

## MODELING AND PARAMETER ESTIMATION OF SPACECRAFT FUEL SLOSH MODE

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### ABSTRACT

The nutation (wobble) of a spinning spacecraft in the presence of energy dissipation is a well-known problem in dynamics and is of particular concern for space missions. Its rate of growth is characterized by the Nutation Time Constant (NTC). For analytical prediction of the NTC this fuel slosh is often modeled using simple mechanical analogs such as pendulums or rigid rotors coupled to the spacecraft. Identifying model parameter values which adequately represent the sloshing dynamics is the most important step in obtaining an accurate NTC estimate. Currently, the identification of the model parameters is a laborious trial-and-error process in which the equations of motion for the mechanical analog are hand-derived, evaluated, and their results are compared with the experimental results. The current research is an effort to automate the process of identifying the parameters of the slosh model using a MATLAB/SimMechanics based computer simulation of the experimental setup.

### 1 INTRODUCTION

Spinning a spacecraft or an upper stage is a well-established method for stabilizing a space vehicle with a minimum of hardware, complexity, and expense. While spinning a deployed spacecraft over its operational lifetime has generally fallen out of style in favor of the more modern three axis stabilized active systems popular today, there still is a community of users that have to deal with spin stabilized upper stage dynamics. Many NASA and DoD payloads are launched on Boeing Delta II expendable launch vehicles with spinning solid rocket third stages. This particular version of the Delta II has been very popular for NASA interplanetary missions. Because of this, NASA's Expendable Launch Vehicle program office at Kennedy Space Center has been investigating ways to improve their understanding and ability to model spinning upper stage dynamics. While this research work has

important near term applications for expendable launch vehicles, it also has significant implications for NASA's future manned space program. Spinning a large manned vehicle (or perhaps segments of one connected by a long tether) is the only practical way to obtain "artificial gravity". Long duration space missions may require some form of artificial gravity to counteract the effects of extended weightlessness on the human body.

Liquid slosh in the fuel tanks of an attached spacecraft has been a long standing concern for space missions with a spinning upper stage. Loss of rotational kinetic energy through the movement of liquid propellants affects the gyroscopic stability of the combined spacecraft and upper stage. Energy loss leads to an ever increasing wobble or "nutation" which can grow to cause severe control issues (Hubert 2003). The more vigorous the slosh the greater the energy loss and hence the greater the nutation. The "nutation angle" is defined as the angular displacement between the principal axis of rotation of the spacecraft and its angular momentum vector and is a measurement of the magnitude of the nutation (Wertz 1978). The amount of time it takes for the nutation angle to increase by a factor of  $e^l$  is defined as the Nutation Time Constant (NTC), and is a key parameter in assessing the stability of the spinning spacecraft during the upper stage burn. The NTC can sometimes be very difficult to calculate accurately during the early stages of spacecraft design.

There is a high degree of uncertainty in predicting the effect of liquid propellant motion in spinning spacecraft. The resulting nutation growth can be excessive and can pose a threat to the mission. Missions have been lost because of excessive and unanticipated nutation growth (ATS-5 in 1969 being an early example). Purely analytical methods of predicting the influence of onboard liquids have been generally unsatisfactory (Hubert 2003). The NTC values provided analytically are quite often significantly different than actual flight values. Hence, there is a need to identify conditions of resonance between nutational motion and liquid modes and to understand the

general characteristics of the liquid motion that cause the problem in spinning spacecraft. The current research is a first step in trying to understand and model certain modes of induced resonance found during experimental testing and during flight. This study will focus on the modeling of fluid motion and will utilize the results obtained to develop a more accurate prediction of the fuel slosh effects on spin stabilized spacecraft.

During the initial design of spacecraft, use of purely analytical means of predicting the influence of onboard liquids has not worked well. Computational fluid dynamics software packages provide some insight, but it turns out that they have several shortcomings. Their complexity and inability to accurately model the coupling effects of sloshing mass on the six degree-of-freedom motion experienced by the spacecraft make their application problematic.

Liquid oscillations in spinning tanks have been studied in the past. Liquid oscillations in spinning fuel tanks produce very different response characteristics compared to those of non-spinning fuel tanks (Greenspan 1969). An energy sink model was originally developed by Thomson (1961) to include the effects of small, passive sources of energy dissipation. This model does not work well for spacecraft fuel slosh energy dissipation if the liquid mass is a large fraction of the total mass of the spacecraft.

Extensive analysis has been done on the different tank shapes and locations, as well as the use of propellant management devices (PMD). A summary of this analysis, like that reported by Hubert (2001) shows the vast differences in possible behaviors of different designs. For the off-spin-axis-mounted, cylindrical tanks with hemispherical end-caps that have been popular in a number of spacecraft programs, a number of relatively simple mechanical models have been developed. Hubert also notes that one of the most difficult aspects of employing such mechanical models is in the selection of appropriate parameters in the model.

Use of mechanical analogs such as pendulums and rotors to simulate sloshing mass is a common alternative to fluid modeling. A homogeneous vortex model of liquid motions in spinning tanks and an equivalent mechanical rotor model was developed by Dodge et al. (1994). An approximate theory of oscillations that predicts the characteristics of the dominant inertial wave oscillation and the forces and moments on the tank are described. According to Dodge et al., the pendulum model simulates a motion that does not involve an oscillation of the center of mass. Therefore, it is not a valid model of inertial wave oscillations. Weils and Dodge (1991) illustrate that the free surface effects can be ignored when the liquid depth is small.

A 3-D pendulum model was proposed by Green et al. (2002). There was evidence of liquid resonance from the experimental data. The resonance was closely tied to the tangential torque and to a lesser degree to the radial torque,

and there was little or no resonance in the force measurements. Green et al., proposed a rotary oscillator concept to simulate the torque resonance in tangential and radial directions. This rotary oscillator model was superimposed on the pendulum model to provide the overall response of liquid oscillation in the tank.

The current research effort proposed is directed toward modeling fuel slosh on spinning spacecraft using simple pendulum analogs. The pendulum analog will model a spherical tank with no PMD's. An electric motor will induce the motion of the pendulum to simulate free surface slosh. Parameters describing the simple pendulum models will characterize the modal frequency of the free surface sloshing motion. The one degree of freedom model will help to understand fuel sloshing and serve as a stepping stone for future more complex simulations to predict the NTC accurately with less time and effort.

## 2 PROBLEM DEFINITION

Various simulation parameters are estimated by matching the pendulum/rotor model response to the experimental response of full sized test tanks in NASA's Spinning Slosh Test Rig (SSTR) located at the Southwest Research Institute (SwRI) in San Antonio, Texas. The experimental set-up of the SSTR is shown in Figure 1. The SSTR can subject a test tank to a realistic nutation motion, in which the spin rate and the nutation frequency can be varied independently, with the spin rate chosen to create a centrifugal acceleration large enough to ensure that the configuration of the bladder (PMD) and liquid in the tank is nearly identical to the zero-g configuration. A complete description of the actual tests, data acquisition and analyses of data for the Contour mission is provided by Green, et al. (2002). The propellant motion is simulated using models with various parameters (inertia, springs, dampers, etc.) and the problem reduces to a parameter estimation problem to match the experimental results obtained from the SSTR.

The SSTR can accommodate a full-sized fuel tank complete with any internal PMD for testing. The SSTR measures and records the force and torque response of the fuel tank to the internal slosh motion of the propellant. It has the capability to identify and characterize slosh resonances. The data from the tests are used to derive model parameters that are then used in the slosh blocks of a MATLAB/SimMechanics-based spacecraft and upper stage simulation. Currently the identification of the model parameters is a laborious trial-and-error process in which the equations of motion for the mechanical analog are hand-derived, evaluated, and compared with the experimental results.

The current research is an effort to automate the process of slosh model parameter identification using a MATLAB/SimMechanics-based computer simulation of the experimental SSTR setup. Different parameter estimation and optimization approaches are being evaluated

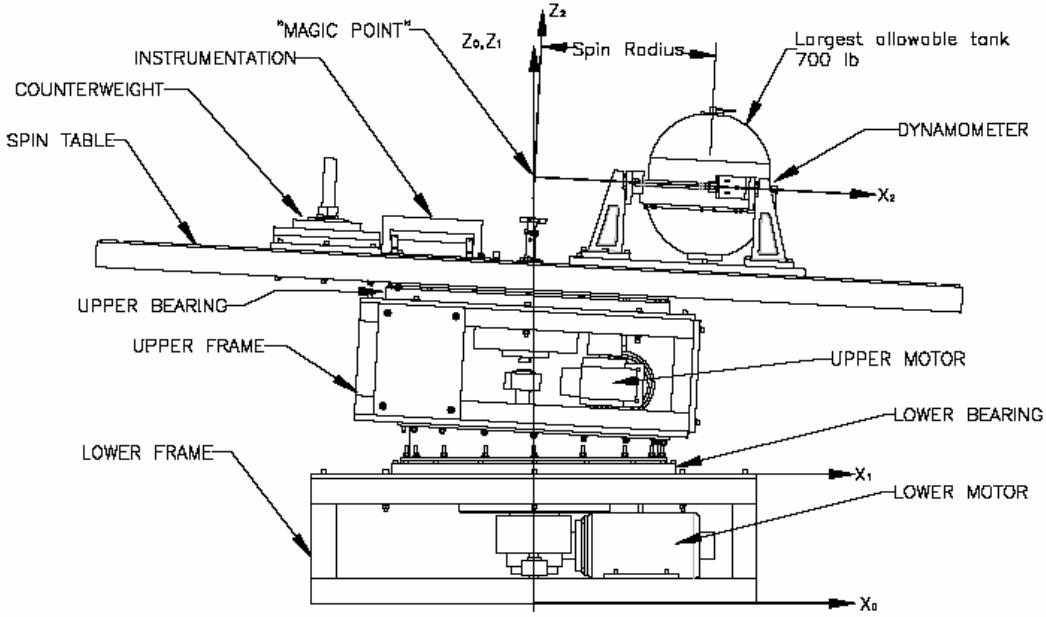


Figure 1: Schematic Diagram of SSTR

and compared in order to arrive at a reliable and effective parameter identification process. To evaluate each parameter identification approach, a simple one-degree-of-freedom pendulum experiment is being constructed and motion will be induced by an electric motor. By applying the estimation approach to a simple system with known characteristics, its effectiveness and accuracy can be evaluated. The same experimental setup can then be used with fluid-filled tanks to further evaluate the effectiveness of the process. Ultimately, the proven process can be applied to the full sized SSTR setup to quickly and accurately determine the slosh model parameters for a particular spacecraft mission.

The problem with modeling the complete SSTR as a starting point is that there is considerable complexity in the SSTR machine itself. By reducing the problem to that seen in Figure 2, a better understanding can be made of the effectiveness of the optimization and estimation approaches and to the fundamental slosh behaviors of the liquid without having to model all of the complexity of the SSTR first (Gangadharan et al. 1991).

The fixed mass represents the amount of propellant that is not undergoing free surface slosh while the free surface fuel slosh mass is represented by the mass attached to the pendulum. This is discussed further in the following section. Motion of the pendulum analog is induced by an electric motor attached to a locomotive assembly. This robust setup can yield very accurate sustained frequencies. Forces are measured on a Sensotek model 31 force transducer rated at +/- 5 lbs.

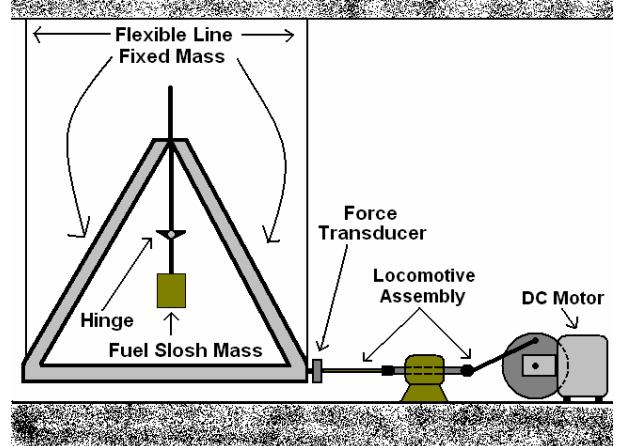


Figure 2: Schematic Diagram of 1-DOF Pendulum Analog

### 3 METHOD OF APPROACH

A spherical tank with no PMD's undergoing free surface slosh is the simplified model for the pendulum parameters. Free surface slosh has a well defined resonant frequency (Hubert 2003). The only sloshing motion assumed to be taking place in this simplified model is a surface wave. The rest of the liquid is essentially at rest and can be treated like a fixed mass. Initial pendulum properties are found by the use of a program developed by Dodge at SwRI. This "Dodge" code predicts the modes of the fuel tank with that of a pendulum. The tank/fuel parameters such as shape, kinematic viscosity, and liquid fill level are provided as

input to the program. An illustration of the tank/pendulum definition along with values for various pendulum parameters for an eight inch spherical tank is in Figure 3.

Using the tank/fuel parameters, the code can then determine the proper pendulum equivalent. The physical parameters given by the code include the liquid's fixed and pendulum masses as well as the pendulum length. First and second mode slosh data is also given by the code. The first mode parameters (sloshing mass) represent the majority of the propellant undergoing free surface slosh while the second mode represents a small correction factor for the first mode. This correction factor is an order of magnitude smaller than the first mode. Therefore, it can be added to the first mode mass with minimal error.

After running the code for several fill levels, several plots can be created from the data. Figure 4 shows the mass distribution for various fill levels for an 8 inch sphere with water as the liquid. For laboratory testing, water is an excellent and frequently used substitute for hazardous propellants. Water's fluid properties (density, viscosity, etc.) are nearly identical to those of hydrazine, the most commonly used propellant. The code predicts that the maximum sloshing mass will occur at approximately 60% fill level. Figure 5 indicates the various pendulum lengths that are required for different fill levels.

While the simulation will test this entire range of fill levels, the 1-DOF pendulum experiment will be limited to test from 60-90% fill levels. This is due to the fixed mass

constraint of the pendulum frame for the lower end and the mass of the pendulum's hardware for the upper end. For the tank experiment, a much wider range will be able to be tested due to the tank being light weight.

Using the code's data distributions along with the geometric/material characteristics obtained from the experimental setup (Figure 2), a computer simulation of the one DOF pendulum analog can be developed using SimMechanics software (Wood and Kennedy, 2003) as illustrated in Figures 7 and 8. Each of the different parts of the simulation model is located in one of the following four groups:

1. This group simulates the electric motor and locomotive arm assembly. The locative arm consists of five different parts starting with the DC motor. These are the flywheel, flywheel linkage, locomotive arm (piston), and the "stinger" as illustrated in Figure 6. The "stinger" is a flexible metal rod designed to absorb forces that are not coincident with the axis of the locomotive arm. The rest of the locomotive arm assembly is assumed to be ridged. Geometric parameters such as component mass and moments of inertia are fixed in this group. Desired Frequencies and simulation running time are also input parameters for this group.

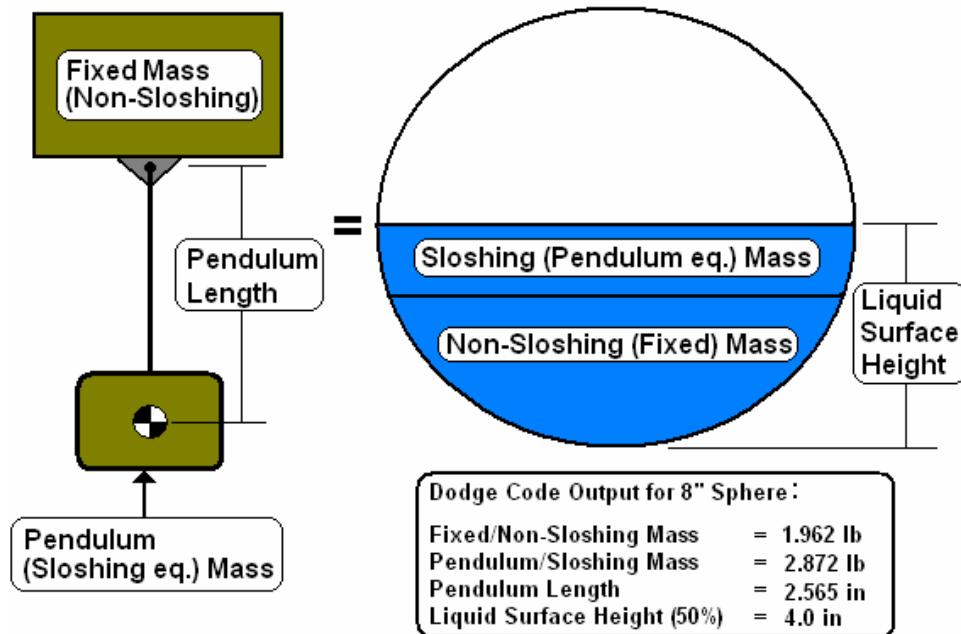


Figure 3: Dodge Code Pendulum/Tank Equivalent and Data

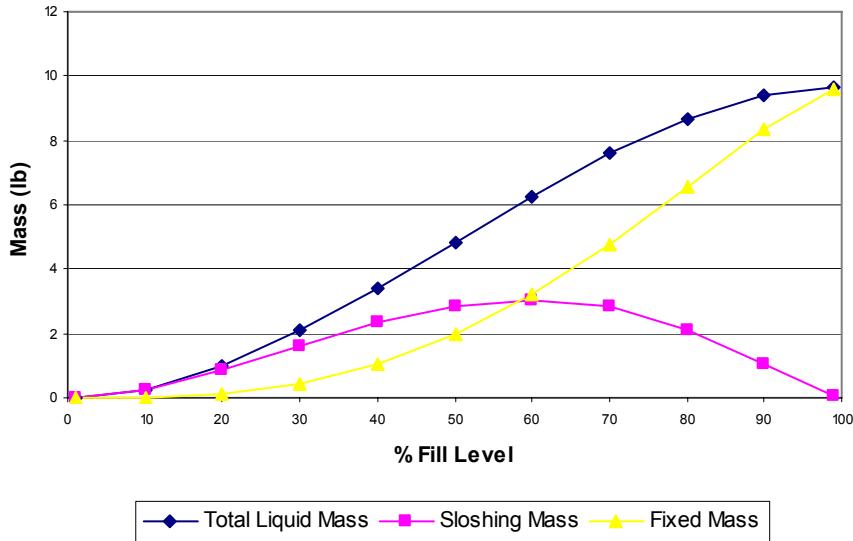


Figure 4: Liquid Mass Distribution for 8" Sphere

#### Pendulum Geometry Information for 8" Sphere

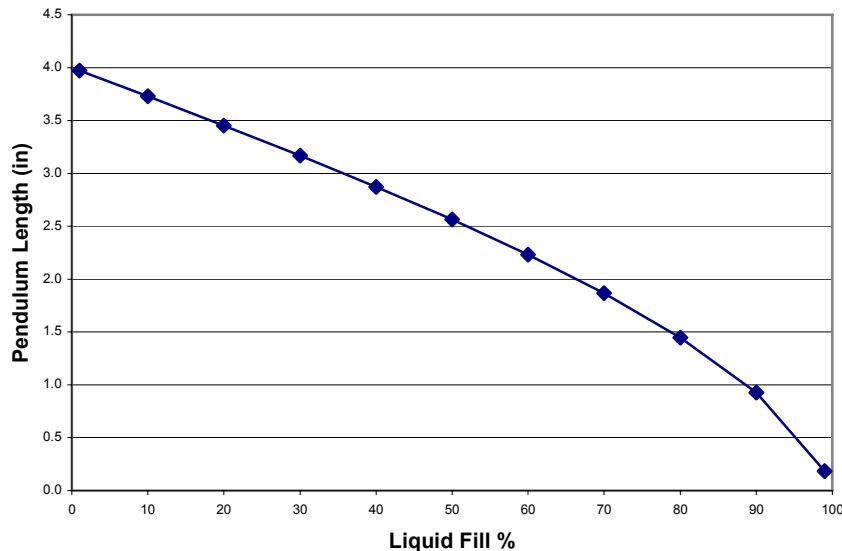


Figure 5: Pendulum Geometry for 8" Sphere

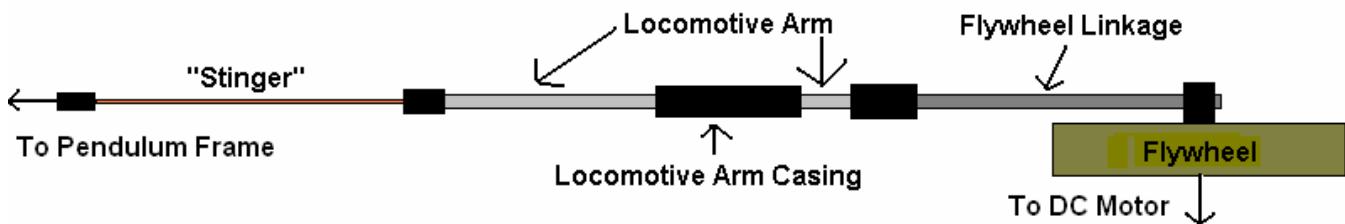


Figure 6: Group 1 Assembly

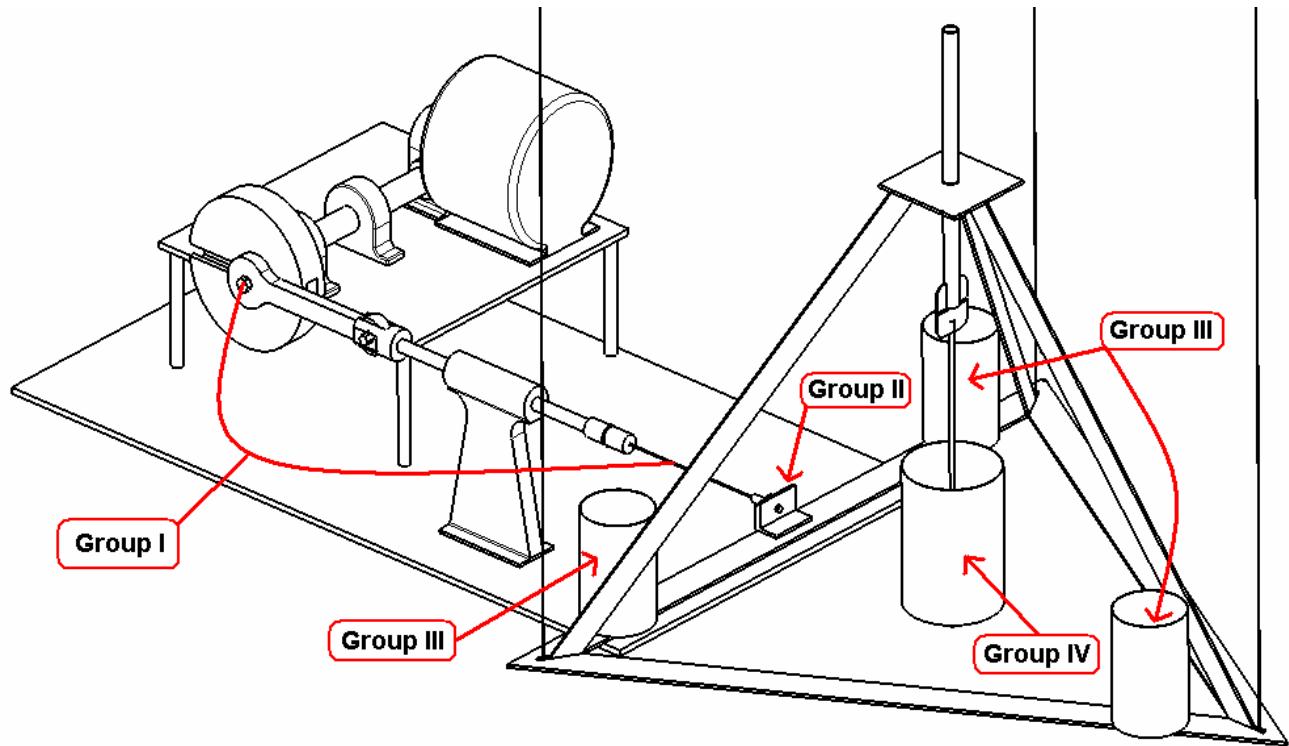


Figure 7: Groups 1-4 Locations

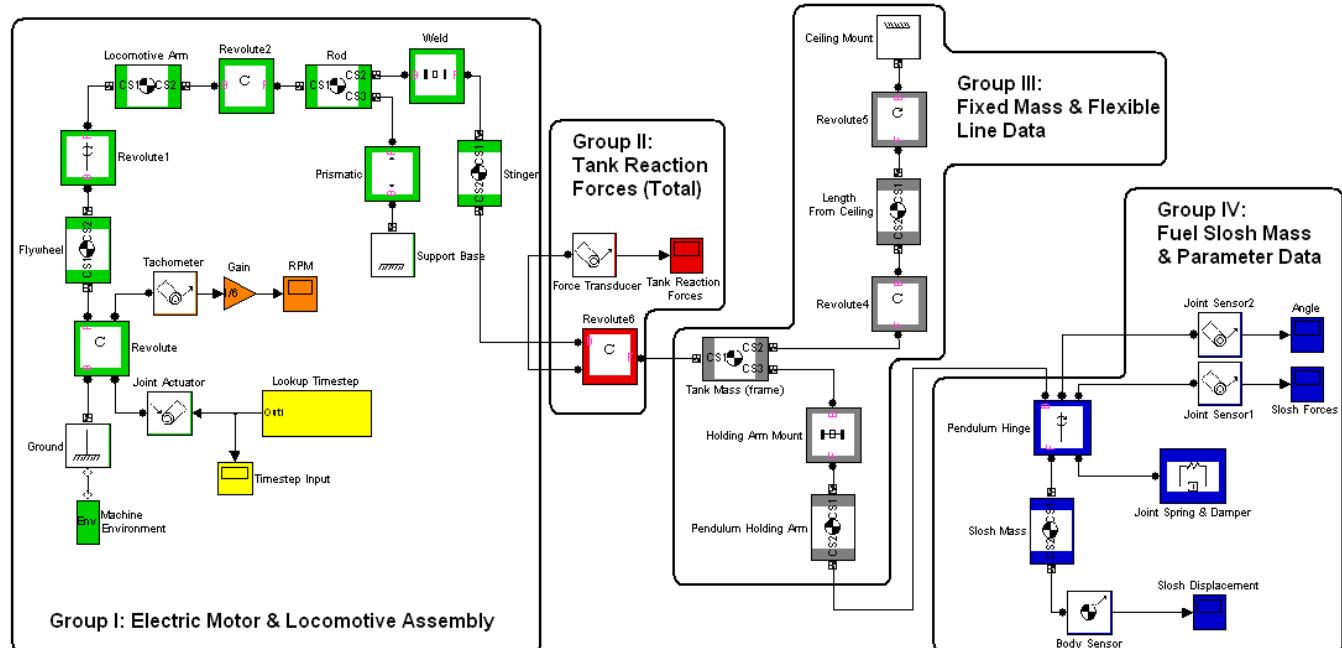


Figure 8: SimMechanics Model of 1-DOF Pendulum

2. The location of the force transducer in Figure 7 is represented by this group. The force transducer connects the pendulum frame to the locomotive arm assembly. The total forces of the fixed and sloshing masses are recorded in this group.
3. Parameters in this group include non-sloshing fuel mass assumed to be fixed along with its geometric properties of the frame assembly illustrated in Figure 7. The frame is constructed out of 1/8" aluminum and weighs of 1.46 kg.
4. Group four simulates the sloshing fuel and is considered to be critical in the parameter estimation process. Fuel mass, hinge damping, and pendulum length are all parameters in this group.

As a proof-of-concept, results from this SimMechanics simulation incorporating the one DOF pendulum analog matches those predicted by the Dodge code for a 60% fill level for a 12 inch sphere as shown in Figures 9 and 10. Figure 9 is a force simulation that would be experienced by the force transducer in the experiment. The simulated locomotive assembly (Group I) is driven by parameters determined by the user. At the start of the simulation ( $t=0s$ ), the frequency is 0.5 hertz (30 RPM) and at time equal to 500 seconds, the frequency is equal to 3.0 hertz (180 RPM). At approximately 250 seconds, the pendulum reaches its first mode at a frequency of approximately 1.71 hertz. This frequency matches the natural frequency prediction of the "Dodge" code. The natural frequency is inversely proportional to the square root of the length of the pendulum. For the case of a 60% fill level, the length is 3.35 in.

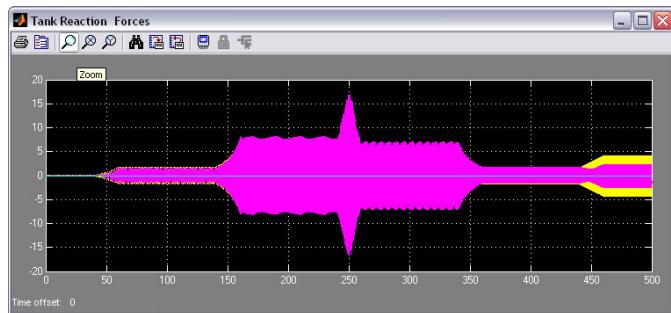


Figure 9: SimMechanics Output for Sphere at 60% Fill Level. [force (lb) vs. time (s)]

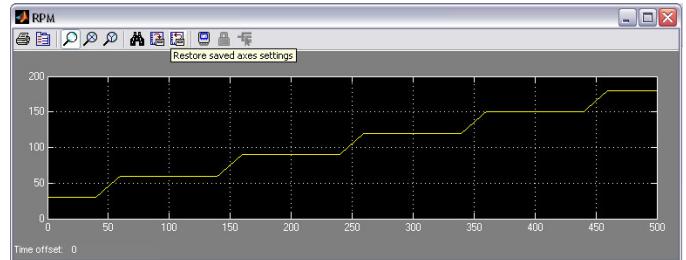


Figure 10: SimMechanics Output for Sphere at 60% Fill Level. [rotational speed (RPM) vs. time (s)]

After confidence is gained with a 1 degree of freedom pendulum model, a spherical tank with propellant properties which match the Dodge parameters (water in this case) will then be tested experimentally to verify the validity of the pendulum analog as shown in Figure 11. The block diagram of the parameter estimation process is illustrated in Figure 12.

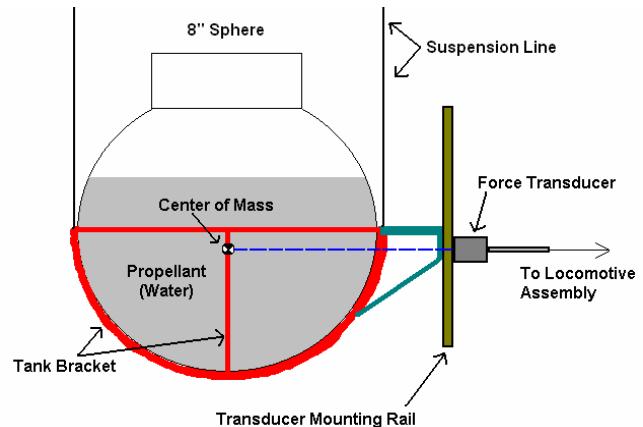


Figure 11: Schematic Diagram of 1-DOF Fuel Analog

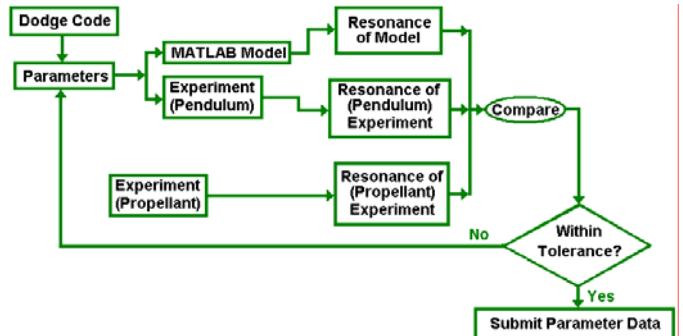


Figure 12: Block Diagram of the Parameter Estimation Process

#### 4 PRELIMINARY DATA

Currently, the experiment and the simulation are being calibrated using fixed masses. This is to verify that the model is accurately representing the experiment's way of oscillating the pendulum frame using a flywheel and locomotive arm. Examples comparing the simulation and experimental data are located on the following page in Figures 13 and 14. As seen from the examples, the main problem in comparing the two data sets is the noise in the experimental data. Currently, several methods of filtering the data are being considered so that it closely matches the simulation data. Once this is achieved, the automated parameter estimation approach can be applied.

#### 5 CONCLUSION

The effects of fuel slosh aboard spinning spacecraft need to be accurately predicted to avoid mission failures. Using a combination of test derived fuel slosh parameters and computer simulations of the spacecraft dynamics, an improvement in the current ability to make predictions of NTC can be achieved. By applying the parameter estimation approach to a simple, accurately modeled system during initial stages of design, its effectiveness and accuracy can be evaluated. The same experimental setup can then be used with fluid-filled tanks to further evaluate the effectiveness of the process. Ultimately, the proven process can be applied to the full sized spinning

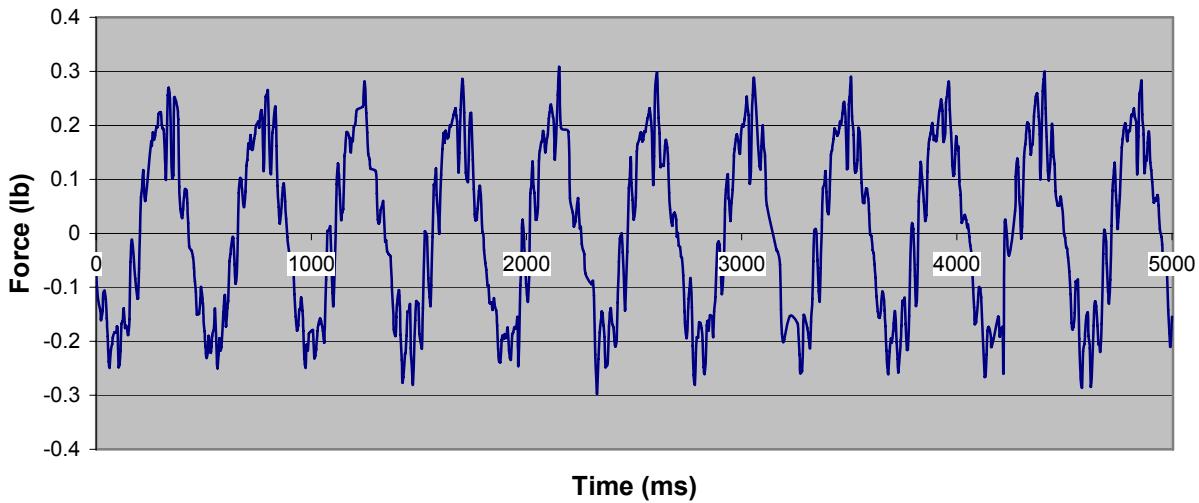


Figure 13: Non-Sloshing Data (Experiment) for 8" Sphere at 2.0 Hz at 80% Fill Level  
Mass = 6.54 lb Length = 0.155 in

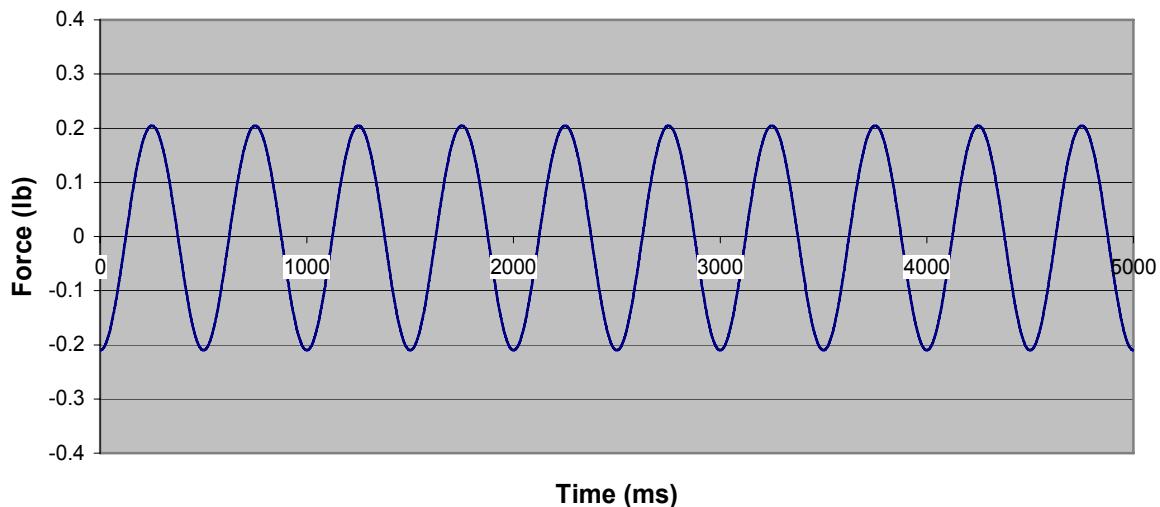


Figure 14: Non-Sloshing Data (Simulation) for 8" Sphere at 2.0 Hz at 80% Fill Level  
Mass = 6.54 lb Length = 0.155 in

experimental setup to quickly and accurately determine the slosh model parameters for a particular spacecraft mission. Automating the parameter identification process will save time and thus allow earlier identification of potential vehicle performance problems. This, in turn, can reduce the cost and schedule penalty associated with needed design changes. Applications of an automated process to find the NTC will benefit all space exploration missions involving a spinning spacecraft. Today, all spinning spacecraft are used for unmanned missions. In the future, manned space exploration missions involving artificial gravity will greatly benefit from the automated parameter identification process. Understanding and being able to confidently predict the stability of the spinning human habitat will be crucial for the success of the manned mission.

## 6 ACKNOWLEDGMENTS

The authors would like to thank ELV Mission Analysis Branch at NASA Kennedy Space Center. Special thanks go to Bora Eryilmaz (The MathWorks, Inc.) and Raphael T. Haftka (University Distinguished Professor at University of Florida) for their guidance and help in this research project.

## REFERENCES

- Dodge, F.T., *Fuel slosh in asymmetrical tank software*.  
Dodge, F.T., Unruh, J.F., Green, S.T., Cruse, M.W., 1994. *A Mechanical Model of Liquid Inertial Waves for Use with Spinning Craft*, Fluid Transients, FED-Vol. 198/PVP-Vol. 291, ASME  
Gangadharan, S.N., Nikolaidis, E., Haftka, R.T. 1991 *Probabilistic system identification of flexible joint models*, AIAA Journal.  
Green, S., Burkey, R., Dodge, F., and Walter, D., Nutation time constant model parameters for the contour spacecraft. Final Report SwRI Project#18.04617 2002.  
Greenspan, H.P. 1969. *The theory of rotating fluids*, Cambridge University Press, Cambridge  
Hubert, C., 2003. Behavior of spinning space vehicles with onboard liquids, Hubert Astronautics,.  
Hubert, C., 2001. Introduction to the dynamics and control of spinning space vehicles, Hubert Astronautics.  
MATLAB Software, The MathWorks Corporation.  
Thomson, W.T. 1961. *Introduction to space dynamics*, John Wiley, New York.  
Weihs, D., and Dodge, F.T. 1991. Liquid motions in nonaxisymmetric, partially filled containers rotating at zero gravity, *AIAA Journal of Spacecraft and Rockets*, Vol. 28.
- Wertz, James R., editor. 1978 *Spacecraft attitude determination and control*, ISBN 90-277-0959-9  
Wood, G.D., and Kennedy, D.C. 2003 *Simulating mechanical systems in Simulink with SimMechanics*, The Mathworks report.

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