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A REAL OPTIONS DESIGN FOR PRODUCT OUTSOURCING

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ABSTRACT

We develop a financial model to assess the option value of outsourcing. We value the real options associated with outsourcing an item using Monte Carlo simulation. This valuation gives decision makers a way to choose the appropriate outsourcing strategy based on an integrated view of market dynamics. A simulation example is used to demonstrate the application of real options to value outsourcing. The simulation program code was written in JavaScript so that the valuation task would be accessible to other users because of its web enabled feature.

1 INTRODUCTION

Increased competition in the global market has caused organizations to realize that the most competitive way of survival is high value. This can often be achieved through increased flexibility. Then the question becomes: Precisely how valuable is flexibility? The financial arena was the original ground for the application of the options-based framework to the valuation of flexibility. More recently, managerial operating flexibility has been likened to financial options.

The goal of our research is to view the flexibility surrounding manufacturing operations using financial options. Nembhard, Shi, and Park (2000) develop a framework for the broad scope of this research activity. In Nembhard, Shi, and Aktan (2001), we consider the decision to introduce statistical process control (SPC) charts to monitor quality.

In this paper, we specifically consider the manufacturing decision to increase flexibility through outsourcing, which is recognized as a source of great competitive advantage (Gupta and Zhender 1994). The classical net present value approach often falls short in the analysis of such decisions, due to its inability to address market dynamics with respect to the key determining variables. We use the options approach to find the value of outsourcing during a specified length of time, considering future uncertainty. The problem is analyzed using Monte Carlo simulation. Results of the simulation are provided with numerical examples. Using the proposed design, a company will be able to answer questions about the long-term value of product outsourcing.

This paper is organized as follows. Approaches for multivariate option valuation are discussed in Section 2. The financial model that will be used to find the option value of outsourcing is defined in Section 3. Section 4 discusses the Monte Carlo simulation. An example and numerical results are given in Section 5. We make some concluding remarks in Section 6.

2 OPTION MODELS

An option is the right, but not the obligation, to take an action in the future (Amram and Kulatilaka 1999). Some options are associated with investment opportunities that are not financial instruments. These operational options are often termed *real options* to emphasize that they involve real activities or real commodities, as opposed to purely financial commodities, as in the case, for instance, of stock options (Luenberger 1998).

A European option gives the right to exercise the option on the expiration date. Here, we formulate the outsourcing problem as a series of n European options, where all options start at time zero, and each option expires at one of the n equally spaced time intervals. In our context, this means that outsourcing may or may not be applied (which is the option) in any time interval.

Black and Scholes (1973) developed a closed form solution for valuing a European option with one variable. In the case of one variable, the binomial lattice approach of Cox, Ross, and Rubinstein (CRR) (1979) is a powerful numerical procedure for valuing options. Boyle (1988) developed an extension of the CRR procedure for option valuation in the case of two state variables.

In our outsourcing model (which will be defined in the next section), there are multiple sources of uncertainty.

Valuing real options for such a model will require an analysis of multiple variables. Boyle, Evnine and Gibbs (1989) developed an n-dimensional extension of the CRR procedure using multinomial lattices. Kamrad and Ritchken (1991) developed a similar multinomial lattice technique for valuing projects for one or more state variables. However, multinomial lattice approaches are not practical for option valuation if we must deal with a large number of variables, because these approaches generate many nodes and require extensive calculations. Monte Carlo simulation provides a good alternative for valuing European options with more than one variable.

3 A FINANCIAL MODEL FOR OUTSOURCING

In this section, we provide a framework for the financial model for product ourscourcing. It parallels Kouvelis (1999) which develops a general framework for evaluating the total expected cost of outsourcing from a global network of suppliers when the purchasing firm faces uncertain exchange rates.

Consider an item that is a part of a final product. The total cost for the items sold has three main sources of uncertainty: unit production cost of the item during the time interval beginning at time t, $S_1(t)$; unit outsourcing price of the item during the time interval beginning at time t, $S_2(t)$; and unit delivery cost of the outsourced item during the time interval beginning at time t, $S_3(t)$. We assume that the demand D is constant.

Assuming there is no switching cost, and the manufacturer can change the decision about outsourcing at each time interval. If the unit outsourcing cost of the item is less than the unit production cost of the item, then total cost reduction $R(t)$ from outsourcing the item during the time interval that begins at time t is

$$R(t) = \left[S_1(t) - S_2(t) - S_3(t)\right] \times D\,.$$

If K denotes the fixed cost of outsourcing per time interval (contracting cost, and other possible costs to make the contract), then net cost reduction $F(t)$ due to the outsourcing option is

$$F(t) = \max\left\{0, \left[S_1(t) - S_2(t) - S_3(t)\right] \times D - K\right\}. \quad (1)$$

Figure 1 shows the relationship between the variables, total production and outsourcing costs, the option value, and its effect on decisions. The three variables affect the total costs. Whether we outsource or not also influences the total costs. If the outsourcing cost of the item is less than the production cost of the item, then we may decide to outsource the item instead of producing it. The option value is found by evaluating the cost reduction that may be possible by outsourcing in future. Depending on the option value, we decide whether to outsource now, possibly later, or never.

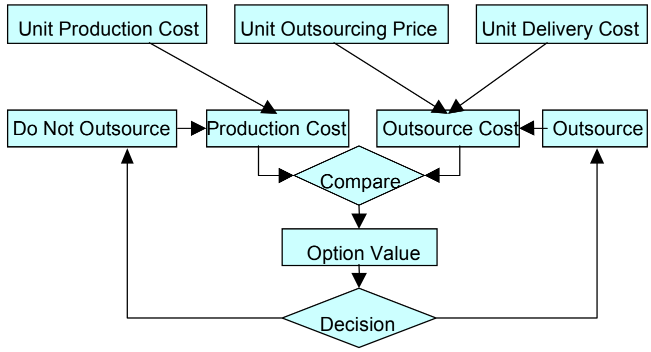

Figure 1: Relation between variables and the option value

4 MONTE CARLO SIMULATION

To value the three-variable outsourcing option, we will use Monte Carlo simulation. Since there are no switching costs, a decision given in a time interval does not affect the decisions in other time intervals. This feature makes it relatively easy to use Monte Carlo simulation in this problem.

Simulation models may be used to give numerous possible paths of evolution for underlying state variables from the present to the final date in the option. In the commonly used Monte Carlo simulation method, the optimal strategy on each path is determined and the payoff is calculated (Amram and Kulatilaka 1999).

Suppose that the process followed by the underlying variable S in a risk-neutral world is

$$\mathrm{d}S = \mu S \mathrm{d}t + \sigma S \mathrm{d}z \quad (2)$$

where z is a Wiener process, μ is the expected return in a risk-neutral world ($\mu = r$), and σ is the volatility. To simulate the path followed by S, we divide the life of the underlying variable into n short intervals of length Δt and approximate Equation (2) as

$$S(t+\Delta t) - S(t) = \mu S(t)\Delta t + \sigma S(t)\varepsilon\sqrt{\Delta t} \quad (3)$$

where $S(t)$ denotes the value of S at time t, and ε is a random sample from a normal distribution with a mean zero and unit standard deviation. This enables the value of S at time Δt to be calculated from the initial value of S, the value at time $2\Delta t$ to be calculated from the value at time Δt, and so on. One simulation trial involves constructing a complete path for S using n random samples from a normal distribution (Hull 1997).

From Ito's lemma (see Hull (1997) for a discussion of Ito (1951)), the process followed by ln*S* is

$$d\ln S = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma\, dz$$

so that

$$S(t+\Delta t) = S(t)\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\,\varepsilon\sqrt{\Delta t}\right] \qquad (4)$$

This equation is used to construct a path for S in a similar way to Equation (3). Whereas Equation (3) is true only in the limit as Δt tends to zero, Equation (4) is exactly true for all Δt (Hull 1997).

In our model, we use Equation (4) to generate values for the three variables $S_1(t)$, $S_2(t)$, and $S_3(t)$. For each t, we calculate the average of max$\{0, [S_1(t)–S_2(t)–S_3(t)]*D–K\}$ values (see Equation (1)) obtained from the simulation runs, and discount that average to the present time. In order to find the estimated option value, we add the discounted averages for all t until T, where T is the expiration time of the option.

Since the three variables may be correlated, we need correlated samples ε_i, (i=1,2,3) from normal distributions (see Equation (4)) where the coefficient of correlation between sample i and sample j is $\rho_{i,j}$. We first sample three independent variables x_i (i=1,2,3), from univariate standardized normal distributions. The required samples ε_i are

$$\varepsilon_i = \sum_{k=1}^{i} \alpha_{ik} x_k \ .$$

For ε_i to have the correct variance and the correct correlation with the ε_j ($1 \le j < i$), we must have

$$\sum_{k=1}^{i} \alpha_{ik}^2 = 1$$

and, for all $j < i$,

$$\sum_{k=1}^{j} \alpha_{ik}\alpha_{jk} = \rho_{i,j} \ .$$

The first sample, ε_1, is set equal to x_1. These equations for the α's can be solved so that ε_2 is calculated from x_1 and x_2; and ε_3 is calculated from x_1, x_2 and x_3 (Hull 1997).

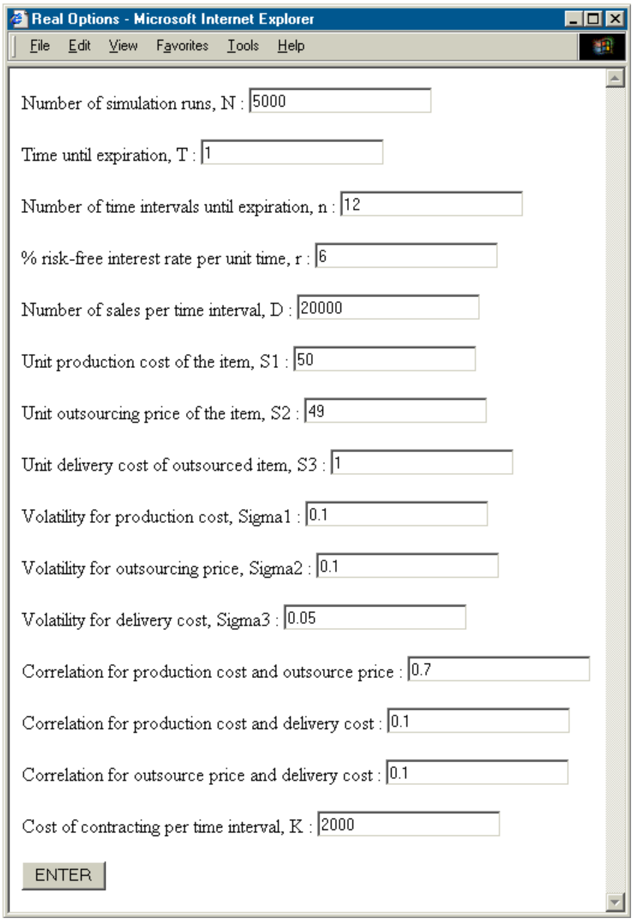

Figure 2: Input window for the simulation

5 A SIMULATION EXAMPLE

The program code for the Monte Carlo simulation was written in JavaScript, so that it can be executed with Microsoft Internet Explorer 3.0 or Netscape Navigator 2.0, and later versions.

First, the user enters all parameter values into the input boxes and these values are checked to ensure that they are valid. In order to store the generated values, the program generates an array for each of the three variables. The first array contains the $S_1(t)$ values for all time intervals until the expiration date of the option. Similarly, the second array contains the $S_2(t)$ values, and the third array contains the $S_3(t)$ values. Then, the value of Equation (1) at time t is calculated for all simulation runs, and the average of those values is found. Then, this average is discounted to the present time. A discounted average is calculated for each time point, i.e., $t,2t\ldots T$. The option value estimate is the sum of these discounted averages.

Figure 2 gives the input window for the simulation program. For this simulation example, the time until option expiration is one year, and length of each time interval is one month since there are 12 intervals. The risk-free interest rate r is 6% per year. The number of simulation runs is 5,000. The other parameters for modelling variables, volititily, and correlation can be seen in the input fields. Microsoft Internet Explorer version 5.5 was used to run the simulation code.

Figure 3 gives the output window where the estimated option value, standard deviation of the option value estimate, and minimum and maximum option values obtained from the 5,000 simulation runs are presented. Also, for each time interval, the fraction of simulation runs where outsourcing resulted in a cost reduction is given as a rate in the output window.

The estimated option value is $247,610. This means that considering outsourcing as an option for one year has an estimated value of $247,610. Standard deviation of the estimated option value is $4,437. Then, a 95% confidence interval for the option value is $247,610±1.96*$4,437 which gives

($238,912 , $256,308).

We see that outsourcing rate is zero at t=0. This means that outsourcing does not reduce the cost now. From t=1 to t=12, outsourcing rate is between 46.92% and 49.78%. Therefore, our chance of outsourcing from the first month to the twelfth month is nearly between 47% and 50%. In other words, with 47 to 50 percent probability, outsourcing will cost less than producing.

We also see that the maximum option value obtained from the 5,000 simulation runs is $2,072,012. This means that cost reductions as much as $2,072,012 may be possible by outsourcing. Minimum option value for the 5,000 simulation runs is $0. This means that market conditions were never favorable for outsourcing in any of the twelve months.

6 SUMMARY

In this paper, we have shown how the value of outsourcing can be determined using a real options framework. The need for this approach is due to the inability of classical net present value methods to address dynamics in the market condition with respect to unit production cost, unit outsourcing price and unit delivery cost for an item.

By connecting the dynamic aspects with the manufacturing operational aspects, we now have a way to address a key issue: the bottom-line cost associated with the outsourcing decision. Monte Carlo simulation was key in this study because for three sources of uncertainty, it was more practical to use than the other methods. Monte Carlo simulation also provided the maximum and minimum option values obtained during the simulation, which represent the best and the worst scenario for the option.

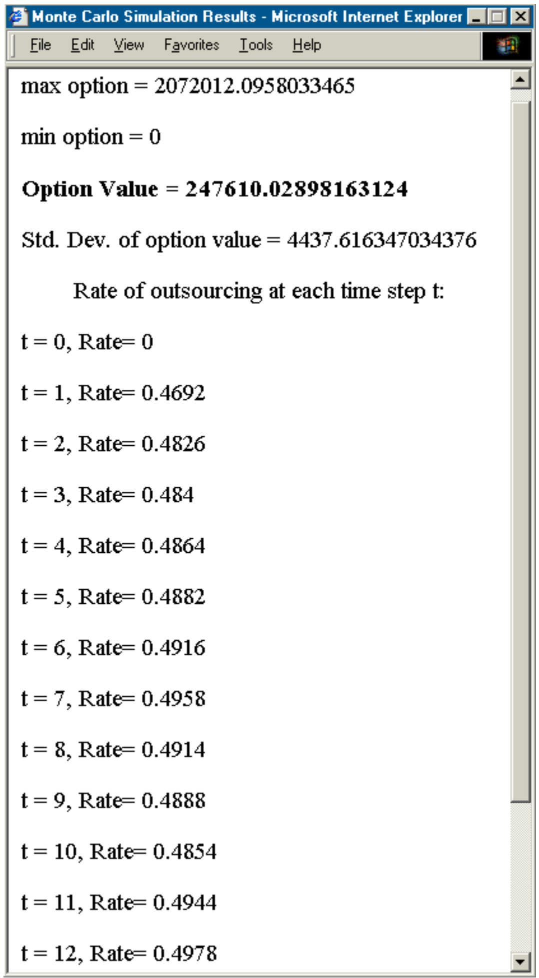

Figure 3: Output window of the simulation

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