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TRACK LOSS AND RECOVERY FOR INTERMITTENTLY EMITTING TARGETS

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ABSTRACT

There is far less literature on passive sensor systems for tracking intermittently emitting targets than for tracking continuously emitting ones. A methodology for evaluating these systems via simulation is proposed, and a prototype model, whose main purpose is to test hypotheses about the tracking system, is discussed.

1 INTRODUCTION

Passive sensor systems can be used in tracking moving objects that emit visible light or other wavelengths in the radio spectrum. Despite the potential uses in signals intelligence (SIGINT), there is less literature on intermittently emitting targets than on continuously emitting ones. Probabilistic models of motion, of emission repetition frequency, and of the frequency of "false alarms" have been used successfully in the design of such systems. Tracks are most likely to be maintained when the system employs an optimal gating or gate function. Such a function—that takes into account measurement error, the false target density, the covariance of the target's position and the probability of detection—might be periodically updated in order to respond to variations in the rate of change in state (position). A simulation of the target(s), the false emissions, and the RF environment can be employed to find the best algorithms for track maintenance and track recovery.

2 BACKGROUND

Tracking system designers would like a design that minimized the likelihood that tracks are lost, due to false updates or to poor representations of the target's (or multiple targets') kinematics. A system's simulation inputs should include the frequency of true and false updates, as well as the transition probabilities between the two states "track maintained" — "track lost". Any intercept whose state satisfies the gating criteria is considered for updating the track. As discussed in Blackman, an assignment algorithm may be used if multiple valid intercepts occur simultaneously.

In this paper we assume that the validation gate involves kinematic metrics only, i.e., that the signal of the object of interest is well understood and the signal metrics are correctly interpreted.

Since gates come in at least two different "shapes", a choice is available. For some computations, a rectangular gate may be simpler; for greater accuracy, the ellipsoidal gate is generally superior. The covariance matrix S of the state x(k|k) determines the optimal spatial validation gate volume. For ellipsoidal gates in N dimensions, Equation (1) maximizes the likelihood that more true intercepts than false intercepts occur within the gate:

$$G_0(k-1) = 2\ln\left[\frac{P_D}{(1-P_D)\beta\cdot(2\pi)^{M/2}\sqrt{|S|}}\right] \qquad (1)$$

where the parameters are as listed after the conclusion.

To maintain a high likelihood that an intermittently emitting target will remain in the gate, the gate volume will be allowed to continuously expand over time whenever the target is assumed to be continuously moving. The volume is therefore a function of time, initial gate size G_0, and the targets dynamics (x, v, a).

2.1 Losing the Track

As a result of false intercepts, the gate may become corrupted. Two of the main byproducts of gate corruption to be examined—and two good areas for research—are the following.

1. What is the probability that a target will remain in its track, despite gate corruption?

2. What is the intercept time for targets that have been determined to be outside the gate?

We can determine the probability of detection of a target inside a validation gate that has been biased through misassociation of false signal intercepts. We consider two cases. In the first case, the target did not move since its last intercept. In this case, the gate will be offset by a distance d due to a sequence of false associations before the true update. In the second case, the target moves to any location within a region determined by some maximum radial velocity. In this case, assume that the probability distribution of target position is uniform within the maximum feasible radius h. Then, the probability of the target's location falling within the gate is the fraction of the feasible target area which overlaps the validation gate.

2.2 Recovering the Track

This mathematical model can also be used to determine the probability that a target that has exited the tracking gate can be recovered. Track recovery can occur in one of two ways.

1. The target re-enters the appropriate validation gate through the combination of target movement and gate walking, given that the system is not aware that the target had been lost.

2. The system determines that the target is outside the gate and initiates a more general search for it.

The first situation can be modeled using the same equation used for calculating the probability of losing the target. Thus $P_G(r_0)$, the probability of the target being in the gate is given by Equation (2).

$$P_G(r_0) = \frac{\int_{-a}^{a} y_2(x)dx - \int_{-a}^{a} y_1(x)dx}{\pi \cdot h_2^{\,2}} \tag{2}$$

$$\int_{-a}^{a} y_2(x)dx = \left.\frac{h_2^{\,2} - h_1^{\,2} + d^2}{2d}x + \frac{x}{2}\sqrt{h_2^{\,2} - x^2} + \frac{h_2^{\,2}}{2}\sin^{-1}\frac{x}{h_2}\right|_{-a}^{a}$$

$$\int_{-a}^{a} y_1(x)dx = \left.\left(\frac{h_2^{\,2} - h_1^{\,2} + d^2}{2d} - r_0\right)x + \frac{x}{2}\sqrt{h_1^{\,2} - x^2} + \frac{h_1^{\,2}}{2}\sin^{-1}\frac{x}{h_1}\right|_{-a}^{a}$$

In Equation (2), a is the length of the semi-major axis of the overlap region, if any, h_2 is the maximum feasible radius at update k-1, h_1 is the radius of the biased validation gate, whose center is offset from the last update by a distance d.

The second situation will require the modeler to calculate the expected time (waiting time) before the more general search succeeds in recovering the target. To save space, this will not be discussed here, but see "Intercept Time, a Primer for EW Systems Designers", *EW Design Engineers' Handbook*, 1990.

2.3 Declaring Lost Target

A surveillance controller should conclude that the intermittently emitting target is no longer in track whenever the rate of associated intercepts i_{actual} is inconsistent with the expected rate $i_{expected}$ implied by prior estimates on P_D, emission rates and durations. Additional indications of a lost target may be afforded by a mismatch between a track's random walk and a prior motion model for the target.

3 SIMULATION ANALYSIS

A prototype model of the tracker was developed using the Foresight™ tool. The main purpose of this prototype is to generate statistical output, test hypotheses about the tracker and ultimately, to determine the credibility of this direction in modeling a complex, multivariate problem.

In five experiments, the model tested the effect of varying a single parameter related to the probability of target detection of the recovery system (Pd). At 50%, the system struggles to recover the target once it is recognized lost. In all of the runs, the target was eventually recovered. At Pd = 80%, the recovery system enabled the tracking system as a whole to track the target around two-thirds of the time. These results are shown in Table 1.

Five more experiments tested the sensitivity of the response variable to varying the spatial density of false targets, β_{SF}, or mbeta_F in the model. As shown in Table 2, the tracking performance falls off gradually with significant increases in the false target density. This is to be expected, since the probability that there are no false intercepts in time t is a negative exponential function, as shown in Equation (3).

$$P(0|t) = e^{-\beta_F V(t)} \tag{3}$$

Two notes about equation (3). First, β_F, rather than β_{SF}, appears because some of the variables are suppressed in the Foresight model. A false intercept will occur as a result of a detection of a false source. This brings in PdF, the probability of detection of false sources. One also has to consider r_F, e_F as independent variables in a detailed calculation of β_F. In any event, the value of β_F is directly proportional to β_{SF}. Second, V(t) is a function of the spatio-temporal volume $V_S(t)$, the mean composite receiver revisit rate and dwell lengths in the SOI band, r_R and e_R. The volume V(t) is proportional to all three of these variables.

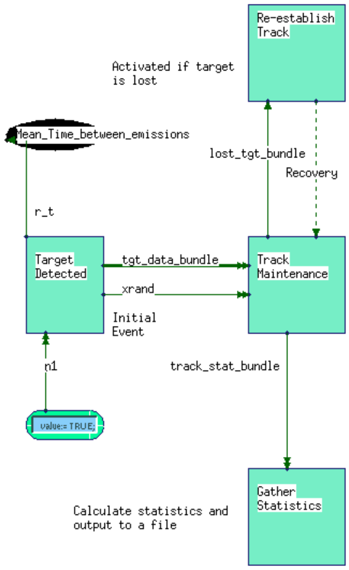

Figure 1: Top Level of the Simulation Model

Table 1: Time Tracked as a Function of Effectiveness of Recovery System

Pd	% Time Tracked
0.5	54
0.6	57
0.7	61
0.8	65
0.9	68

Table 2: Time Tracked as a Function of False Target Density

mbeta_F	% Time Tracked
0.64	65
0.74	62
0.94	55
1.34	52
2.14	49
3.74	41

One might say that Equation (3) and Table 2 describe a model in which a graph of the response variable, percentage of time tracked, is fairly flat with respect to changes in input variables r_R, e_R ,$V_S(t)$, β_{SF}, PdF, r_F, and e_F. That is a true statement about the model; however, in real-world scenarios, there may be so much variation in receiver and false source parameters that there will be some dramatic differences in the response variable.

4 CONCLUSION

These experiments begin to show the interplay between the recovery subsystem and the tracking system itself. The set-up for these experiments determined the relative inefficiency of the simulated system for tracking the target. In other words, the tracking system has severe imperfections, or in a different interpretation, there is an extreme amount of clutter, that causes the target to be lost every 40 or 50 minutes.

The fact that in all of these experiments, the target is always recovered in time to restore the track is an artifact of the settings for the 15 or so other parameters. The most developed part of the model so far is the track maintenance module, shown in Figure 2 below. Further work on the recovery module may yet yield a model that need no longer be called a “prototype”.

APPENDIX A: PARAMETER LIST

P_D = Probability of Detection
r_T, e_T = Mean Emission Rate and Duration (True Int.)
T_3 = Mean Period of Receiver Sweep (Wide Area Search)
t_3 = Mean Duration of Receiver Sweep (Wide Area Search)
$T_0 = (r_T)^{-1}$ = Mean Interval between True Int.
r_R, e_R = Mean composite receiver revist rate and dwell length in signal-of-interest band (Local Area Search)
$V_S(t)$ = Spatio-temporal Gate Volume
β_{SF} = Spatial Density of False Sources in the SOI band PdF = Mean Prob. of Detection of False Sources
r_F, e_F = Mean Emission Rate and Duration (False Int.)
G_0 = Optimal Spatial Validation Gate Volume
$|S|$ = Determinant of the Covariance Matrix

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AUTHOR BIOGRAPHY

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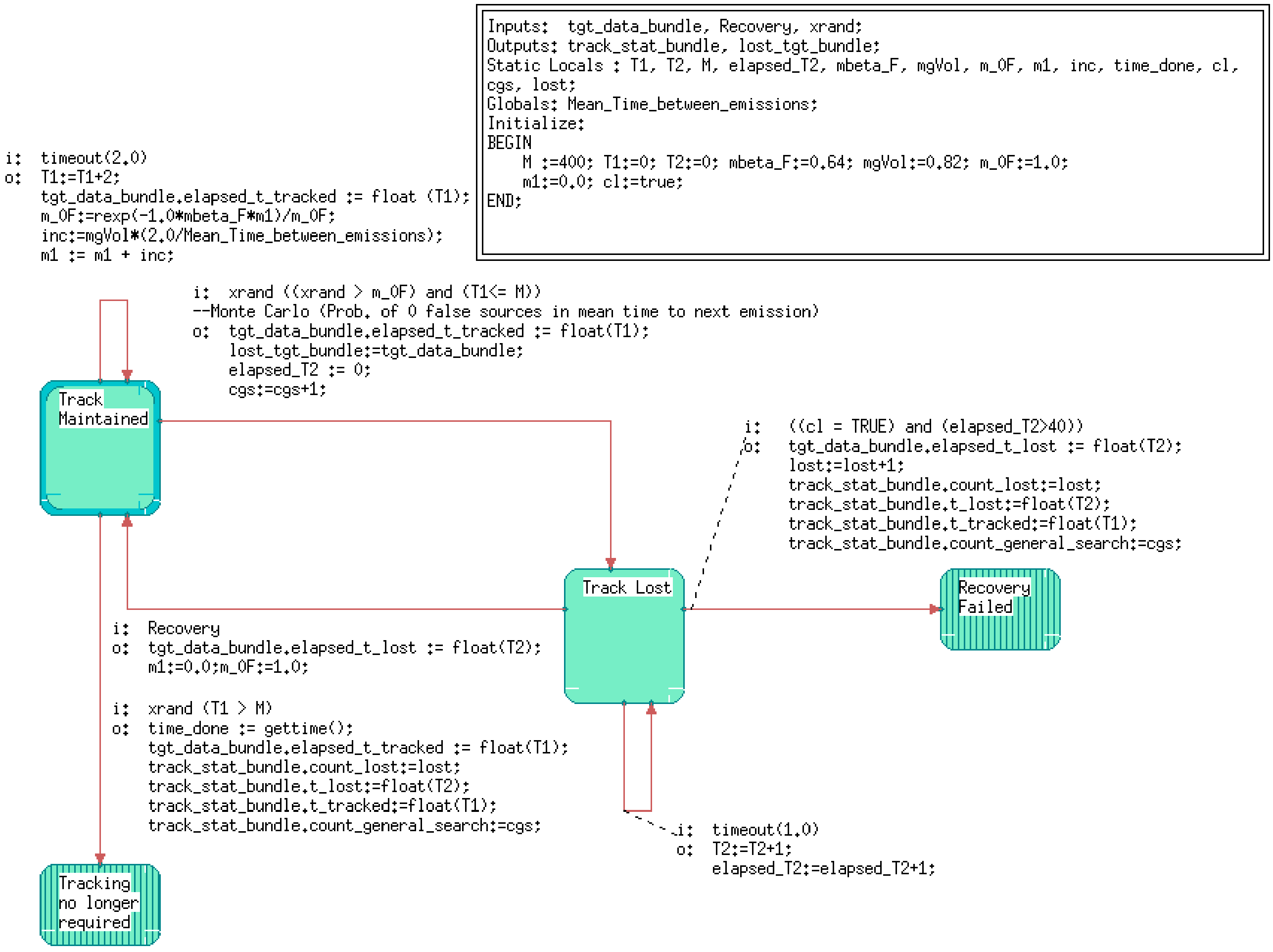

Figure 2: Track Maintenance Module