

## OPTIMAL LOT-SIZING IN A TWO-STAGE SYSTEM WITH AUTO-CORRELATED ARRIVALS

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### ABSTRACT

Most research on lot-size optimization has concentrated on single-stage batch production systems. However, in practice it is of interest to optimize performance over multiple processing stages, where stages are not independent. The models in this study consider two stages, where multiple products are produced using the processing stages sequentially. The objective is to minimize total lot flow times across both stages by selecting the optimal lot sizes for each product, subject to these remaining constant across both stages. Analytical relationships can be developed but rely on the assumption of lot interarrival time independence. This is clearly not the case in most manufacturing problems. Better solutions can be obtained experimentally using simulation and response surface methods. Alternatively, an approach has been developed that allows dynamic feedback to be used in adjusting analytical relations to compensate for auto-correlation. Results using this approach compare well with those obtained experimentally.

### 1 INTRODUCTION

Since the lot-sizing problem is fundamental to batch production system performance, lot-sizing approaches to guide manufacturing practise are highly desirable. Lot sizes can be treated as decision variables if queuing theory is used to describe the behavior of lots of parts flowing through a capacity-constrained system. It is then of primary interest to determine lot sizes that will minimize flow times and work-in-process (WIP) inventory, thus enhancing the performance of the manufacturing system.

The objectives of this research are three fold. The first objective is to demonstrate an experimental approach for determining optimal lot sizes using discrete-event simulation and response surface methods. A two-stage system processing multiple product types is assumed. The second objective is focused on investigating analytical models for predicting lot flow times and determining lot sizes that will minimize average flow times. Lot interarrivals are de-

scribed by the first two moments of a general distribution, with lot interarrival times assumed to be independent. Lot service times are also described by the first two moments of a general distribution, where service times include a setup time for each new lot of parts and a processing time based on the lot size. Analytical lot size optimization may be used in attempting to minimize average lot flow times across multiple stages. However, performance is shown to be poor when compared to experimental results. This is due to auto-correlation effects. The final objective is to demonstrate a methodology that overcomes the limitations of assuming independent lot interarrival times. The approach is to use dynamic performance feedback to adjust queuing relationships so auto-correlation is compensated for. This methodology is an extension to the recent work by Enns and Li (2004) on single-stage problems.

### 2 LITERATURE REVIEW

Since the 1980's, several researchers have dealt with stochastic lot interarrival times and the prediction of lot queue or flow times. Karmarkar was one of the earliest to examine the effects of lot-sizing policies using queuing models. Several papers were published describing the impact of the lot sizes on flow times and work-in-process (WIP) inventory. Analysis of the single-product, single-stage problem (Karmarkar, 1987) dealt primarily with  $M/M/1$  and  $M/G/1$  queuing assumptions. Later extensions included multiple product, multiple stage models.

The single-stage lot sizing problem of most relevance to manufacturing is one that allows general interarrival time assumptions. Only approximate  $GI/G/1$  queuing relationships can be used since no closed form solution exists. These approximations are based on the mean and variance of the interarrival and service time distributions. Examples can be found in Whitt (1983) and Buzacott and Shanthikumar (1993). When the entities in queue are considered to be lots of parts, the lot size can be considered a decision variable.

Lambrecht and Vandaele (1996) derived the approximate relationships between lot size and expected flow times for a single product, single stage  $GI/G/1$  model. Optimal lot sizes were determined using a steepest-descent algorithm. As well, the variance of lot flow times was considered. Lambrecht, Ivens and Vandaele (1998) extended the development of this lot-sizing approach, as part of a scheduling procedure called ACLIPS, to multiple product types moving through multiple stages.

Enns and Choi (2002) derived sets of equations, based on differentiation, to obtain optimal lot sizes for  $GI/G/1$  approximations where multiple products were considered. These relationships were used to study the performance of a production system using Material Requirements Planning (MRP). Fowler, et. al (2002) investigated lot-size optimization in a multiple product, multiple stage production environment through the use of queuing relationships and Genetic Algorithm search techniques.

Finally, Enns and Li (2004) considered the problem of auto-correlation between lot interarrival times in manufacturing systems. It was demonstrated that lot size optimization based on  $GI/G/1$  assumptions worked poorly in a multiple product, single stage environment where lot arrivals were derived from accumulated independent customer demand. A methodology was developed that used dynamic performance feedback to adjust queuing relationships so as to compensate for auto-correlation effects.

### 3 A SIMPLE TWO-STAGE MODEL

This research extends the work of Enns and Li (2004) to consider optimal lot sizing across multiple serial stages. A simple problem environment was developed to facilitate experimentation, as shown in Figure 1. There are two outlets at which individual customer orders are placed, with each outlet providing a different product. The customer order interarrival times at the outlets are assumed to have a coefficient of variation of  $c_{c,j}$ , where  $j$  is the stock keeping unit (SKU) or product type. These customer orders are batched until a quantity of  $Q_j$  orders have been received. The lot-size orders are then released and take some time to arrive at the first capacity-constrained resource. This order placement delay has a coefficient of variation of  $c_{o,j}$ . When the orders are received, they are placed in queue. The merged arrival stream for the orders to this first stage has a coefficient of variation of  $c_{a,1}$ . The lots at the first stage, designated as  $i$  equal to 1, are processed in first-come-first-serve (FCFS) order.

When all the parts in the lot have been processed, the lot is shipped to a second stage, designated as  $i$  equal 2, for an additional processing step. A transit delay is incurred during shipping. The processing of each lot-size order at each resource requires a setup time and a processing time for each unit in the order. The coefficient of variation of lot-size service times is designated as  $c_{s,i}$ . Once the lot-size

orders have been processed at both stages, they are shipped to the outlet that placed the order. The coefficient of variation of lot interdeparture times is designated as  $c_{d,i}$ , while the coefficient of variation of lot transit times is shown as  $c_{fj}$ .

The objective is to determine lot size quantities,  $Q_j$ , that will minimize the replenishment time, defined as the time from when a lot-size order is placed to the time it is received. If the lot sizes are too small, there will be too many setups incurred at the capacity-constrained resources. Utilization will be high and long queue times will result. If the lot sizes are too large, the resources will be committed to one product type for too long and other orders will have to wait longer than necessary, causing excessive average lot flow times.

In the scenario considered, all order and lot transit times are assumed to be independent of the lot sizes and therefore have no effect on minimizing replenishment cycle times. However, since these transit time delays are stochastic, the distribution of these times will affect the variability of lot interarrival times to the queues. In other words, as the coefficient of variation,  $c_{a,i}$ , increases due to more variable transit times, the average queuing delays will also increase. Since transit times affect the choice of optimal lot sizes only through  $c_{a,i}$ , the optimal lot sizes are simply those that minimize the average weighted lot flow times at the capacity-constrained resources, defined as the sum of queue times and lot processing times.

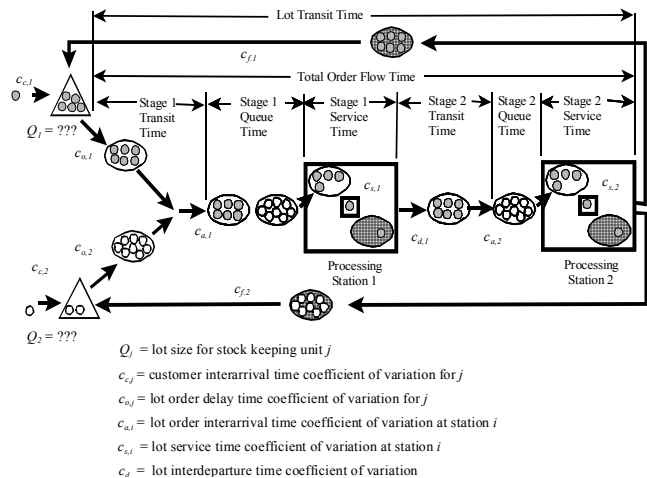


Figure 1: Diagram of Experimental Scenario

In this paper the problem is illustrated using an example based on the following set of assumptions. The mean demand rates for the product types at each outlet,  $D_1$  and  $D_2$ , are assumed to be 44 and 50 units per period, respectively. The customer order interarrivals are described by a Poisson process, with each order quantity being 1. Therefore, the coefficients of variation for the customer interarrival times,  $c_{c,j}$ , are 1. However, the lot-size order releases

will have a lower coefficient of variation and will be auto-correlated if the lot size exceeds 1. The order placement delays are all assumed to be lognormal, with a mean of 5 periods and a standard deviation of 1. Therefore, the  $c_{o,j}$  values are 0.20.

The setup times at the first stage,  $\tau_{1,1}$  and  $\tau_{1,2}$ , are assumed to be 0.30 and 0.20 periods per lot for product types 1 and 2 respectively, while the part processing rates,  $P_{1,1}$  and  $P_{1,2}$ , are 120 and 140 units per period, respectively. These setup and processing times were considered to be deterministic. Once a lot of parts is completed at the first stage, it is transported to the second stage. The transit time distributions for lots of both product types traveling to the second stage are assumed to be lognormal; each with a mean of 2.0 and a standard deviation of 0.6. The setup times at the second stage,  $\tau_{2,1}$  and  $\tau_{2,2}$ , are assumed to be 0.25 and 0.30 periods per lot for product types 1 and 2 respectively, while the part processing rates,  $P_{2,1}$  and  $P_{2,2}$ , are 150 and 110 units per period, respectively.

#### 4 EXPERIMENTAL LOT-SIZE OPTIMIZATION

The first stage in analysis is to determine the optimal lot size combination,  $Q_1$  and  $Q_2$ , experimentally using discrete-event simulation and response surface methods (RSM). A two-stage simulation model was developed using ARENA 5.0 (Kelton, et al., 2002). Experiments were then run using the central composite design (CCD) shown in Figure 2. A total of 26 runs were made, using the number of replications at each design point indicated within brackets in Figure 2. A warmup of 1000 periods was used to reach steady-state conditions, with data collection continuing for 100,000 periods in each run.

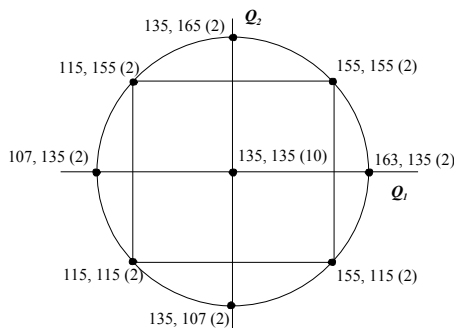


Figure 2: Central Composite Design

The average total lot flow time results at the capacity-constrained resources were analyzed using Design Expert® 6.0. A quadratic model, shown as Equation (1), fit the results nicely. Lack-of-fit was not significant at the 95% confidence level and the resulting model had an  $R^2$  value of 97.4%. Figure 3 shows a normal probability plot of the residuals.

$$W = 9.317 - 3.337(E - 02)Q_1 - 4.275(E - 02)Q_2 + 7.183(E - 05)Q_1^2 + 10.603(E - 05)Q_2^2 + 10.702(E - 05)Q_1Q_2 \quad (1)$$

The response surface generated by this model is shown in Figure 4. It is obviously quite flat along some lines of orientation through the optimal, indicating there will be a variety of lot size combinations performing well. Figure 5 shows a contour plot of the surface around the optimal.

The optimizer in Design Expert® was used to determine the best lot size combinations to minimize the mean lot flow times (Montgomery, 2001). These values were found to be 132 and 135, with a predicted average total flow time of 4.23 periods per lot. This optimal lot size combination, was used in running five additional replications. The average total lot flow time was 4.233, with average flow times of 2.213 and 2.020 at stages 1 and 2 respectively. The observed  $c_{a,i}$  values were 0.690 and 0.567 respectively. The utilization levels at stages 1 and 2 were 0.898 and 0.942 respectively.

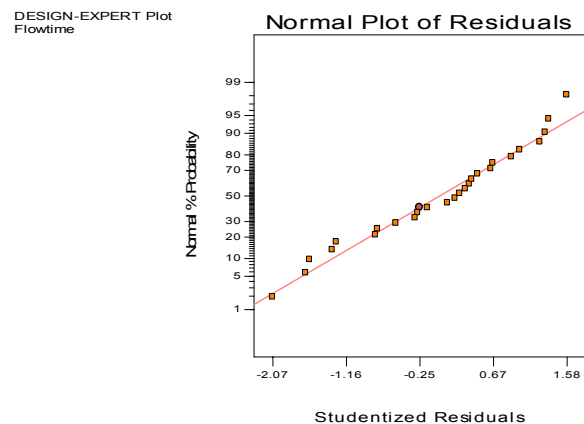


Figure 3: Normal Probability Plot of Residuals

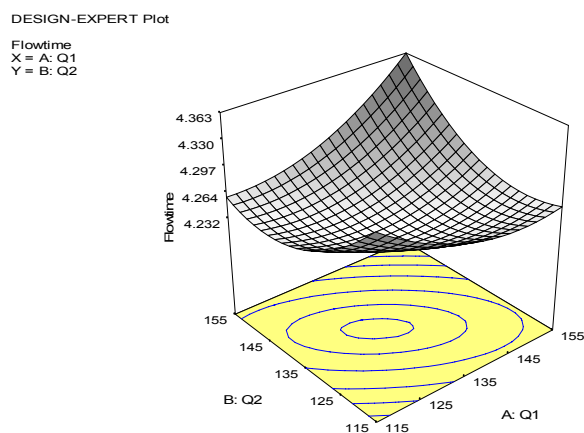


Figure 4: Response Surface for Flow Times

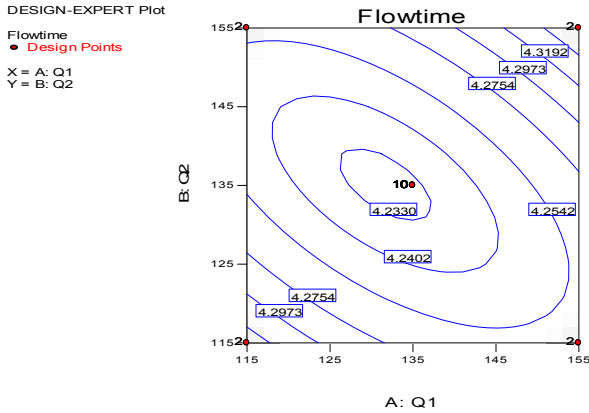


Figure 5: Contour Plot of Lot Flow Times

### 5 GI/G/1 LOT-SIZE OPTIMIZATION

Approximate lot-sizing relationships to minimize mean lot queue or flow times have been developed in previous research. However, these are based on restrictive assumptions about the interarrival times. If general interarrival time distributions are used, they are usually based on the assumption of independent arrivals. This is clearly not the case in many applications involving lot sizes. Therefore, it is necessary to evaluate if these relationships still provide reasonable flow time estimates. Good estimates of flow times facilitate finding optimal lot sizes to minimize flow times.

When the lot interarrival time distribution is assumed to be general, it is usually satisfactory to describe it in terms of the first two moments; the mean and standard deviation. In this case, the following approximations can be used to estimate mean flow times,  $W_{Tot}$ , across a two-stage system.

$$W_{Tot,t} = W_{1,t} + W_{2,t} = \sum_{i=1}^m W_{i,t} \quad (2)$$

For each stage  $i$ , the following flow time approximation can then be substituted.

$$W_{i,t} = W_{q(i,t)} + \bar{x}_{i,t} = \bar{x}_{i,t} \frac{(c_{a(i,t)}^2 + c_{s(i,t)}^2)}{2} \frac{\rho_{i,t}}{1 - \rho_{i,t}} + \bar{x}_{i,t} \quad (3)$$

where  $W_{q(i,t)}$  is the weighted mean queue time at stage  $i$ ,  $\bar{x}_{i,t}$  is the weighted mean lot service time,  $c_{a(i,t)}$  is the coefficient of variation for lot interarrival times,  $c_{s(i,t)}$  is the coefficient of variation for lot service times and  $\rho_{i,t}$  is the utilization rate. If the parameters of the system are allowed to change through time, the subscript  $t$  indicates the values are for the current time. These relationships are based on steady-state GI/G/1 queuing assumptions, which means the interarrival times are independent.

When the entities in queue represent lots of parts, the weighted mean lot service time, including setup times, for  $n$  product types processed at stage  $i$  is given by the following,

$$\bar{x}_{i,t} = \frac{\sum_{j=1}^n \frac{D_j}{Q_{j,t}} \left[ \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right]}{\sum_{j=1}^n \frac{D_j}{Q_{j,t}}} \quad (4)$$

where  $j$  is the product type index,  $D_j$  is the demand rate,  $Q_j$  is the product type lot size,  $P_{i,j}$  is the part processing rate at resource  $i$ , and  $\tau_{i,j}$  is the lot setup time at resource  $i$ .

The utilization rate for the stage  $i$  resource, including setup times, is then given by the following,

$$\rho_{i,t} = \sum_{j=1}^n \left[ \frac{D_j}{Q_{j,t}} \left( \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right) \right] \quad (5)$$

This value is constrained to be less than 1 under steady-state conditions.

If it is assumed the lot setup times and part processing times are deterministic, the squared coefficient of variation for the lot service times at each stage  $i$  is expressed as follows,

$$c_{s(i,t)}^2 = \frac{\sum_{j=1}^n \frac{D_j}{Q_{j,t}} \left[ \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right]^2 \left( \sum_{j=1}^n \frac{D_j}{Q_{j,t}} \right)^{-1}}{\bar{x}_{i,t}^2} - 1 \quad (6)$$

The objective is to solve for the lot sizes,  $Q_j^*$ , that will minimize the average total lot flow times. This can be done by taking the partial differential of Equation (2) with respect to each  $Q_j$ , setting the resulting set of equations equal to 0 and then solving them simultaneously for  $Q_j^*$  (Enns and Choi, 2002). The resulting set of equations is shown as Equation (7).

$$\begin{aligned} \frac{\partial(W_{Tot,t})}{\partial Q_{j,t}} &= \frac{\partial(W_{1,t})}{\partial Q_{j,t}} + \frac{\partial(W_{2,t})}{\partial Q_{j,t}} \\ &= \sum_{i=1}^2 (AA_{i,t} + BB_{i,t} + CC_{i,t} + DD_{i,t}) \end{aligned} \quad (7)$$

where,

$$AA_{i,t} = \frac{\left\{ \frac{2\rho_{i,t} \left( \frac{-D_j \tau_{i,j}}{Q_{j,t}^2} \right) \sum_{j=1}^n \frac{D_j}{Q_{j,t}} + \rho_{i,t}^2 \frac{D_j}{Q_{j,t}^2}}{\left( \sum_{j=1}^n \frac{D_j}{Q_{j,t}} \right)^2} \right\} (c_{a(i,t)}^2 - 1)}{[2(1 - \rho_{i,t})]^2},$$

$$BB_{i,t} = \frac{\left[ \frac{-D_j}{Q_{j,t}^2} \left( \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right)^2 + 2 \frac{D_j}{Q_{j,t} P_{i,j}} \left( \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right) \right] 2(1 - \rho_{i,t})}{[2(1 - \rho_{i,t})]^2},$$

$$CC_{i,t} = - \frac{\left\{ \frac{\rho_{i,t}^2}{\left( \sum_{j=1}^n \frac{D_j}{Q_{j,t}} \right)} (c_{a(i,t)}^2 - 1) + \sum_{j=1}^n \frac{D_j}{Q_{j,t}} \left( \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right)^2 \frac{D_j \tau_{i,j}}{Q_{j,t}^2} \right\}}{[2(1 - \rho_{i,t})]^2},$$

$$DD_{i,t} = \frac{\left[ \frac{D_j}{Q_{j,t} P_{i,j}} - \frac{D_j}{Q_{j,t}^2} \left( \tau_{i,j} + \frac{Q_{j,t}}{P_{i,j}} \right) \right] \sum_{j=1}^n \frac{D_j}{Q_{j,t}} + \rho_{i,t} \frac{D_j}{Q_{j,t}^2}}{\left( \sum_{j=1}^n \frac{D_j}{Q_{j,t}} \right)^2}.$$

This nonlinear set of equations can be solved readily using various software, such as the Solver in Excel®.

When the simulation was run with the optimal lot sizes based on the response surface model developed previously,  $c_{a,i}$  values of 0.690 and 0.567 were observed. If these values are used in Equation (7), the best lot sizes are found to be 185 and 161, with a predicted total lot flow time of 7.997. These lot sizes are significantly larger than the optimal lot sizes of 132 and 135 obtained experimentally in the previous section. Therefore, it can be concluded that violating the assumption of independence cannot be ignored when optimizing lot sizes with auto-correlated lot interarrival times.

While it is possible to obtain optimal lot sizes experimentally, this is impractical in real batch production environments. As well, the present analysis reveals it is insufficient to simply observe the lot interarrival time coefficients of variation,  $c_{a,i}$ , and compute the optimal lot sizes using *GI/G/1* relationships. Therefore, development of a practical approach that uses queuing relationships but takes auto-correlation into account is desirable.

## 6 DYNAMIC LOT-SIZE OPTIMIZATION

It is difficult to deal with the problem of auto-correlated data analytically. Auto-regressive models can be used to analyze the behaviour but queuing relationships that allow lot-size optimization with correlated data have not been developed. Therefore, an alternative approach developed by Enns and Li (2004) is applied. This approach assumes the *GI/G/1* relationships might prove satisfactory for lot-size optimization if  $c_a$  could be replaced by some other suitable parameter which is not actually the interarrival time coefficient of variation. In other words, it is assumed the form of Equation (2) is suitable and that Equation (7) could be used to find near optimal lot sizes if appropriate adjustments could be made to the lot interarrival time parameter,  $c_{a,i}$ . This adjusted parameter will be designated as  $c'_{a,i}$ .

The strategy is to use a dynamic feedback approach, implemented in a test bed where ARENA® is linked to Excel® through the use of Visual Basic for Applications® (VBA). This involves taking the terms in Equation (2) related to  $W_{q(i,t)}$ , replacing  $c_{a,i}$  with  $c'_{a,i}$ , and rearranging them as shown in Equation (8).

$$c'_{a(i,t)} = \sqrt{\frac{2W_{q(i,t)}(1 - \rho_{i,t})}{\bar{x}_{i,t} \rho_{i,t}}} - c_{s(i,t)}. \tag{8}$$

A dynamic estimate of queue time,  $W_{q(i,t)}$ , can be obtained using exponential smoothing. Every time a lot is completed at each capacity-constrained resource, the observed queue time for the lot is used to update  $W_{q(i,t)}$ . This value is then fed over to the Excel® spreadsheet program where  $W_{q(i,t)}$  is plugged into Equation (8), along with the current values of  $\bar{x}_{i,t}$ ,  $\rho_{i,t}$  and  $c_{s(i,t)}$ . The value of  $c'_{a(i,t)}$ , which might be termed the implied lot interarrival time coefficient of variation, is then solved for. In other words, this adjusted coefficient of variation value is the one that would result in the observed flow times, given independent lot interarrival times. This implied lot interarrival time coefficient of variation,  $c'_{a(i,t)}$ , is then used in solving for the current optimal lot sizes,  $Q_{j,t}^*$ , using Equation (7). As well, the values of  $\bar{x}_{i,t}$ ,  $\rho_{i,t}$ , and  $c_{s(i,t)}$  are updated, based on the new  $Q_{j,t}^*$  values. Equations (4)-(6) are used for this purpose. Finally, the new lot sizes are dynamically fed back to the ARENA® simulation program to determine the lot size quantities for any new order releases.

This feedback approach for dynamic lot sizing was applied to the previous problem. An exponential smoothing constant of 0.05 was used and five replications were run. Each replication included a warm-up period of 1,000 and data collection over 40,000 time units.

The average  $c'_{a(i,t)}$  values used in lot size computations were 0.338 and 0.276 for stages 1 and 2 respectively. These are considerably less than the previous observed  $c_{q(i,t)}$  values. Figure 6 shows a typical plot of the dynamic  $c_{q(i,t)}$  values through time. The time-averaged values of  $Q_{j,t}$  were 154 and 133 for product types 1 and 2 respectively. Figure 7 shows a typical plot of the dynamic lot sizes. The average lot queue times,  $W_{q(i,t)}$ , were 0.717 and 0.809, while the average lot flow times,  $W_{i,t}$ , were 2.054 and 2.217 at stages 1 and 2 respectively. The average total lot flow time of 4.271 compares well with the average total lot flow time of 4.233 observed using the optimal lot sizes of 132 and 135 obtained experimentally. The observed utilization levels were 0.885 and 0.932 at stages 1 and 2 respectively.

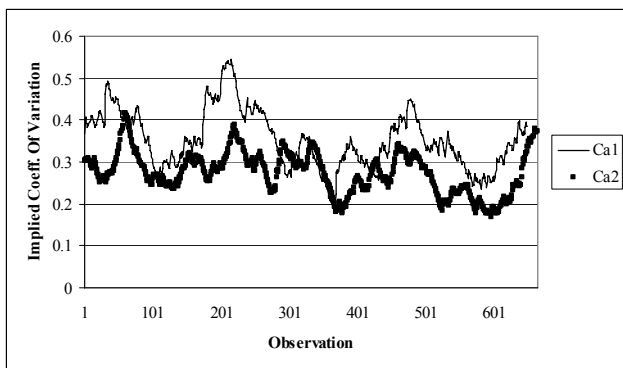


Figure 6: Implied Interarrival Time Coeff. of Variation

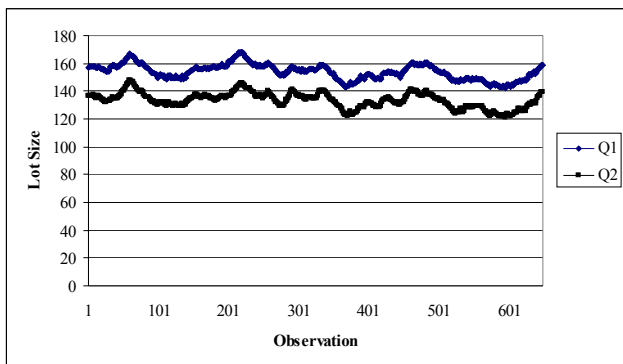


Figure 7: Dynamic Lot Sizes

The observed mean lot flow times using the dynamic lot-sizing approach based on feedback were very close to the mean lot flow times obtained using the optimal lot sizes determined experimentally. The flow time value of 4.271 obtained using dynamic lot sizing is around 1.0% higher than the flow time value of 4.233 obtained using static lot sizes of 132 and 135. Therefore, it can be concluded that the dynamic lot-sizing approach works well for lot size optimization in this serial-stage problem. In general, the ap-

proach would appear to be suitable when common lot sizes are required across multiple serial stages.

## 7 CONCLUSIONS

This purpose of this research was to further investigate a dynamic feedback approach to adjust queuing relationships for auto-correlation. Previous work dealt with a single-stage problem. The current study has extended the results by showing the approach can also be effectively applied to serial-stage problems where common lot sizes are desired across stages. The queuing relationships exploited to optimize lot sizes are based on summing the relationships for single-stage problems, while constraining lot sizes at each stage to be equal. This simple approach can be exploited since lot interarrival times at each stage are monitored, with adjustments made accordingly. Therefore, interdeparture time distributions from upstream stages do not need to be explicitly considered. The approach was found to perform well for the sample problem tested and would appear to be both robust and practical.

Further research should consider more complex scenarios. As well, there is an opportunity to use models based on this approach to better understand the impact bottleneck resources have on performance. Of particular interest would be studying the use of this lot-size optimization approach to help mitigate bottleneck effects.

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