Proceedings of the 1998 Winter Simulation Conference
D.J. Medeiros, E.F. Watson, J.S. Carson and M.S. Manivannan, eds.

SIMULATION OPTIMIZATION OF AIR TRAFFIC DELAY COST

Nathan L. Kleinman

Options and Choices, Inc. (OCI)
Cheyenne, Wyoming 82009, U.S.A.

Stacy D. Hill
Victor A. Ilenda

The Johns Hopkins University
Applied Physics Laboratory
Laurel, Maryland 20723, U.S.A.

ABSTRACT

The cost of delay is a serious and increasing problem in the airline industry. Air travel is increasing, and already domestic airports incur thousands of hours of delay daily, costing the industry $2 billion a year. One strategy for reducing total delay costs is to hold planes for a short time at the gate in order to reduce costly airborne congestion. In a network of airports involving thousands of flights, it is difficult to determine the amount to hold each flight at the gate. This paper discusses how the optimization procedure simultaneous perturbation stochastic approximation (SPSA) can be used to process delay cost measurements from air traffic simulation packages and produce an optimal gate holding strategy. As a test case, the SIMMOD air traffic simulation package was used to model a simple four-airport network.

1 INTRODUCTION

Air traffic delay is a rapidly increasing problem in the United States. The National Transportation Research Board reported that in 1990, over 20,000 hours of delay were incurred at each of 21 airports nationwide (Peterson et al. 1995). Airlines report that delay costs their industry $2 billion annually (roughly the same amount as the industries total losses in 1991), and the Federal Aviation Administration (FAA) expects the demand for air travel to increase 25% by the year 2000 (Vranas et al. 1994). Congestion is increased further by airlines' desire to use "hub" airport systems. Here, airlines schedule large numbers of flights from outlying airports so that they arrive at the hub airport at approximately the same time. Passengers are then exchanged, and a new group of flights leaves the hub airport, again at roughly the same time. As an example, Odoni (1987) cites Atlanta, a major hub for Delta Airlines, where at least six times a day "banks" of 100 flights arrive and depart within approximately one-hour periods.

Clearly the need exists for reducing air traffic delay. Current options include constructing new airports or runways, encouraging or constraining airlines to spread out arrivals and departures more evenly at congested airports in order to reduce "peak" period congestion, and using larger aircraft in order to transport more passengers per flight. But these methods are either very expensive or unlikely to be implemented soon (Vranas et al. 1994).

Vranas et al. (1994) state that "ground holding policies" offer a more promising way to reduce delay costs. Air traffic delay can be divided into three categories: holding at the gate, delay while taxiing, and airborne delay. Ground holding policies attempt to assign a small amount of delay to each flight prior to leaving the gate in order to reduce network-wide congestion and, particularly, to reduce the much more costly airborne delay

Determining the amount of gate delay to assign to each flight on a particular day for an entire network of airports is a nonlinear optimization problem of high dimension. Software packages exist which simulate (some with a high degree of detail) many aspects of flights and airport operations in a network of airports. But these packages do not optimize; they only take a ground holding policy from the user and output the associated delay cost. To set up and solve a detailed stochastic programming problem that models a network of airports and finds an optimal ground holding policy would be intractable. Vranas et al. (1994) describe an integer programming model for the multi-airport ground-holding problem. However, an integer programming approach would not model the activity of the air network in as much detail as would simulation packages. For instance, their model does not account for taxiway or airspace congestion. Furthermore, it may be necessary to optimize objective functions that are highly nonlinear and for which the form is unknown and only noisy measurements are available. In 1993 researchers at The MITRE Corporation (Helme et al. 1993) stated that their current methods for determining ground-holding policies were optimal if the following restrictive (and unrealistic) assumptions were made:

aircraft travel times are deterministic, airway capacities are infinite or nonrestrictive, and an aircraft's flight plan includes only one destination. They found no optimization method at that time which did not invoke at least two of these assumptions. Helme (1992) proposed a multi-commodity minimum cost flow method which allowed somewhat random capacities and negated the second and third assumptions. But, this method used deterministic take-off times, a linear objective function, and discrete delay times (problem size increases dramatically with finer discretizations). Hence, an optimization method is needed which incorporates the high level of modeled detail and stochastic nature of air traffic simulation packages.

Simultaneous perturbation stochastic approximation (SPSA) is an iterative technique for finding local optima of nonlinear objective functions for many types of systems (Spall 1992). (See Fu and Hill (1997) for an application of the algorithm to optimize queueing systems.)

SPSA is a Kiefer-Wolfowitz stochastic approximation algorithm (similar to finite difference-based stochastic approximation), which only requires measurements (possibly noisy) of an objective function to form gradient estimates and converge to a local optimum. SPSA differs from finite differences stochastic approximation (FDSA) in that SPSA only requires two objective function evaluations per gradient estimate, while FDSA requires 2*p* evaluations, where *p* is the number of system parameters being estimated. This gives SPSA a significant advantage in problems of high dimension, especially when evaluating the objective function is expensive or time-intensive. Further improvements in SPSA can be achieved by using the method of "common random numbers" to reduce the variance of the gradient estimation error (Kleinman et al. 1996).

SPSA appears to be an ideal tool for solving the ground-holding problem. This paper discusses how SPSA can be used in conjunction with an air traffic simulation package to improve ground-holding policies in a network of airports. Because SPSA only requires objective function measurements, a detailed simulation package can be used to estimate the delay cost associated with a particular ground-holding policy. Each iteration SPSA will generate a new ground-holding policy, based on previous delay cost measurements, until a desired level of improvement is reached.

The purpose of this paper is to introduce and describe an SPSA-based method for solving air traffic-related optimization problems. In Section 2 we outline the form and characteristics of the SPSA algorithm and explain how SPSA can be combined with air traffic simulation output to form an iterative optimization method. In Section 3 we describe the air traffic network simulation package SIMMOD and explain why this particular simulation tool is a good candidate for supplying simulated delay cost measurement data.

Section 4 describes a 168-dimensional test case for this method which involves a simple hypothesized network consisting of four airports. We start with a flight schedule and an initial ground-holding policy which assigns no gate-delay to every flight, and then we use the SPSA method to find a ground holding policy which improves the total delay cost in the network. This test illustrates how planners can use this method to improve existing ground-holding policies. Note that although these results are for a fictitious network of airports, the method is general and can be applied in an identical manner to a simulation model of any network of real airports.

2 SIMULTANEOUS PERTURBATION STOCHASTIC APPROXIMATION

Let $\theta \in R^p$ be a vector whose components represent system parameters we wish to control (each flight's ground-holding delays, for example). Suppose that $L(\theta)$ is the objective function we wish to optimize. The goal is to find a zero of the gradient of this objective function. That is, we want a θ such that

$$g(\theta) \equiv \frac{\partial L}{\partial \theta} = 0$$

The SPSA algorithm attempts to find a local minimizer θ^* by starting at a fixed $\hat{\theta}_0$ and iterating according to the following scheme:

$$\hat{\theta}_{k+1} = \hat{\theta}_k - a_k \hat{g}_k\left(\hat{\theta}_k\right) \tag{1}$$

Here $\{a_k\}$ is a *gain* sequence of positive scalars satisfying the conditions in Spall (1992) (in particular, $a_k \to 0$ and $\sum_1^\infty a_k = \infty$), and $\hat{g}_k$ is an estimate of the gradient g whose *l*-th component is defined as

$$\hat{g}_{kl} = \frac{y_k^+ - y_k^-}{2c_k \Delta_{kl}} \tag{2}$$

Here, $y_k^\pm$ represents a (perhaps noisy) measurement of $L\left(\hat{\theta}_k \pm c_k \Delta_k\right)$. The sequence $\{c_k\}$ is a sequence of positive scalars such that $c_k \to 0$, and Δ_k is a vector of *p* mutually independent random variables, the *perturbations*, satisfying certain conditions. For example, the components of Δ_k could be independent Bernoulli distributed random variables, whose outcomes ± 1 are equally likely. (Sadegh and Spall (1996) discuss the optimal choice of the perturbations.)

Observe that the numerator in Equation (2) is the same for each component of the gradient estimate. Thus, only *two* measurements of the objective function $L(\theta)$ are required to obtain the SPSA gradient estimate at each iteration. To illustrate, the cost delay values $y_k^{\pm}$ are the total delay cost values obtained by performing two simulations of the air traffic network using $\hat{\theta}_k \pm c_k \Delta_k$, where $\hat{\theta}_k$ is the current estimate of the best ground-holding policy θ^*. The (two) cost delay values would then be used in Equation (2) to obtain an estimate of the gradient. A new estimate $\hat{\theta}_{k+1}$ of the best ground-holding policy is then obtained from Equation (1). The process is repeated until a desired level of improvement in delay cost is reached.

Kleinman et al. (1996) introduced a version of SPSA which employs common random numbers (CRN). The method is implemented by using the same random number seeds to drive the simulations which yield $y_k^{\pm}$ each iteration. This can reduce the variance of the gradient estimate in Equation (2) and thus reduce the variance of the SPSA error estimates.

The SPSA algorithm is very general and can be applied in many different kinds of objective functions. The flexibility of the algorithm stems from the fact that only (noisy) objective function are required.

3 SIMMOD

The objective function measurements required by the SPSA algorithm can come from a real system or from a computer simulation which models a real system, depending on the purpose of the optimization. Although SPSA can be used for real-time control, our purpose here, in particular with the example in Section 4, is to outline a method for making decisions on policies minutes, hours, or days prior to the time of implementation. Thus, detailed simulations are appropriate means for obtaining objective function measurements.

As mentioned in the introduction, a number of simulation packages exist which model activity in a network of airports. For our tests we chose the simulation package SIMMOD, developed for the FAA. SIMMOD is widely used to model detailed aspects of inter- and intra-airport activity. SIMMOD models and tracks individual aircraft of different types as they move in a network of airports from gate to taxiway to runway to airspace and ultimately to the final destination gate.

In the air, speed control, vectoring, and several different holding procedures are automatically implemented, as needed, to maintain proper separation distances between different types of aircraft. SIMMOD also takes into account route and sector capacity constraints as well as wind speeds. It also models detailed aspects of arrival and departure procedures, including ground separation constraints, runway selection, missed approaches, and takeoff and landing runway distances and times.

Before running simulations, the user inputs information about the airspace, airfields, and simulation events. The user is allowed to set up gates, taxipaths, runways, and airspace routes as he wishes.

4 NUMERICAL EXAMPLE OF THE SPSA OPTIMIZATION METHOD

The purpose of performing the numerical example described in this section is to illustrate an application of the SPSA optimization method. As shown above, this method is very flexible. It can accommodate many different objective functions and varying degrees of simulation complexity, depending on the needs and resources of the user. Hence, the numerical results obtained for the following example problem serve as qualitative evidence of the usefulness and applicability of the SPSA optimization method.

4.1 Problem Setup

In this problem we use SPSA to find a ground-holding policy that significantly reduces the total delay cost for aircraft movement within a network of airports. The network in our example is comprised of four simplified airports labeled 1, 2, 3, 4. Air routes exist between all airport pairs, except the pair (2,4). Each airport consists of one runway, one gate, and taxipaths leading from the gate to each end of the runway. The structure of the airspace and airfields and the location of the nodes and links were input to SIMMOD by means of a graphical input program within SIMMOD. Node characteristics, such as air and taxiway holding strategies, capacity, and altitude; link characteristics, such as capacity, passing restrictions, and average speed for different aircraft types; and other information, such as takeoff, landing, and in-flight separation distances, were input to SIMMOD as well.

Next, a flight schedule was created. The schedule consisted of 168 flights, with 30 to 54 departing from each of the four airports over a three hour period. On average, 7 flights were scheduled to begin loading at each of the runways on each of the half hours. (Starting seven flights at the same time at the same airport creates a congestion “peak” and results in a less than optimal taxi-way delay.) Over the three-hour period the runways averaged 14 departures per hour, roughly the same rate as in the sample project provided with the SIMMOD software (a simulation of the San Diego airport). In addition, during the third hour, arrivals of earlier flights competed with the later departures for use of the same runways. The last flights finished landing and unloading approximately seven simulated hours after the earliest flights began.

4.2 Objective Function and Parameter Definition

During each simulation SIMMOD keeps track of the amount of time aircraft are required to hold (due to congestion ahead), both on the taxiways and in the air. This information is given as output so that an objective function value can be calculated. Specifically, we wish to minimize the total cost of delay in the network of airports described above, given the flight schedule and an initial ground-holding policy for the 168 flights. Geisinger (1989) of the FAA reported that for the average flight, taxiway delay cost 2.38 times more than air traffic controller-induced gate-holding delay, and airborne delay cost 3.86 times more than gate-holding delay per hour. (According to Geisenger (1989), the 1986 hourly values for gate-holding and airborne delays were $591 and $2,283, respectively.) Let θ_i be the number of minutes of air traffic controller-induced gate hold for flight i and $\theta = (\theta_1, \ldots, \theta_p) \in R^p$. Our total delay cost objective function is

$$L(\theta) = m_g(\theta) + 2.38 m_t(\theta) + 3.86 m_a(\theta) \qquad (3)$$

where $m_g(\theta)$, $m_t(\theta)$, and $m_a(\theta)$ are, respectively, the total number of minutes of gate, taxiway, and airborne delay throughout the 168 flights. Note that at each iteration $m_g(\theta) = \theta_1 + \cdots + \theta_{168}$. Also, $m_g(\theta)$ is a controlled quantity, whereas the other two terms in Equation (3) are obtained as output from the simulations.

4.3 RESULTS

In the simulation, normal air link capacities were used (approximately as restrictive as the aircraft separation constraints which required at least five miles between two airplanes on an air link). We started with a ground-holding policy which assigned no ground-holding to each flight, that is, $\theta_i \equiv 0$. Then, using the process described in Section 2, the SPSA optimization algorithm found a ground-holding policy which improved the total delay cost in the system.. Twenty runs, with 30 iterations in each run, were performed. To reduce estimation error, we employed the common random numbers version of SPSA (Kleinman et al. 1996).

The average initial objective function value $L(\theta)$ was 8796 and the average final value was 7618. (The average initial air delay per flight was 11.313 minutes, and the average final air delay was 9.568 minutes.) Table 1 summarizes some results at intermediate iterations. The values in the table are the averages of the cost delay values $y_k^{\pm}$ (over the 20 simulations) at the iterations specified in column 1.

Table 1: Average Delay Cost Function Values

Iteration	Parameter	
	y^+	y^-
5	8700.5	8657.0
10	8553.6	8622.1
15	8193.2	8328.4
20	8204.1	8230.5
25	8110.4	7886.0
30	7830.3	7909.0

For each run, the gain sequences $\{a_k\}$ and $\{c_k\}$ in Equations (1) and (2) were defined as follows: $a_k = a \cdot (k + B)^{-\alpha}$ and $c_k = c \cdot k^{-\gamma}$, where a = 0.00001, B = 10.0, $\alpha =$ 0.602, c = 0.05, and $\gamma = .0.101$. (See Spall (1992) and Kleinman et al. (1996) for discussions of the gain sequences).

All simulations were run on Pentium Pro 200 Mhz computer in a Windows 95 operating environment. Each 30-iteration run took (approximately) 3 hours and 15 minutes to run.

5 CONCLUSIONS

This paper outlines the stochastic optimization algorithm SPSA and describes its usefulness as a tool for solving air traffic optimization problems. Since the SPSA algorithm requires only measurements of the objective function to be optimized, detailed air traffic simulation software can be used to accurately model an air traffic network and provide system performance measurements. The combination of a powerful optimization technique and detailed simulation models provides planners with a tool that is potentially much better than current optimization techniques.

To illustrate this method, this paper describes how the SPSA optimization method can be used in conjunction with the SIMMOD air traffic simulation software package to find ground-holding policies which yield improved network-wide delay costs. The results of a test case involving a network of four simple airports illustrates the application of the approach.

One direction for future study would be to use this optimization method to find optimal ground-holding policies for actual flights in a simulated network of real airports. Another direction of study would be to consider optimizing other objective functions such as minimizing fuel consumption or to control parameters other than ground-holding times.

ACKNOWLEDGEMENTS

This work was partially supported by U.S. Navy Contract N00039-95-C-0002 and the JHU/APL IR\&D Program.

REFERENCES

Fu, M. C. and S. D. Hill. 1997. Optimization of discrete event systems via simultaneous perturbation stochastic approximation. *Transactions of the Institute of Industrial Engineers, Special Issue: Computer Simulation in Industrial Engineering Research and Practice* 29:233-243.

Geisinger, K. 1989. Airline delay: 1976-1986. Technical Report FAA-APO-88-13, Federal Aviation Administration, Washington, D.C.

Helme., M. P. 1992. Reducing air traffic delay in a space-time network. *IEEE International Conference on Systems, Man, and Cybernetics* 1:236—242.

Helme, M. P., K. Lindsay, S. V. Massimini, and G. Booth. 1993. Optimization of traffic flow to minimize delay in the national airspace system. Technical Report MP93W14, The MITRE Corporation.

Kleinman, N. L., J. C. Spall, and D. Q. Naiman. 1996. Simulation-based optimization with stochastic approximation using common random numbers. *Management Science*. Submitted.

Odoni, A. R. 1987. The flow management problem in air traffic control. In *Flow Control of Congested Networks*, ed. A. R. Odoni, L. Bianco, and G. Szego, 269-288. Berlin: Springer-Verlag.

Peterson, M. D., D. J. Bertsimas, and A. R. Odoni. 1995. Models and algorithms for transient queueing congestion at airports. *Management Science* 41:1279-1295.

Sadegh, P. and J. C. Spall. 1996. Optimal random perturbations for stochastic approximation using a simultaneous perturbation gradient approximation. *IEEE Transactions on Automatic Control*. Submitted.

Spall, J. C. 1992. Multivariate stochastic approximation using a simultaneous perturbation gradient approximation. *IEEE Transactions on Automatic Control* 37:332-341.

Spall, J. C. and J. A. Cristion. 1994. Nonlinear adaptive control using neural networks: Estimation with a smoothed simultaneous perturbation gradient approximation. *Statistica Sinica* 4:1-27.

Vranas, P. B. M., D. Bertsimas, and A. R. Odoni. 1994. Dynamic ground-holding policies for a network of airports. *Transportation Science* 28:275-291.

AUTHOR BIOGRAPHIES

NATHAN L. KLEINMAN completed his Ph.D. in mathematical sciences at The Johns Hopkins University in 1996. Subsequently, he taught mathematics at Brigham Young University, and in 1998 he became a Decision Support Analyst at OCI. Dr. Kleinman has published articles on optimization of air and roadway traffic networks and has submitted articles for publication on methods of analyzing and improving stochastic optimization algorithms. He has been a member of the Phi Kappa Phi Honor Society and is a member of INFORMS.

STACY D. HILL received the B. S. and M. S. degrees from Howard University in 1975 and 1977, respectively, and the D. Sc. degree in control systems engineering and applied mathematics from Washington University in 1983. Since 1983, Dr. Hill has been on the Senior Staff of the Johns Hopkins University, Applied Physics Laboratory where he has been project and technical lead in developing, testing, and applying statistical techniques and software, and has led systems analysis and modeling projects. He has published papers on diverse topics in statistics and engineering, including subjects such as simulation, optimization, and parameter estimation

VICTOR A. ILENDA received the B.S.E.E. degree from Case Institute of Technology (now Case Western Reserve) in 1963 and the M.S.E.E. degree from Ohio State University in 1970. Mr. Ilenda is a member of the Principal Professional Staff of the Johns Hopkins University, Applied Physics Laboratory, where, since 1970, he has worked on various aspects of submarine launched ballistic missile programs. Prior to joining the Johns Hopkins University, Mr. Ilenda worked for Battelle Memorial Institute and a division of General Motors specializing in inertial guidance systems and radiation effects on electronic systems. He is a member of the IEEE and AIAA.