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EFFECT OF IMPLEMENTATION TIME ON REAL OPTIONS VALUATION

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ABSTRACT

Exercising real options often requires an implementation time, whereas financial options can be exercised instantly. Neglecting the implementation time needed to exercise a real option causes overvaluing that option. We develop lattice and Monte Carlo simulation techniques to value real option problems, where exercising the option requires an implementation time. We present the application of the proposed techniques on a global supply chain network problem with exchange rate uncertainty and value the flexibility to switch between manufacturing options for a firm that has operations in different countries.

1 INTRODUCTION

Increased competition in the global market has caused organizations to realize that the most competitive way of survival is high value. This can often be achieved through increased flexibility. The financial arena was the original ground for the application of the options-based framework to the valuation of flexibility. More recently, managerial operating flexibility has been likened to financial options.

The goal of our research is to view the flexibility surrounding manufacturing operations using real options. McDonald and Siegel (1985) presented option pricing techniques to study the investment problem of a firm which has the option to shut down or change level of production. Brennan and Schwartz (1985) presented a problem where the option was to open, close, or abandon a mine. Majd and Pindyck (1987) used options analysis to derive optimal decision rules and to value investment decisions where construction proceeds flexibly and can be adjusted with the arrival of new information. Hodder and Triantis (1993) presented a general framework for modeling and evaluating investments which involve flexibility to switch between alternative states of operation. Kogut and Kulatilaka (1994) modeled the operating flexibility to shift production between two manufacturing plants located in different countries using a stochastic dynamic programming model. Huchzermeier and Cohen (1996) developed a stochastic dynamic programming formulation for the valuation of global manufacturing strategy options with switching costs, where a firm maximizes its expected, discounted, global, after-tax value through the exercise of product and supply chain network options through exploitation of flexibility contingent on exchange rate realizations. Dasu and Li (1997) studied the structure of the optimal policies for a firm operating plants in different countries. They determine the structure of the optimal policies for deciding when and by how much to alter the production quantities. Smith and McCardle (1998) presented how stochastic dynamic programming and option pricing theory can be profitably integrated to evaluate oil properties where production rates and oil prices both vary stochastically over time and, at any time, the decision maker may terminate the production or accelerate production by drilling additional wells. Nembhard, Shi, and Aktan (2001) studied the option value of being able to switch between the states of producing or outsourcing an item, using Monte Carlo simulation. Nembhard, Shi, and Aktan (2002) valued real options associated with the flexibility to apply statistical process control charts to monitor quality in a production process.

In these studies, the common assumption is that the option can be implemented immediately when the exercise decision is given, i.e., there is no time lag between the decision time and the implementation. However, in real manufacturing operations, this assumption does not truly hold. Manufacturing operations need some time to be executed.

In this paper, we specifically consider the time lag between the exercising decision and the implementation of the option. We develop lattice and Monte Carlo simulation techniques that can be used in option valuation analysis when there is time lag between the decision and the implementation of the options. We value the flexibility to switch between global manufacturing options for a firm that has operations in different countries, and maximizes its expected discounted value through the exercise of the manufacturing options in an environment of uncertain exchange rates. Using the proposed techniques, a company will be able to get better estimates for the option value, and to get optimal decisions considering the effect of time lag between the decision and the implementation of the option.

This paper is organized as follows. Section 2 gives the supply chain network model for a global. Section 3 gives a lattice technique that estimates the value of flexibility and gives the optimal decisions. Section 4 presents a Monte Carlo simulation approach that estimates the value of flexibility and gives the optimal decisions. We make some concluding remarks in Section 5.

2 THE SUPPLY CHAIN NETWORK AND DECISION IMPLEMENTATION STRATEGIES

We use a supply chain network model for a global firm that has suppliers, production plants, and markets in two foreign countries. There is a supplier in each of the two foreign countries. There is one production plant in each country. At the same time, each of these two countries is a market region for the final product. The supply chain network for the problem is shown in Figure 1.

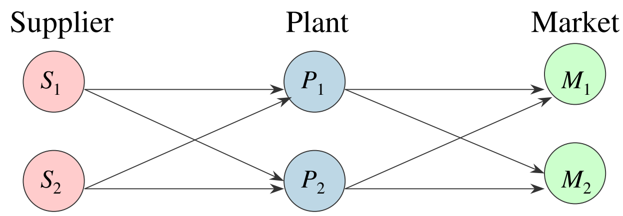

Figure 1: The Supply Chain Network

The firm defines a number of manufacturing options, where the suppliers, plants, and market regions are selected. Twelve manufacturing options are defined based on possible connections in the supply chain network. These twelve options are shown in Figure 2. Each option shows the connections for the suppliers, plants, and markets. In each decision point, the firm will select one of the manufacturing options. If there is a switch between the previous and the current decisions, a switching cost results. Each switching cost is defined depending on which suppliers, plants, and market regions must be changed to exercise the switch. Switching costs between each of the twelve manufacturing costs are given in Table 2 in the Appendix. Parameters of the problem such as the unit raw material cost for each supplier, unit manufacturing cost in each plant, capacity of each plant, unit selling price, and the demand in each market region are also given in the Appendix.

The total time horizon for the problem is one year. We divide the time horizon into time intervals of equal length.

The firm will exercise one of the twelve manufacturing options in each time interval. Four time intervals are defined so that each time interval is three months.

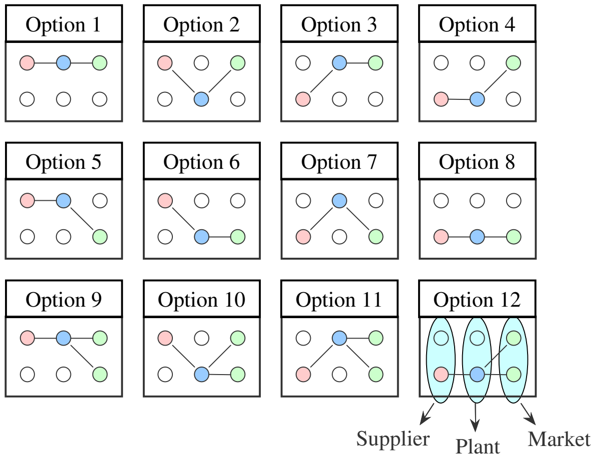

Figure 2: Manufacturing Options

We assume that there is a time lag between the switching decision and the implementation of the switch. This means that when a new manufacturing strategy is selected, it cannot be implemented immediately. For example, when the company wants to switch the supplier, all raw material from the new supplier cannot be received instantly; when a switch is desired for the production plants, required amount of production in the new plant cannot be completed instantly; and when a switch is desired for the market regions, final product cannot reach to the new market regions instantly.

In our first analysis, we assume that the implementation of the switch cannot begin in the time interval that the decision is made; rather it can be implemented in the next time interval. Then, we make a second analysis assuming that a portion of the new option can be implemented in the time interval that the decision is made, and the rest of the new option is implemented in the next time interval.

Exchange rates between the home country and the two foreign countries are the sources of uncertainty in the problem. The exchange rate e_i (i = 1,2) between the home country and the foreign country i follows geometric Brownian motion as

$$\frac{de_{i,t}}{e_{i,t}} = \mu_i dt + \sigma_i dz_i \tag{1}$$

where μ_i is the drift of the exchange rate changes for foreign country i, σ_i is the volatility of the exchange rate for foreign country i, dz is a standard Wiener disturbance term. The expected changes in the exchange rates are set to $\mu_i = \exp((r\text{-}r_i)\Delta t)$, where r and r_i are the risk free rate of interest in home country and country i, respectively.

In the next section, we present a solution to the problem using a lattice technique.

3 A LATTICE TECHNIQUE FOR VALUING FLEXIBILITY

Kamrad and Ritchken (1991) developed a multinomial lattice technique for valuing options including multiple state variables. We have two state variables in our problem, which are the exchange rates e_1 and e_2 between the home country and two foreign countries. We use the lattice structure of Kamrad and Ritchken to model the exchange rate movements. Assume that the exchange rate e_i can move up with the rate of u_i, or move down with the rate of d_i such that $d_i = 1/u_i$, or stay constant at each time interval. Five possible movements for the two exchange rates are shown in Table 1.

Table 1: Possible Movements and Their Probabilities

Change in e_1	Change in e_2	Probability
u_1	u_2	p_1
u_1	d_2	p_2
d_1	d_2	p_3
d_1	u_2	p_4
0	0	p_5

Probabilities of movements p_1 through p_5 are given as

$$p_1 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right) + \frac{\rho}{\lambda^2}\right\},$$

$$p_2 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right) - \frac{\rho}{\lambda^2}\right\},$$

$$p_3 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(-\frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}\right) + \frac{\rho}{\lambda^2}\right\},$$

$$p_4 = \frac{1}{4}\left\{\frac{1}{\lambda^2} + \frac{\sqrt{\Delta t}}{\lambda}\left(-\frac{\mu_1}{\sigma_1} + \frac{\mu_2}{\sigma_2}\right) - \frac{\rho}{\lambda^2}\right\},$$

$$p_5 = 1 - \frac{1}{\lambda^2},$$

where Δt is the length of each time interval, ρ is the correlation for the two exchange rates, and λ is a constant larger than 1.

Valuing a real options problem with switching costs is more difficult than valuing one without switching costs. If there is no switching cost between the options, each time interval can be evaluated independently from other time intervals. Then, we simply maximize the profit in each time interval without considering the decisions in other time intervals. When there are switching costs, options exercised in successive time intervals have connections with each other, and the current decision influences the later ones because of the switching costs. Hence, we cannot separate the problem into time intervals where the decisions are independent from each other. When switching costs exist, we apply a dynamic programming approach to maximize the profit.

Figure 3 shows the structure of the connections in the lattice. There are five nodes emerging from each node. The first element in parenthesis is the state of e_1, and the second element is the state of e_2. We assume that a decision can only be implemented in the next time interval, so the decision given in a node will be implemented in the immediately following nodes. At each node, we must select the best option that will be implemented in the next time interval. In order to do that, we will select the option that maximizes the expected profit from the following five nodes.

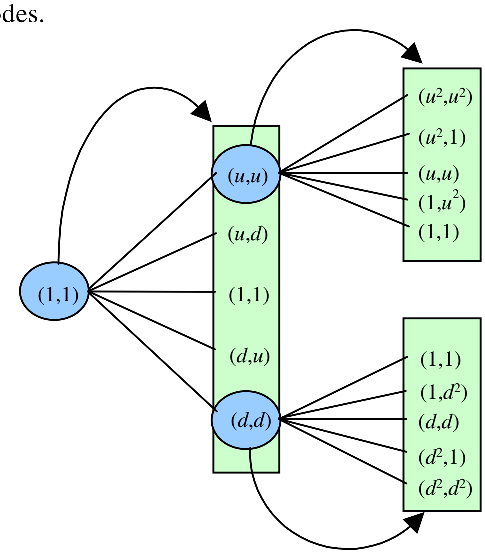

Figure 3: Connections in the Lattice

For the dynamic programming formulation, we need to define the best option at each node for every possible previous option. We begin from the last time interval and go back one time interval at each iteration. The optimal solution is obtained when we reach the first node. A complete set of decisions that maximize profit can be found by backtracking in the dynamic program.

Figure 4 shows the expected profits for an example where there is no switching and costly switching. If there is no switching, the same option will be used at all time intervals. However, when switching is possible, the best option will be selected at each time interval considering the switching costs. Then, value of flexibility that stems from being able to switch the manufacturing strategy is the difference between the expected profits of costly switching and no switching. The upper line in Figure 4 shows the expected profits when switching between options is possible and the decision is implemented in the current time interval. The middle line shows the expected profits when switching between options is possible but the decision is implemented in the next time interval. The lower line shows the expected profits when there is no switching between options. The horizontal axis shows which manufacturing option has been used prior to the first time interval in the problem.

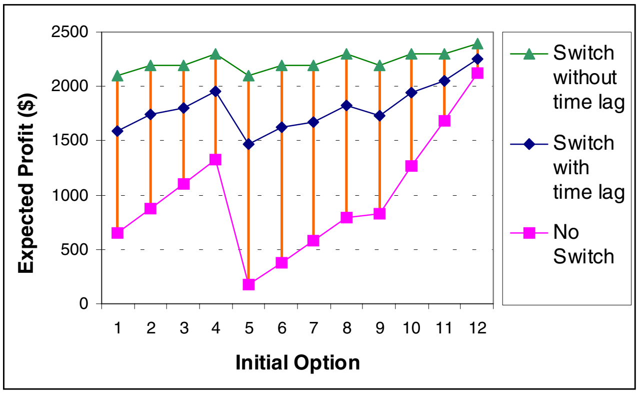

Figure 4: Expected Profits with and without Switching

As stated in the introduction, we are most concerned with the effects of time lag implementation. If the decisions can only be implemented with a time lag, and if we ignore this limitation and solve the problem as if there was no time lag for implementation, then we will overestimate the value of flexibility. This overestimation is the amount between the upper line and the middle line in Figure 4.

An advanced form of the time lag problem is the one where a portion of the option is implemented in the time interval that the decision is given, and the rest of the option is implemented in the following time interval. For example, in our context, only a portion of the new demand can be supplied in the time interval that the switching decision is given. We assume that the suppliers can satisfy the demand in the following time intervals until a switch is made.

Let Q be the portion implemented in the current time interval, where $0 \leq Q \leq 1$. Then, in the following time interval, the remaining 1-Q will be implemented. Figure 5 shows the expected profits for $Q = 1$, $Q = 0.5$, and $Q = 0$. A value of $Q = 1$ means that the option can be fully implemented in the current time interval (i.e., there is no time lag between the decision and the implementation) and we get the expected profits shown in the upper line. A value of $Q = 0.5$ means that 50% of the option is implemented in the current time interval and we get the expected profits shown in the middle line. A value of $Q = 0$ means that none of the option can be implemented in the current time interval (i.e., the switch can only be implemented in the next time interval so there is a time lag of one time interval), and we get the expected profits shown in the lower line.

Again we note that it is important to appreciate the effects of time lag implementation and also the degree to which a part of the decision must be deferred. We can make a more conservative statement about the expected profit assuming a one period lag. As we are able to refine

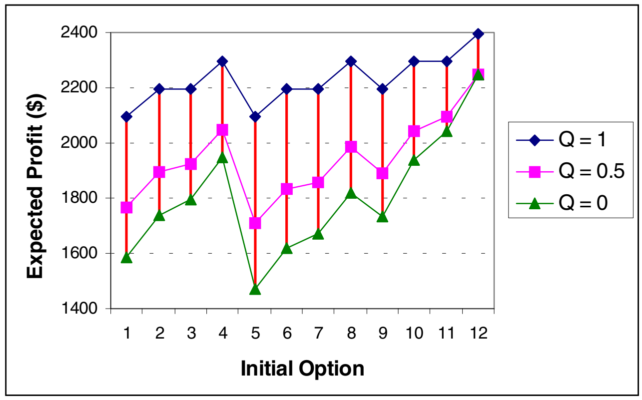

Figure 5: Expected Profits for Different Time Lags

our understanding of the true portion of the option that can be implemented immediately, we can get a better statement of the expected profit.

The lattice technique used to determine the expected profit in this example was suitable because there were only two state variables. When there are more than two state variables, the lattice structure gets quite large with just a few periods. In such cases, Monte Carlo simulation is a good alternative. In the next section, we develop a Monte Carlo simulation technique that can be used in such problems.

4 A MONTE CARLO SIMULATION TECHNIQUE FOR VALUING FLEXIBILITY

Boyle (1977) introduced a Monte Carlo simulation method for asset pricing of European options. This approach, however, cannot be used for problems with switching costs since the problem cannot be treated as a bundle of European options with different expiration dates. Broadie and Glasserman (1997) developed a simulation algorithm for estimating the prices of American-style assets. In order to develop valid error bounds on the true option value, they introduced two estimators, one biased high and one biased low, but both asymptotically unbiased as the computational effort increases. These estimators are based on simulated lattices. The simulated lattices are parameterized by b, the number of branches per node. State variables are simulated at the finite number of possible decision points, i.e., exercise times.

In this section, we propose a Monte Carlo simulation technique that can be used to value a problem with nonzero switching costs, multiple time points to switch between the options, and time lags between the decision and the implementation. In the proposed technique, we use a simulation tree with three branches per node. On this simulation tree, we will apply the dynamic programming algorithm that we used for the lattice in Section 3. For each time point, the best decision for each possible previous policy is determined. During the backward recursion of the dynamic programming, we use the expected profit of the three branches to select the best manufacturing option to be implemented in those branches. In other words, we define the best decision in a node so that the implementation of this decision in the next time interval gives the maximum expected profit. Since the simulated values for the state variables are used for calculations, the dynamic programming takes advantage of knowledge of the future to overestimate the option value. Hence, this estimate is biased high.

In order to find the low estimator, we use the following approach. We use the first branch to determine a policy, and we apply that policy to the third branch. Since we apply the decision to the wrong branch, the expected profit tends to be biased low. We use the average of the high-biased and low-biased simulation estimates to estimate the true value.

To simulate the path followed by e_i, we approximate Equation (1) as

$$e_i(t+\Delta t) = e_i(t)\exp\left[\left(\mu_i - \frac{\sigma_i^2}{2}\right)\Delta t + \sigma_i \varepsilon \sqrt{\Delta t}\right] \qquad (2)$$

Applying this technique on the supply chain network problem that was solved in Section 3 by using a lattice, we obtained the simulation results given in Figure 6. For each initial option, we made 1,000 simulation runs.

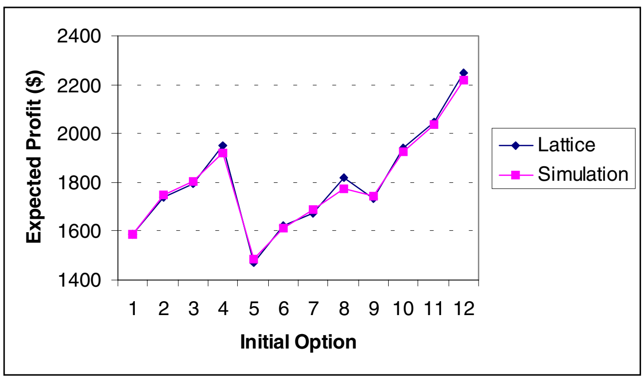

Figure 6: Comparison of Lattice and Simulation Estimates

It has been shown that lattice methods give close results to the analytical solutions (Nembhard, Shi, and Aktan 2002, Boyle 1988, Boyle, Evnine, and Gibbs 1989, Kamrad and Ritchken 1991, Amin and Khanna 1994). We see that the Monte Carlo simulation estimates are close to the lattice estimates, which implies that the proposed Monte Carlo simulation technique yields close results to the analytical solution for this problem.

Many manufacturing system models contain more than two state variables. If there are more than two state variables in the problem, lattice techniques become difficult to apply. If we want to use lattice approach in such problems, structure of the lattice gets too complex and the number of possible jumps at each node gets too large. However, the proposed Monte Carlo simulation technique does not require a different tree structure when the number of state variables increases. The same tree structure is used for any number of state variables; the only difference is the number of random numbers generated. The main advantage of the proposed Monte Carlo simulation approach is its simple application on real options problems with three or more state variables.

5 SUMMARY

In this paper, we have proposed two techniques to value the real options problems in manufacturing enterprises when decisions in the system cannot be implemented immediately. We have presented the application of the proposed technique on a supply chain network with uncertain exchange rates, where costly switching decisions for the suppliers, production plants, and market regions require some time lag to implement.

We proposed lattice and Monte Carlo simulation techniques that can be used to value real options problems with implementation time lags and costly switching decisions. In such problems, there are multiple options. At each time interval, switching between options is possible, and each switch results in a switching cost. We have compared the results of the proposed Monte Carlo simulation technique with the results of the lattice technique. The comparison has shown that the proposed Monte Carlo simulation technique yields close estimates for the true option value.

Real options valuation techniques proposed in this paper give decision makers a way to choose the appropriate manufacturing enterprise strategy based on an integrated view of the market dynamics. Overall, the manufacturing enterprise maximizes its expected discounted profit through effective supply chain network decisions.

APPENDIX

Parameters for the supply chain network:

r = 5% (risk-free interest rate in the home country)
r_1 = 4% (risk-free interest rate in country 1)
r_2 = 2% (risk-free interest rate in country 2)
m_1 =$1.10 (unit raw material cost of supplier 1)
m_2 = $1.00 (unit raw material cost of supplier 1)
c_1 = $1.05 (unit manufacturing cost at plant 1)
c_2 = $1.00 (unit manufacturing cost at plant 2)
R_1 = $2.30 (price of the firm's output in market region 1)
R_2 = $2.2 (price of the firm's output in market region 2)
D_1 =1,100/quarter (demand in market region 1)
D_2 =1,000/quarter (demand in market region 2)
CAP_1 = 2,500/quarter (capacity of plant 1)
CAP_2 = 2,400/quarter (capacity of plant 2)

$\sigma_1 = 0.2$ (volatility for exchange rate of country 1)
$\sigma_2 = 0.133$ (volatility for exchange rate of country 2)
$\rho =$ 0.3 (correlation for the exchange rates)

Table 2: Switching Costs between the Manufacturing Options

From \ To	1	2	3	4	5	6	7	8	9	10	11	12
1	0	100	100	200	80	180	180	280	100	200	200	300
2	110	0	210	100	190	80	290	180	210	100	310	200
3	110	210	0	100	190	290	80	180	210	310	100	200
4	220	110	110	0	300	190	190	80	320	210	210	100
5	80	180	180	280	0	100	100	200	100	200	200	300
6	190	80	290	180	110	0	210	100	210	100	310	200
7	190	290	80	180	110	210	0	110	210	310	100	200
8	300	190	190	80	220	110	110	0	320	210	210	100
9	80	180	180	280	80	180	180	280	0	100	100	200
10	190	80	290	180	190	80	290	180	110	0	210	100
11	190	290	80	180	190	290	80	180	110	210	0	100
12	300	190	190	80	300	190	190	80	220	110	110	0

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