PROJECT SCHEDULING USING STATE-BASED PROBABILISTIC DECISION NETWORKS

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ABSTRACT

This paper presents probabilistic CPM scheduling using an add-on program to the STROBOSCOPE simulation system that allows dependence and correlation between activity durations as well as control over precedence and the selection of paths. Activity durations and sequencing can be defined in terms of the dynamic information that becomes available as a project evolves and includes the actual start date and duration of activities already started. An example highway project illustrates the modeling power of this approach.

1 INTRODUCTION

It is widely recognized that construction planning and scheduling using CPM can be substantially enhanced and become more effective and more realistic by recognizing ahead of time the uncertainty in activity durations and their dependence on the dynamic state of the project as it evolves. Several tools have been developed for this purpose. PERT was a first step by allowing probabilistic activity durations. Monte-Carlo simulation can tackle the merge-event bias problem (Van Slyke 1963). GERT (Pritsker 1977) can additionally model uncertainty in the precedence of activities. VERT (Moeller & Digman 1981), MUD (Carr 1979) and DYNASTRAT (Morua Padilla 1986) can model correlation between activity durations and to a limited extent recognize the state of the project in progress.

None of the existing tools, however, have the necessary flexibility and power to model uncertainty in the duration of activities as a true function of the state of the project, nor can they model the underlying process-level operations through concurrent simulation. The CPM add-on for STROBOSCOPE presented here overcomes these limitations and serves as an example of how STROBOSCOPE can be used as a development vehicle for special-purpose simulators and more sophisticated project-level modeling tools.

2 STROBOSCOPE AND THE CPM ADD-ON

STROBOSCOPE is an acronym for STate and ResOurce Based Simulation of COnstruction ProcEsses. It is a simulation programming language designed specifically for modeling construction operations based on three-phase activity scanning and activity cycle diagrams. The STROBOSCOPE language is described in (Martinez 1996). Example applications can be found in (Ioannou & Martinez 1996a, 1996b, 1996c) and (Martinez & Ioannou 1994, 1995).

STROBOSCOPE can be extended via add-ons written in a high-level compiled language such as FORTRAN or C++. STROBOSCOPE can also be controlled from other applications via OLE automation. A STROBOSCOPE add-on is a 32-bit MS Windows dynamic-link library that extends the STROBOSCOPE language with new statements, functions, and variables. The code within the add-on can call back into the simulation engine using STROBOSCOPE's Application Programming Interface (API) described in (Martinez 1996).

The CPM add-on described in this paper is loaded into STROBOSCOPE with the `LOADADDON` statement:

LOADADDON C:\…\CpmAddOn.dll;

Once loaded, its statements, functions and variables can be used as if they were a standard part of the language. The new statements are shown in Table 1 and the new state variables are listed in Table 2. Of these, the second argument to CPMACTIVITY in Table 1 is of particular importance because it defines an activity's duration. This argument can be any allowable expression and can include functions that sample from probability distributions as well as variables that access the state of project or the state of any concurrent process simulation. Thus, the expression used to define the activity duration can be as complex as necessary, calling any number of functions and variables. The parameters of functions that sample from probability distributions can also include expressions or variables to allow easy modeling of conditional and correlated distributions.

Table 1: Statements Registered by the CPM Add-On

CPMACTIVITY *ActivityName* [*Duration*] [*Cost*];   Defines a CPM activity with the given name. Its cost and duration are sampled from the given expressions.
PRECEDENCE *PredecessorName SuccessorName*;   Indicates that the successor cannot start until the predecessor is finished.
DURATION *ActivityName Duration*;   (Re)defines the duration expression for CPMActivity.
CPMACTCOST *ActivityName Cost*;   (Re)defines the cost expression for CPMActivity.
CPMREPLICATE *Number of Replications*;   Simulates the CPM network the number of times indicated and produces a report as described in the CPMREPORT statement.
DOCPM;   Performs a single forward and backward CPM pass.
CPMREPORT;   Prints the 90% Confidence Interval on project duration and cost, and a report showing the average duration, cost, early and late dates, floats, and criticality for each CPM Activity.

Table 2: Variables defined by the CPM Add-On

Variable Prototype	Value Returned
CPMActivity.Duration	The sampled activity duration used in the current run.
CPMActivity.EarlyStart CPMActivity.EarlyFinish CPMActivity.LateStart CPMActivity.LateFinish	The early/late start/finish for CPMActivity in the current run.
CPMActivity.Started CPMActivity.GoingOn CPMActivity.Finished	TRUE if CPMActivity (has started)/(is going on)/(has finished) in the current run.
CPMActivity.FreeFloat CPMActivity.TotalFloat	The activity's free/total float in the current run.
CPMActivity.Critical	TRUE if CPMActivity is critical in the current run.

3   EXAMPLE HIGHWAY PROJECT

A highway with the profile and plan shown in Figure 1 will serve as an example for using the CPM add-on. This project is 11,600 ft (2.2 mile) long and has been adapted from (Brand et al 1964). Table 3 shows the names of the activities, the three-point estimates for the (optimistic, most-likely, and pessimistic) activity durations (a, M, b), the mean duration based on the PERT approximation $(a+4*M+b)/6$, the variance, and the standard deviation of the activity duration given by the PERT approximation $(b-a)/6$. The last column lists the direct predecessors of each activity and defines the connectivity of the network.

The construction work for this highway is naturally divided into two sections based on the location of the balance points for earthmoving (cut and fill). As shown in Figure 1, the western section extends from Station 42 to Station 100 and the eastern section extends from Station 100 to Station 158. Within each section the amount of cut equals the amount of fill. Assuming that the necessary resources are available, the majority of the work in each section can be performed independently and simultaneously while minimizing haul distances. The division of the work is also reflected in the definition of activities shown in Table 3. They include material and equipment procurement, cut-and-fill earthmoving, the construction of one double-barrel and two single-barrel concrete box culverts, the placement of subbase, paving, landscaping, and guardrail installation.
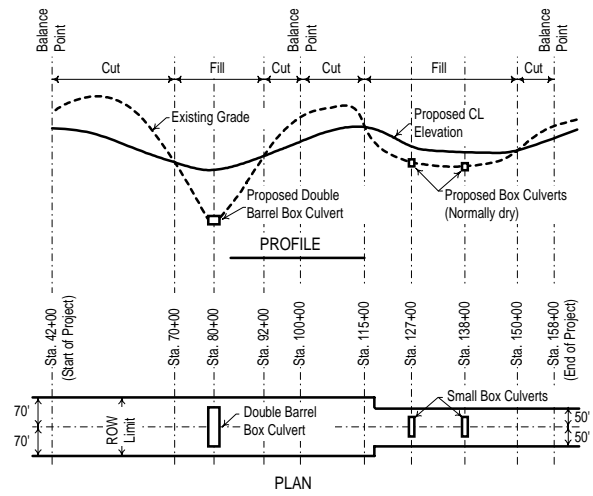

Figure 1: Plan and Profile for Highway Construction

The precedence network for this project is shown in Figure 2. Notice that activity names begin with the corresponding activity numbers from Table 3 (for easy identification) and cannot include spaces.

3.1 PERT Network Analysis

We shall begin the analysis of this project by using the PNET methodology to compute the cumulative probability of project completion (Ang et al 1975).

Table 3: Activity List and Duration Data in Days for Highway Construction Project

| No.	Activity Description	a	M	b	Mean	Var	SD	Preceded by
2	Setup batch plant	0.5	2	3.5	2	0.25	0.5	
3	Order & deliver paving mesh	2	5	8	5	1	1	
4	Deliver rebar for double barrel culvert	2.5	5.5	11.5	6	2.25	1.5	
5	Move in equipment	1.5	3	4.5	3	0.25	0.5	
6	Deliver rebar for small box culvert	1	4	25	7	16	4	
7	Build double barrel culvert	4	10	16	10	4	2	4
8	Clear & grub from Sta. 42 to Sta. 100	1	2.5	7	3	1	1	5
9	Clear & grub from Sta. 100 to Sta. 158	2.5	7	11.5	7	2.25	1.5	5
10	Build box culvert at Sta. 127	1	4	13	5	4	2	6
11	Build box culvert at Sta. 138	0.5	2	9.5	3	2.25	1.5	6
12	Cure double barrel culvert	3	9	15	9	4	2	7
13	Move dirt between Sta. 42 & Sta. 100	2.5	4	11.5	5	2.25	1.5	8, 12
14	Start moving dirt between Sta. 100 & Sta. 158	1.5	3	4.5	3	0.25	0.5	9
15	Cure box culvert at Sta. 127	1.5	6	28.5	9	20.3	4.5	10
16	Cure box culvert at Sta. 138	2	5	14	6	4	2	11
17	Order & stockpile paving material	0.5	2	3.5	2	0.25	0.5	2
18	Place subbase from Sta. 42 to Sta. 100	3.6	6.1	14	7	3	1.73	13
19	Finish moving dirt between Sta. 100 & Sta. 158	1	4	13	5	4	2	14, 15, 16
20	Pave from Sta. 42 to Sta. 100	4	10	16	10	4	2	3, 17, 18
21	Place subbase from Sta. 100 to Sta. 158	2	4.53	21.87	7	11	3.31	19
22	Cure pavement from Sta. 42 to Sta. 100	2.5	5.5	11.5	6	2.25	1.5	20
23	Pave from Sta.100 to Sta. 158	3	6.75	30	10	20.3	4.5	3, 17, 21
24	Cure pavement from Sta. 100 to Sta.158	2.5	5.5	11.5	6	2.25	1.5	23
25	Place shoulders from Sta. 42 to Sta. 100	1	2.5	7	3	1	1	22
26	Place shoulders from Sta. 100 to Sta.158	1	2.5	7	3	1	1	24
27	Place guardrail & landscape	2.5	4	11.5	5	2.25	1.5	25, 26
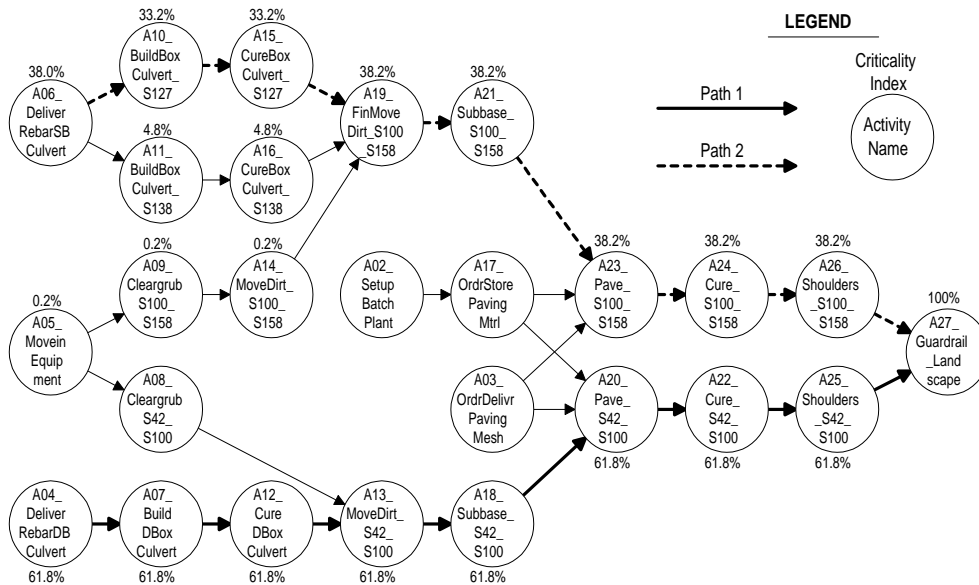
Figure 2: Precedence Network for Highway Construction Project

The basis of this approach are the following two inequalities that provide a lower and upper bound for the true cumulative probability of project completion:

$$\prod_I P[T_I \leq t] \ \leq \ P[T \leq t] \ \leq \ \min_I P[T_I \leq t] \quad (1)$$

Here the random variable T represents the duration of the project and T_I represents the duration of path I.

For the standard PERT problem where a network has a single dominant path, the above upper and lower bounds coincide (i.e., they give the exact same values). When no single dominant path exists, however, any one of several competing paths (with overlapping probability density functions) may determine the duration of the project. In this case, it is not clear ahead of time as to which of these paths will actually be the longest, and thus prove to be critical. This phenomenon is known as *merge-event bias* and the true probability for completion of the project is somewhere between the lower and upper bounds given by Eq. (1). Whether it is closer to the upper or the lower bound depends on the degree of correlation between the durations of the competing paths. If path durations are independent then the true probability equals the lower bound. As positive correlation increases, the true probability moves closer to the upper bound.

Positive correlation between path durations is most often due to the fact that several paths share the same activities. When shared activities are long or short, all paths to which they belong tend to be long or short.

The basic idea in PNET is to eliminate from the lower bound in Eq. (1) *those paths that share enough activities to be highly correlated with (and thus represented by) other (longer) paths in the network.*

This procedure is best illustrated with an example. We begin by constructing a list of all paths in the network and arranging them in descending order of their mean duration. In this case, the network for the highway project consists of 9 paths ordered as shown in Table 4:

The standard deviation of path duration, SD, is the square root of the variance computed by assuming independence between the duration of activities h that belong to each path I:

$$\acute{o}_I^2 = \sum_{h \in I} \acute{o}_h^2 \quad (2)$$

A useful shortcut at this point is to eliminate from further consideration all paths that are clearly too short to assume the role of critical path, such as 6, 7, 8, and 9.

Table 4: Paths Ordered by their Expected Duration

Path I	Activities h in path I	Mean	SD
1	4,7,12,13,18,20,22,25,27	61	5.00
2	6,10,15,19,21,23,24,26,27	57	9.00
3	6,11,16,19,21,23,24,26,27	52	7.93
4	5,9,14,19,21,23,24,26,27	49	6.59
5	5,8,13,18,20,22,25,27	42	4.00
6	3,20,22,25,27	29	3.24
7	3,23,24,26,27	29	5.17
8	2,17,20,22,25,27	28	3.16
9	2,17,23,24,26,27	28	5.12

The fact that paths 1-5 are contenders, however, does not mean that they are independent. It is easy to show that the correlation coefficient between the durations of two paths I and J that share activities is given by:

$$\tilde{n}_{IJ} = \frac{\displaystyle\sum_{k \in (I \cap J)} \acute{o}_k^2}{\acute{o}_I \acute{o}_J} \quad (3)$$

The numerator in this expression is the sum of the variances of the activities k that belong to both paths I and J. The denominator is the product of the standard deviations of the durations of paths I and J from Table 4.

This expression is first used to compute the correlation coefficient between path 1 and every other path:

$$\rho_{12} = 0.05001 \qquad \rho_{13} = 0.05671$$
$$\rho_{14} = 0.06825 \qquad \rho_{15} = 0.73755$$

Paths with correlation coefficients greater than 0.5 are *represented* by path 1 and can be eliminated. In this case we can eliminate path 5 but not paths 2, 3, and 4.

The pairwise calculation of correlation coefficients is now repeated by substituting path 2 (the longest remaining path) in the role of path 1:

$$\rho_{23} = 0.79432 \qquad \rho_{24} = 0.68632$$

Since both correlation coefficients exceed 0.5 we can eliminate paths 3 and 4 as represented by path 2.

Thus, only paths 1 and 2 are long enough to be critical and at the same time sufficiently independent to be included in the calculation of the lower bound. The two paths are shown with heavier precedence links in the network of Figure 2 for easy identification.

In conclusion, the probability of project completion according to PNET can be approximated by:

$$P[T \leq t] \approx P[T_1 \leq t] \cdot P[T_2 \leq t] \quad (4)$$

A graph of this approximation appears in Figure 3. The effect of merge-event bias is also shown in this figure as the difference between the PNET curve and that given by standard PERT with a dominant longest path:

$$P[T \le t] \approx P[T_1 \le t] \qquad (5)$$

Clearly, Eq. (5) is the same as the upper bound in Eq. (1) and can lead to significant overstatement of the true probability of completing the work by a given deadline. In fact, the lower and upper bounds defined by Eq. (1) when considering all paths 1-5 define an envelope for the true probability of project completion given by:

$$\prod_{I=1}^{5} P[T_I \le t] \;\le\; P[T \le t] \;\le\; P[T_1 \le t] \qquad (6)$$
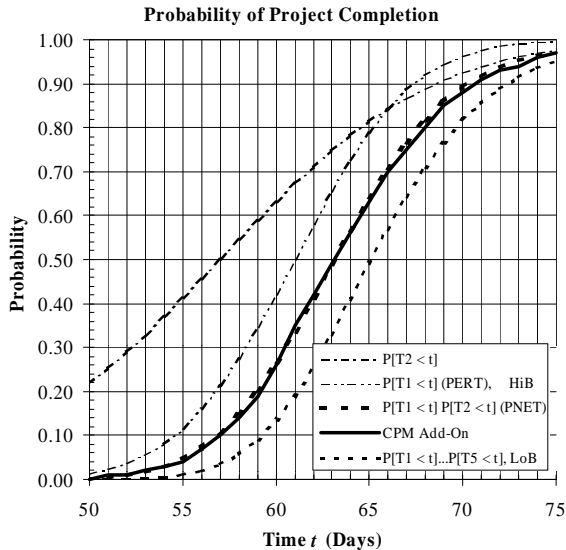

Figure 3: Probability of Project Completion

As shown in Figure 3 this envelope is quite wide and does not provide an accurate estimate for the probability of project completion. By eliminating three of the five terms in the lower bound of Eq. (5), the PNET estimate given by Eq. (4) is not only within this envelope but is also almost identical to the STROBOSCOPE simulation results, a fact that illustrates PNET's accuracy.

3.2 Basic Simulation of Highway Network

At the simplest level a probabilistic CPM network can be analyzed using *ProbSched*, a graphical pre- and post-processor for the CPM add-on that eliminates the need to write any simulation source-code manually. *ProbSched* has been implemented as an add-on to Visio (a programmable and extensible drawing program for Windows 95/NT) that

facilitates the creation of intelligent schematic drawings by using preprogrammed drag-and-drop graphics. *ProbSched* includes a custom library of predefined graphical shapes that can be dragged and dropped onto the drawing page where they can be sized, positioned, and connected to form an activity network as shown in Figure 2. Double-clicking any of the activity nodes activates a custom dialog box for the specification of the appropriate distributions and data for sampling the duration and cost of the corresponding activity. Right-clicking anywhere on the drawing page activates a menu for specifying global parameters, such as the seed and the number of replications. The same menu also allows starting the simulation. This activates STROBOSCOPE, generates the necessary source code and passes it to the simulation engine through OLE automation, runs the specified number of replications, receives the simulation results, and plots them as a statistical bar chart in a separate Visio drawing.

While *ProbSched* provides all the necessary facilities for basic simulation modeling of probabilistic networks, it does not currently allow for more advanced techniques that require access to source-code level programming. In cases that need greater control, such as some of the examples presented below, it is necessary to write the source code for a simulation model using the STROBOSCOPE IDE. For example, the basic simulation input file for the highway construction project consists of statements such as the following:

```
VARIABLE nReplications 1000;
SEED 9111964; STREAMS 27; / If needed
LOADADDON CpmAddOn.dll;
CPMACTIVITY A02_SetupBatchPlant
    sPert[0.5,2,3.5,2];
CPMACTIVITY A03_DeliverPavingMesh
    sPert[2,5,8,3];                \ etc.
PRECEDENCE A02_SetupBatchPlant
    A17_ProcurePavingMtrl;
PRECEDENCE A03_DeliverPavingMesh
    A23_Pave_S100_S158;    \ etc.
CPMREPLICATE nReplications;
```

This source-code consists almost entirely of statements defined by the CPM add-on. The only standard STROBOSCOPE statements are VARIABLE, SEED, STREAMS, and LOADADDON. When this model is processed by the STROBOSCOPE simulation engine on a 400 MHz Pentium II under Windows NT 4, it produces the output shown in Figure 4.

As shown at the top of this report, STROBOSCOPE has performed 1,000 replications of this project. In each replication it sampled a duration for each activity (based on the corresponding probabilistic expression) and used these to perform the standard CPM calculations. Thus, in

```
Number of replications performed : 1000
Average Project Duration      : 63.61
Std. Dev. of Project Duration : 5.66
90% Confidence Interval for the Expected Project Duration: [63.315 - 63.905]
```

Activity	Time	ESD	LSD	EFD	LFD	FF	TF	%Crit
A02_SetupBatchPlant	2.00	0	33.56	2.00	35.56	0	33.56	0.0%
A03_OrderDeliverPavingMesh	5.02	0	32.58	5.02	37.60	26.90	32.58	0.0%
A04_DeliverRebarDBCulvert	5.97	0	2.83	5.97	8.80	0	2.83	61.8%
A05_MoveinEquipment	3.00	0	14.09	3.00	17.09	0	14.09	0.2%
A06_DeliverRebarSBCulvert	7.17	0	5.73	7.17	12.90	0	5.73	38.0%
A07_BuildDBoxCulvert	9.90	5.97	8.80	15.87	18.70	0	2.83	61.8%
A08_CleargrubS42_S100	3.01	3.00	24.76	6.02	27.77	18.93	21.76	0.0%
A09_CleargrubS100_S158	7.01	3.00	17.67	10.02	24.68	0	14.67	0.2%
A10_BuildBoxCulvert_S127	5.05	7.17	13.36	12.21	18.41	0	6.20	33.2%
A11_BuildBoxCulvert_S138	2.95	7.17	18.67	10.11	21.61	0	11.50	4.8%
A12_CureDBoxCulvert	9.07	15.87	18.70	24.94	27.77	0	2.83	61.8%
A13_MoveDirt_S42_S100	4.94	24.94	27.77	29.88	32.71	0	2.83	61.8%
A14_MoveDirt_S100_S158	2.98	10.02	24.68	13.00	27.66	9.04	14.67	0.2%
A15_CureBoxCulvert_S127	9.25	12.21	18.41	21.47	27.66	0.57	6.20	33.2%
A16_CureBoxCulvert_S138	6.05	10.11	21.61	16.16	27.66	5.88	11.50	4.8%
A17_ProcurePavingMtrl	2.04	2.00	35.56	4.04	37.60	27.89	33.56	0.0%
A18_Subbase_S42_S100	7.02	29.88	32.71	36.90	39.73	0	2.83	61.8%
A19_FinMoveDirt_S100_S158	5.00	22.04	27.66	27.04	32.67	0	5.62	38.2%
A20_Pave_S42_S100	10.00	36.90	39.73	46.90	49.73	0	2.83	61.8%
A21_Subbase_S100_S158	7.08	27.04	32.67	34.12	39.75	0	5.62	38.2%
A22_Cure_S42_S100	5.97	46.90	49.73	52.87	55.70	0	2.83	61.8%
A23_Pave_S100_S158	9.92	34.12	39.75	44.04	49.66	0	5.62	38.2%
A24_Cure_S100_S158	5.98	44.04	49.66	50.02	55.64	0	5.62	38.2%
A25_Shoulders_S42_S100	2.96	52.87	55.70	55.83	58.66	2.83	2.83	61.8%
A26_Shoulders_S100_S158	3.01	50.02	55.64	53.03	58.66	5.62	5.62	38.2%
A27_Guardrail_Landscape	4.95	58.66	58.66	63.61	63.61	0	0	100.0%

```
Execution Time = 2.894 seconds
```

Figure 4 – Simulation Output for Highway Construction Project

each replication it performed a forward pass to compute the Early Start Date (ESD) and Early Finish Date (EFD) and then a backward pass to compute the Late Finish Date (LFD) and the Late Start Date (LFD). Based on these, it determined each activity's Total Float (TF) and Free Float (FF) and identified the critical path. This process was repeated 1,000 times, and the average of the results is reported in Figure 4. For example, the column "Time" shows the average duration of each activity based on 1,000 samples. As expected, these values are very close to the expected activity durations shown in Table 3. The column "%Crit" shows each activity's *Criticality Index* (i.e., the fraction of the number of times out of a 1,000 the activity was critical).

More detailed statistics can be obtained by appending the REPORT statement at the end of the source-code file. This produces the standard STROBOSCOPE report that includes the minimum, maximum, average, and standard deviation for each variable shown in Figure 4.When *ProbSched* is used to initiate the simulation process, STROBOSCOPE returns the results shown in Figure 4 back to the Visio add-on, which in turn uses them to create automatically the statistical bar chart shown in Figure 5.

The Criticality Indices listed in Figure 4 also appear next to each activity in Figure 2. An examination of this network indicates that there is no dominant path and that each of paths 1 through 4 served as the critical path in at least some replications as shown in Table 5.

Table 5: Path Criticality Indices

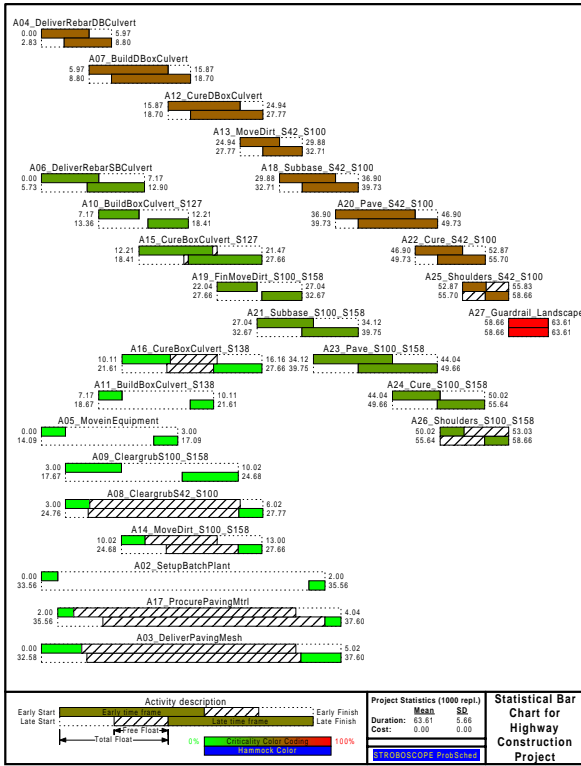| I	Activities h in path I	Mean	SD	% Crit.
1	4,7,12,13,18,20,22,25,27	61	5.00	61.8
2	6,10,15,19,21,23,24,26,27	57	9.00	33.2
3	6,11,16,19,21,23,24,26,27	52	7.93	4.8
4	5,9,14,19,21,23,24,26,27	49	6.59	0.2

Figure 5: Statistical Bar Chart

The fact that there is no single dominant critical path explains why no activity (except 27) has TF = 0. Activity 25, for example, belongs to path 1 and was critical in 618 of the 1,000 replications (TF = 0). In the remaining 382 runs it was non-critical and had TF > 0. Thus, the reported TF=2.83 is the average of 618 zeros and 382 non-zero values. This shows that in general the TF has a *mixed* probability distribution with a probability mass at TF = 0 and probability density for TF > 0.

3.3 Dynamic Scheduling Decisions

A simple illustration of a typical problem that is very difficult to solve analytically, but can be modeled quite easily with the CPM add-on, is that of precedence links that could point in either direction. It is not unusual in a highway project, for example, for the contractor to have only one paving machine. In this case, the two paving activities (20 and 23) cannot overlap and must be performed serially, one after the other. The problem is that we do not know beforehand which can be started first and which must follow. Merging the two paving activities into one does not solve the problem because it is not necessary to finish both subbase activities (in the east and west sections) in order to start paving, nor is it necessary to finish all paving before curing and shoulder work can begin.

This is an example of a dynamic decision that must be made while the project is going on. A simple way to model this using the CPM add-on is by *redefining* the durations of the two paving activities (X, Y) to add the extra time one of them must wait until the *other* activity (the one that has already started) finishes:

```
VARIABLE XWaitsForY 'Y.Started?
        Max[Y.EarlyFinish-SimTime,0]:0';
VARIABLE YWaitsForX 'X.Started?
        Max[X.EarlyFinish-SimTime,0]:0';
DURATION X XWaitsForY+Pert[aX,MX,bX];
DURATION Y YWaitsForX+Pert[aY,MY,bY];
```

Inserting the above code right before the statement CPMREPLICATE produces the results shown in Table 6 under the heading DL (dynamic links). Table 6 also shows the results for static precedence links as well as for the base case of two pavers to allow comparison.

Table 6: Effect of Correlation on Project Duration

	Two Pavers	One Paver		
		DL	23->20	20->23
Replications	1000	1000	1000	1000
Average Proj. Dur.	63.61	67.11	68.96	71.09
Std. Dev. Proj. Dur.	5.66	6.39	7.92	6.55
90% CI Lo Bound	[63.31	[66.78	[68.54	[70.75
90% CI Hi Bound	63.90]	67.44]	69.37]	71.43]

3.4 Correlated Activity Durations

Another typical problem is that of modeling correlation between activity durations caused by common factors such as the weather, site conditions, the use of the same equipment and crews, etc. Correlated activity durations can also be modeled easily with the CPM add-on. For example, in the case of two activities X and Y that are normally distributed with correlation coefficient ρ (*rho*), we can first sample X and then Y as follows:

```
DURATION X Normal[mX,sX];
DURATION Y Normal[mY+(X.Duration-mX)*
            rho*sY/sX,sY*Sqrt[1-rho^2]];
```

Using the highway project as an example, it is interesting to investigate the effect of correlation between two activities that belong to parallel paths versus that caused by two correlated activities that follow each other serially. Table 7 shows the effect of perfect positive correlation between the two subbase activities 18 (path 1) and 21 (path 2), and between subbase activity 18 (or 21) and the following paving activity 20 (or 23).

The introduction of positive correlation between the two subbase activities makes paths 1 and 2 to tend to vary together and this reduces the average project duration from

63.43 to 63.27 and raises the standard deviation a little from 5.75 to 5.91. In contrast, positive correlation between a subbase activity and the paving activity that follows increases the variance for both paths 1 and 2 and this increases the average project duration from 63.43 to 64.10 and the standard deviation from 5.75 to 6.71. Having both types of correlation increases the variance for both paths 1 and 2 but also makes them move together (in lock-step) even more. This reduces the average duration from 63.43 to 63.07 and increases the standard deviation from 5.75 to 7.20.

Table 7: Effect of Correlation on Project Duration

$\rho_{18, 21} =$	0	1	0	1
$\rho_{18, 20} = \rho_{21, 23} =$	0	0	1	1
Replications	1000	1000	1000	1000
Average Proj. Dur.	63.43	63.27	64.1	63.07
Std. Dev. Proj. Dur.	5.75	5.91	6.71	7.2
90% CI Lo Bound	[63.13	[62.96	[63.75	[62.70
90% CI Hi Bound	63.73]	63.57]	64.45]	63.45]

4   CONCLUSION

The CPM add-on and its source code in C++ are included with STROBOSCOPE and are available from http://grader.engin.umich.edu and http://strobos.ce.vt.edu. The CPM add-on illustrates STROBOSCOPE's powerful API and the ease of extending the language. It is also a solid starting point for more sophisticated project-level analysis tools. Moreover, the CPM add-on is especially very useful for teaching and research as a probabilistic scheduling tool. In fact, the CPM add-on was the primary tool for verifying a heuristic scheduling approach in a recently completed Ph.D. dissertation (Wang 1996).

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