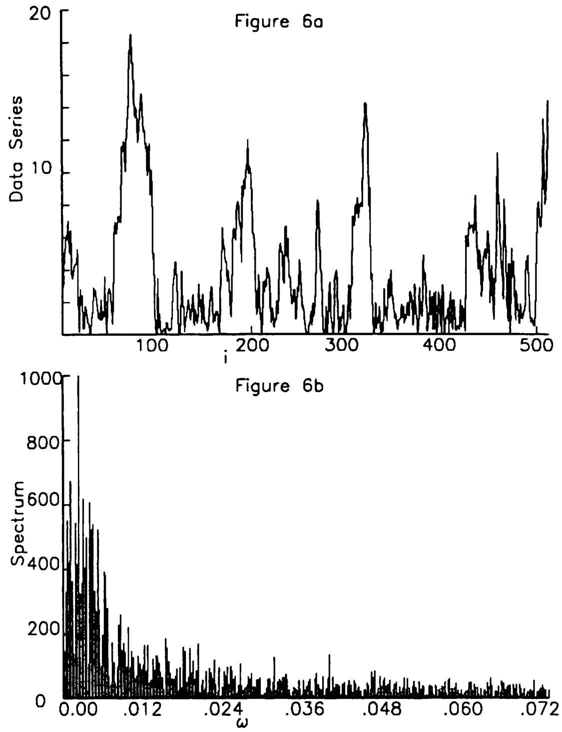

Figure 6: Data Series and corresponding spectrum obtained by naively using the global-simulation clock as the time index, as given in section 2. Input $\omega = 0.007812$

are needed when the sampled function is *deterministic*. If the sampled function is *stochastic* the added randomness inherent in the samples serves to further smear the spectral energy peaks. Therefore, in order to perform spectral analysis of samples from a time-varying stochastic process, equally spaced samples are necessary.

We can further demonstrate the need for equally spaced samples by performing a FDE of a FIFO $M/M/1$ queue. Customers arrive according to a non-stationary Poisson process with the arrival rate given by

$$\lambda(t) = \lambda(0) + \alpha \sin(2\pi\omega t) \tag{2}$$

where $\lambda(0)$ represents the nominal arrival rate, α is the amplitude of oscillation, and ω is the frequency of oscillation. The service rate is fixed at $\nu = 1$. The wait experienced by each customer is selected as the system response of interest.

The wait times for 4096 customers were recorded, with $\lambda(0) = 0.55$, $\alpha = 0.45$ and $\omega = 0.007812$ and the corresponding DFT computed. Figure 6 represents the results of this experiment. Consistent with Jacobson's results, the response spectrum (Figure 6b) does not have a distinct spike at the frequency of oscillation. This unsatisfactory result is caused by sampling non-uniformity. The samples are observed at arrivals (or departures); these times are random and clearly not uniformly spaced in time.

The conclusion here is that if the global simulation clock is the oscillation index then it is not sufficient to naively sample at the processing time of each customer. As Jacobson et. al. used this sampling scheme they failed to obtain satisfactory results in using the global simulation clock. In the next section of this paper we demonstrate that the global simulation clock can be used as the oscillation index—provided the response data is properly sampled and Fourier analyzed.

3 A NEW METHODOLOGY

As discussed in section 2, a major issue that has yet to be resolved for performing frequency domain analysis is the selection of a suitable oscillation index. In this section we present a solution to this index selection problem.

As observed in the previous section, sampling the system response at random event times results in non-uniformly spaced samples. Instead, if we divide the (simulation) time interval of interest into equally spaced subintervals and sample by counting the number of system response events in each subinterval, we obtain uniformly spaced samples. Spectral (Fourier) analysis can then be performed. This methodology uses the global simulation clock as the oscillation index.

Let $\{N(t),\ t \geq 0\}$ be a non-homogeneous, periodic counting process, whose rate function $\lambda(t) \geq 0$ is varied sinusoidally with time. That is, $N(t)$ represents the number of events that occur in $(0, t]$. Let $m(t)$ be the expectation function for the counting process; i.e. $m(t) = E[N(t)]$. If $m(t)$ is differentiable for all t then

$$m(t+\delta) - m(t) = \int_t^{t+\delta} \lambda(y)dy \tag{3}$$

provided $dm(t)/dt$ is bounded on $[t, t+\delta]$ and is continuous for all but finitely many points in $[t, t+\delta]$ [Law and Kelton, 1982]. Hence we have

$$\lambda(t) \approx \frac{m(t+\delta) - m(t)}{\delta} = \frac{\beta(t,\delta)}{\delta} \tag{4}$$

where $\beta(t,\delta) = m(t+\delta) - m(t)$ is the expected number of events in the interval $(t, t+\delta]$.

From Equation (4) it follows that we can obtain an estimate of $\lambda(\mathrm{t})$ by dividing the time line $t > 0$ into subintervals of small length δ and accumulating the number of events that occur in each subinterval, averaged over several replications. In this way, in a simulation study, if the rate function of a system parameter is oscillated at an assigned frequency then the rate function of a response parameter of interest can be estimated. At least two methods can be used for accumulating the number of events in each subinterval.

3.1 Histogram Method

This method attempts to approximate Equation (4) by building a histogram for the event counts. A histogram of the event counts is created for each replication. At the end of the simulation, a DFT of the histogram is computed to produce the required spectral estimate. Several replications are performed and to reduce noise the ensemble average of the histogram counts is used as the basis for the Fourier analysis. The choice of the number of bins is important. With large bin sizes the events will not be sufficiently sampled and the corresponding spectral estimate will be corrupted by aliasing. Very small bins on the other hand may result in empty bins. For the purpose of the simulations presented later, the time axis was scaled so that a unit bin size could be used.

As a simple illustration of the histogram method, a non-stationary Poisson process was simulated using the thinning method [Law and Kelton, 1982]. The rate function was given by Equation (2) with $\lambda(0) = 0.55$ and $\alpha = 0.45$. The elapsed simulation time was 4096.

Figures 7a and 8a illustrate a portion of the resulting ensemble averaged histogram corresponding to two different values of ω. In each case, the histogram is a somewhat noisy estimate of $\lambda(t)$. The noise level could be reduced, if desired, by averaging additional replications. However, as Figures 7b and 8b illustrate, additional noise suppression is not necessary because—as desired—the corresponding spectral estimates exhibit a distinct spike at the oscillation frequency of the rate function (ω).

3.2 Differentiation Method

The histogram method just illustrated is, essentially, an integration (low-pass filtering) technique. An alternate approach is to observe $N(t)$ at uniformly spaced sampling times $t = \delta, 2\delta, 3\delta \cdots$ for some small increment δ. These samples can then be ensemble averaged over several replications with the result numerically differenced to yield an estimate of $\lambda(t)$ at the sampling times. These estimates can then be Fourier analyzed. We refer to this method as the differentiation method. Our experience to date, however, has been that the differentiation method tends

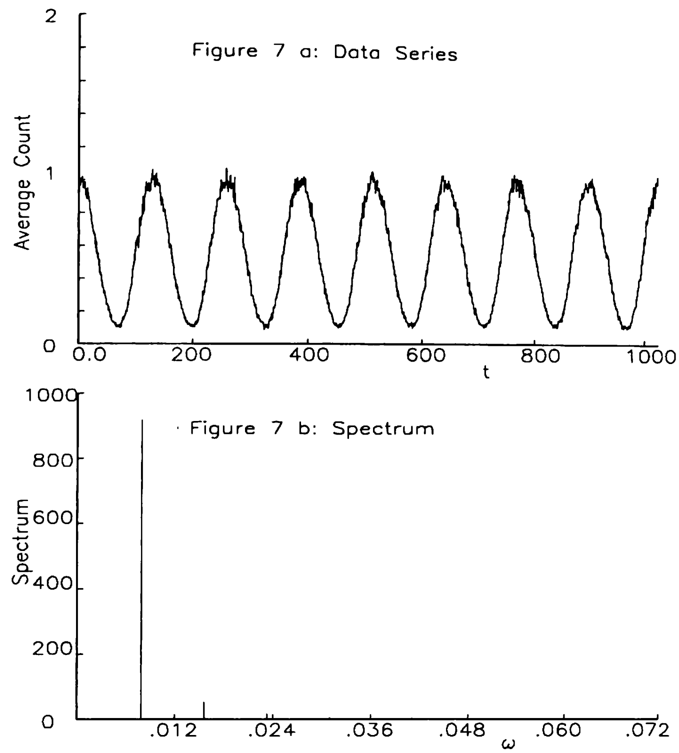

Figure 7: Data series and corresponding spectrum obtained by using the Histogram method as given in section 3.1. $\omega = 0.007812$; $\delta t = 1.0$. Simulation time 4096.0

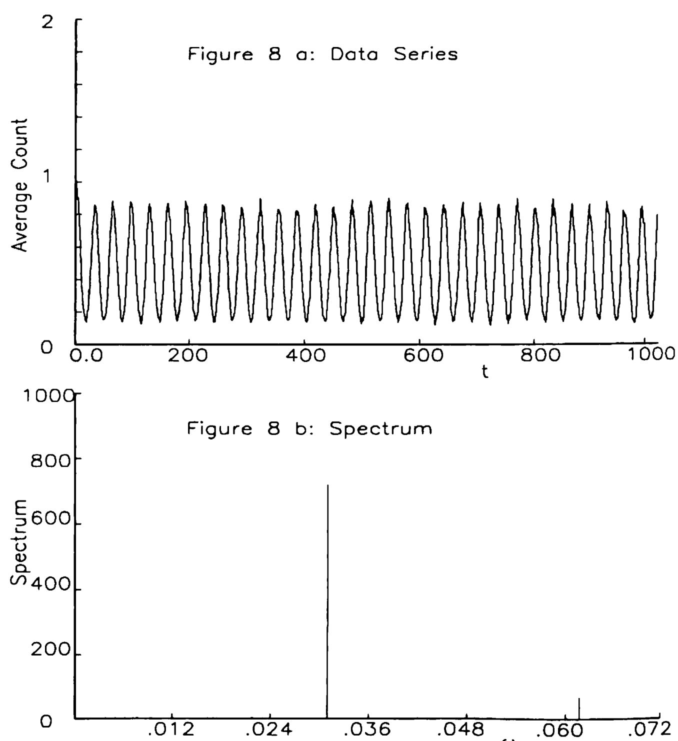

Figure 8: Data series and corresponding spectrum obtained by using the Histogram method as given in section 3.1. $\omega = 0.031250$; $\delta t = 1.0$. Simulation time 4096.0

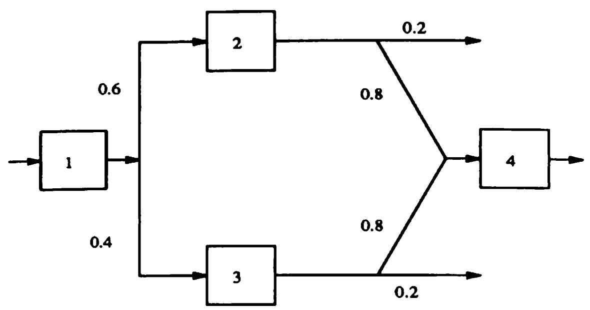

Figure 9: A Network of Single server FIFO Queues

to produce more noisy spectral estimates than the histogram method. For that reason, in the two example to follow the histogram method is used.

4 EXAMPLES

This section illustrates two applications of the histogram method to perform a FDE for a single-server queuing system (Figure 1) and a network of queues (Figure 9).

4.1 Single-Server FIFO Queue

In vivid contrast to the failed spectral analysis presented in Figure 6, Figures 10 and 11 illustrate the result of using the histogram method to analyze the departure rate from the single-server queue discussed in section 2. Two different oscillation frequencies $\omega = 0.007812$ and 0.031250 were simulated. In each case, the response spectra exhibits a distinct spike at the oscillation frequency, clearly indicating the effectiveness of uniform sampling implemented via the histogram method.

4.2 Network of FIFO queues

As a second example, consider the network of queues in Figure 9. Each node is a single-server $M/M/1$ FIFO queue with service rate $\nu = 2$. External arrivals only occur at node 1 according to a non-stationary Poisson process with rate given by Equation (2). As before $\lambda(0) = 0.55$ and $\alpha = 0.45$. The oscillation frequency is $\omega = 0.031250$. After exiting one node, a job selects its next destination (another node or outside world) consistent with the transition probability associated with each path. Once again the departure rate at each node was selected as the response parameter and the histogram method was used to obtain the required response spectrum. Figure 12 depicts the response spectra for each node. As in the

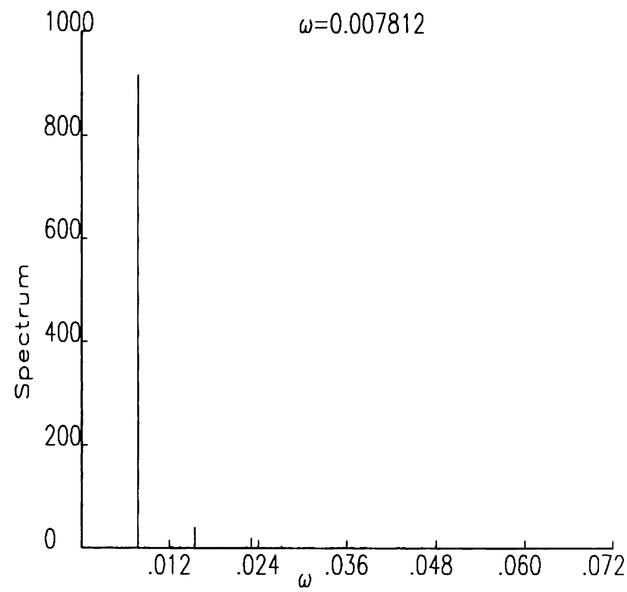

Figure 10: Spectrum of Departure Histogram; $\lambda(0) = 0.55$; $\alpha = 0.45$; $\omega = 0.007812$; Avg. Utilization=0.55

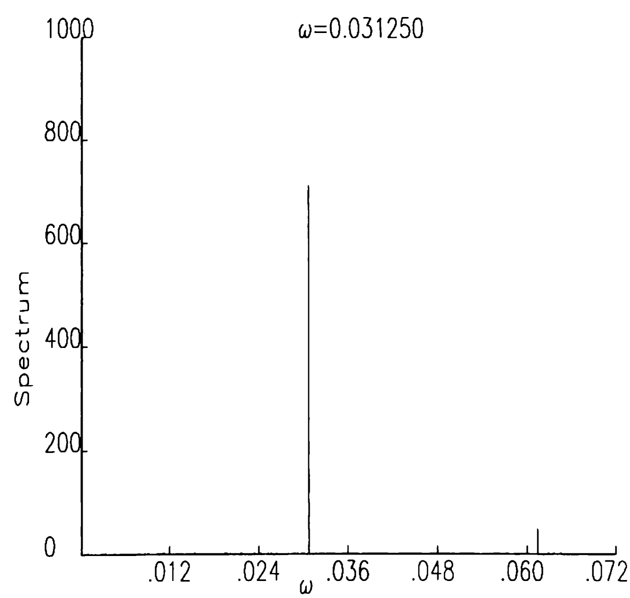

Figure 11: Spectrum of Departure Histogram; $\lambda(0) = 0.55$; $\alpha = 0.45$; $\omega = 0.031250$; Avg. Utilization=0.55

much simpler case of a single-server queue, the response spectra at each node exhibits a distinct spike at the oscillation frequency, clearly indicating the effectiveness of uniform sampling implemented via the histogram method.

5 CONCLUSIONS

In the paper we have discussed why the global simulation clock time is the most natural choice for the FDE oscillation index. The reasons behind the failure of past efforts to reference all the frequencies in a FDE to the global simulation clock are also presented. A new methodology for performing frequency domain analysis of discrete-event simulation experiment is proposed. The new methodology is based upon the Fourier (spectral) analysis of stochastic systems and uses the global simulation clock time as the oscillation index. The histogram method for implementing this methodology is discussed. Using this method it is possible to analyze stochastic systems subject to non-stationary inputs - a difficult analytical problem. The effectiveness of this method is demonstrated by applying it to a single-server FIFO queue and a network of single-server queues. As claimed, the method is efficiently implementable, resilient to noise and lends itself efficiently to any simulation model being analyzed using frequency domain methods.

ACKNOWLEDGEMENTS

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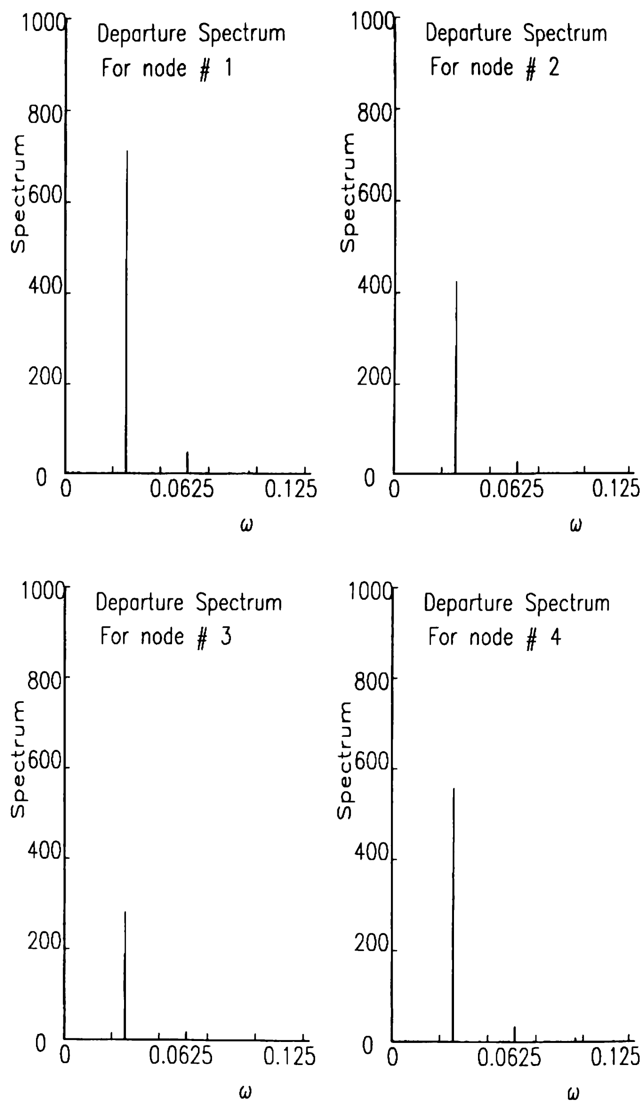

Figure 12: Spectrum of Departure Histograms for network in Figure 9; $\lambda(0) = 0.55$; $\alpha = 0.45$; $\omega = 0.031250$; Avg. service rate per node = 2.0; Avg. Utilization per node =0.275